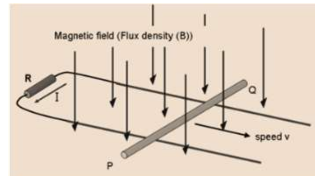
# INDUCTIVE SENSORS

**Part I – Self inductance**
**Part II – Mutual inductance**
**Part III – DC magnetic field blood flow meter**
**Part IV – AC magnetic field blood flow meter**




Michael Fraraday
1791-1867



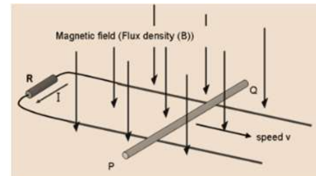
## Parts I-II

- Based on the **electromagnetic induction principle,** non-electric quantities, such as displacement, stress, flux, vibration, can be converted into variations of self-inductance $L$ or mutual inductance $M$ of the coil, which will be finally output as voltage or current through a measuring (conditioning) circuit. This kind of device is called an inductive sensor.
- Based on the conversion mode from non-electric parameters to voltage, inductive sensors can be classified as self-inductance sensors and mutual inductance ones.

## Parts III-IV

- The moving conductive blood in a static or dynamic magnetic field generates an electrical signal
- The flowmeter converts mechanical movement (flow of blood) into an electrical signal (voltage), thanks to Faraday's Law.

# INDUCTIVE SENSORS




## Part I – Self inductance

Respiratory inductive plethysmography

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Respiratory Inductive plethysmography (RIP) is the most frequently used, established and accurate method to **estimate the lung volume changes from respiratory movements.**

RIP has been used in many clinical and academic research studies in a variety of domains including polysomnographic (sleep), psychophysiology, psychiatric research, anxiety and stress research, anesthesia, cardiology and pulmonary research (asthma, COPD, dyspnea).

**Dual band respiratory inductance plethysmography** described in Part I can be used to characterize various measures of complex respiratory patterns (respiratory rate, tidal volume, peak inspiratory flow)

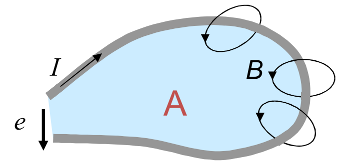
Definition: Plethysmography measures changes in volume in different parts of the body.

'plesthymos': increasing, enlarging

# Self inductance

- A conductor carrying a time-varying **current *I*** produces a magnetic **field *B*,** which generates a **magnetic flux $\Phi$** across a **surface *A***:

$$\Phi = \int_S B \cdot dA$$




- **Faraday's laws of induction-** variation of $\Phi$ will induce an electromotive force (*e*); **Lenz' law:** the induced *e* will always have a direction such that the magnetic field it produces opposes the change in magnetic flux that caused it (indicated by sign -):

$$e = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

- ***L* is the (self)inductance of the conductor:** $L = \frac{\Phi}{I}$

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When an electric current flows through a conductor, it generates a **magnetic field (*B*)** around the conductor. The strength of this magnetic field depends on the current's magnitude and follows any changes, as described by **Ampère's circuital law**.

**The magnetic flux (*Φ*)** is the net number of B-field lines passing through a surface and is calculated by integrating the B-field over the surface (see eq.).

**Faraday's law of induction** states that any change in the magnetic flux through a circuit induces an electromotive force (e) in the circuit, which is proportional to the rate of change of flux. The induced voltage is in a direction that opposes the change in current that created it, following **Lenz's law**.

**The inductance (*L*)** is the ratio between the induced voltage and the rate of change of the current. **Inductance is a property of a conductor or circuit due to its magnetic field, which tends to oppose changes in current through the circuit**.

Any alteration to a circuit that increases the total magnetic field through it, produced by a given current, increases the inductance, because *L* is also equal to the ratio of magnetic flux to current temporal variation

Interestingly, **inductance can be used as a sensor to measure changes in volume. When the area (*A*) changes, the flux also changes, which causes a change in *L*.** Therefore, **measuring the change in inductance can be used to determine changes in volume.**

## Change of *L* when uniform field *B* across varying surface *A*

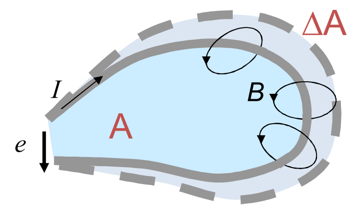
- Variations of A (frequency $f_A$) are negligible with respect to frequency of magnetic field B: $f_A \ll f_B$

$$L = \frac{\Phi}{I} \qquad \Phi = \int_S B \cdot dA$$

- Orientation($\alpha$) of A with respect to B is stable:

(1) $$\Phi = B \cdot A \cos\alpha, \quad L = \frac{B \cdot A}{I} \cos\alpha$$

(2) $$L = Lo + \Delta L = \frac{B \cdot A}{I} \cos\alpha + \frac{B \cdot \Delta A}{I} \cos\alpha$$




- Measurement of *L* possible using the *resonant frequency* of an *oscillatory circuit including L*



Here we assume that:

(i) area A increases by small ΔA;

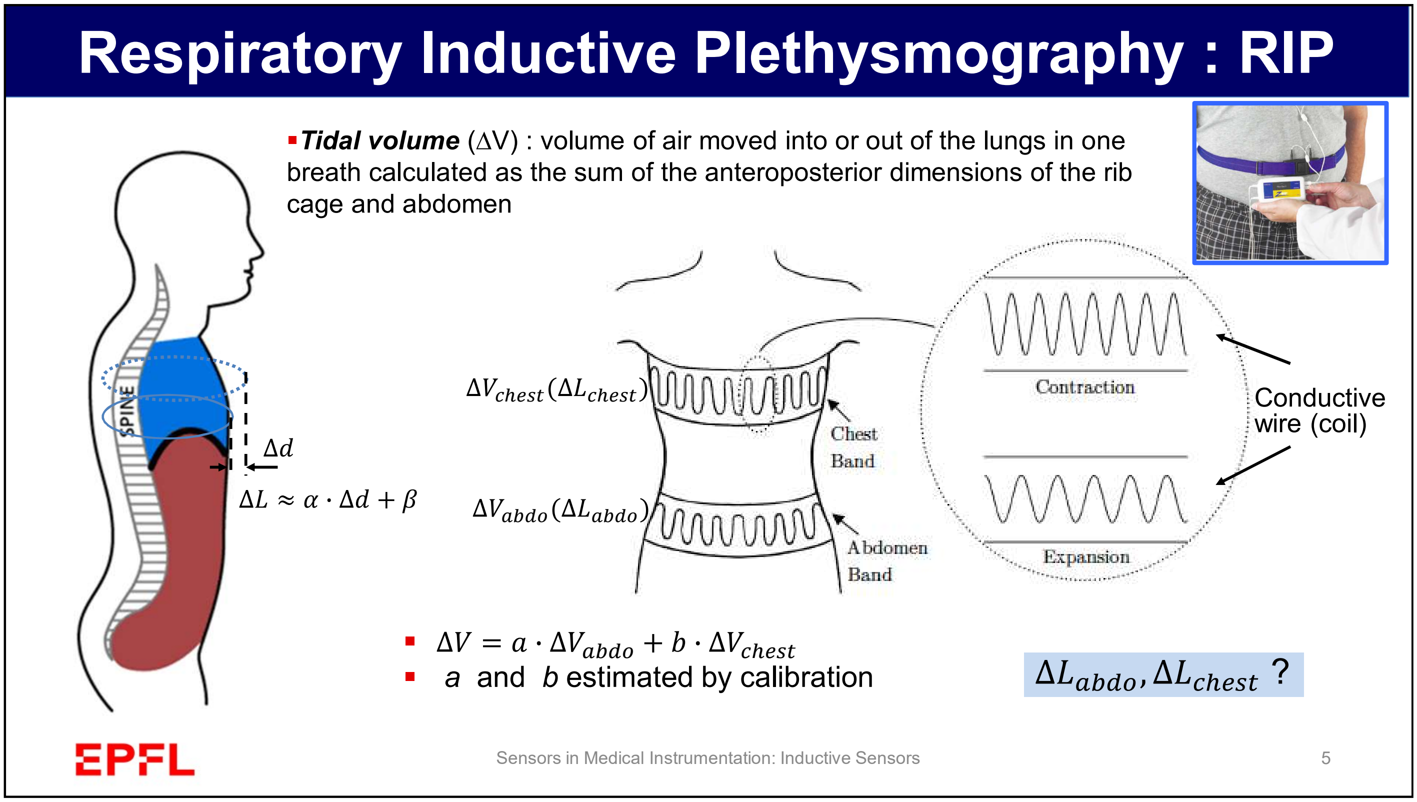
(ii) variation of A (frequency) is much lower than variation of magnetic field B;

(iii) orientation of the surface A with respect to the magnetic field B is stable.

- With these assumptions, the flux generated by this current can be expressed by the relation (1)
- The change of the area will have as effect a change of the inductance according to relation (2)

So, **to measure the change of this inductance, we need to have a circuit that is sensitive to the inductance, as for example an LC oscillatory circuit where the resonant frequency change with the inductance. By measuring the resonant freq. and its changes, we can deduce the change of *L*, and from this the change of area *A*.**

This is the basic principle of respiratory inductive plethysmography (RIP), where the conductor that changes area is in the form of belts — one placed around the chest and one around the abdomen. The variation in cross-sectional area $A$A (and thus in inductance) is related to breathing, with the frequency of $A$A variations corresponding to the respiration rate.




In RIP, two elastic belts, into which a zigzagging (coiled) wire (for expansion and contraction) are used, one worn around the chest, and the other worn around the abdomen, resembling two inductance loops (see fig). **Based on the principle of Faraday's Law, an alternating current applied through a loop of wire with high frequency and low amplitude generates a magnetic field normal to the orientation of the loop**. The frequency of the alternating current is set to be more than twice the typical respiratory rate in order to achieve adequate sampling of the respiratory effort waveform.

During inhalation, the volume of the chest increases while the volume of the abdomen decreases, and the opposite happens during exhalation. Why 2 belts ? -> By placing one belt around the **chest/rib cage** and one around the **abdomen**, RIP captures the full range of respiratory motion allowing **more accurately estimate lung volume changes**, especially when calibrated.

**Measurement principle**: The change in inductance of the belts around body corresponds to a change in diameter (Δd), → ΔL is proportional to Δd plus a constant bias or offset. So, to estimate the volumes of the chest and abdomen, the device measures the changes in inductances $\Delta L_{abdo}$, $\Delta L_{chest}$.

The device provides a weighted average of the two volumes, **and the weights are estimated through calibration**. The variation of the volume $\Delta V$ corresponds to the respiratory parameter called **'tidal volume'.**

**Important Notes:**

**Calibration** is necessary: This usually needs a **reference breathing maneuver** (like breathing into a spirometer once) to correctly match the belt signal to real volumes.

**Advantages**:

**Non-invasive** (no tubes, no discomfort).

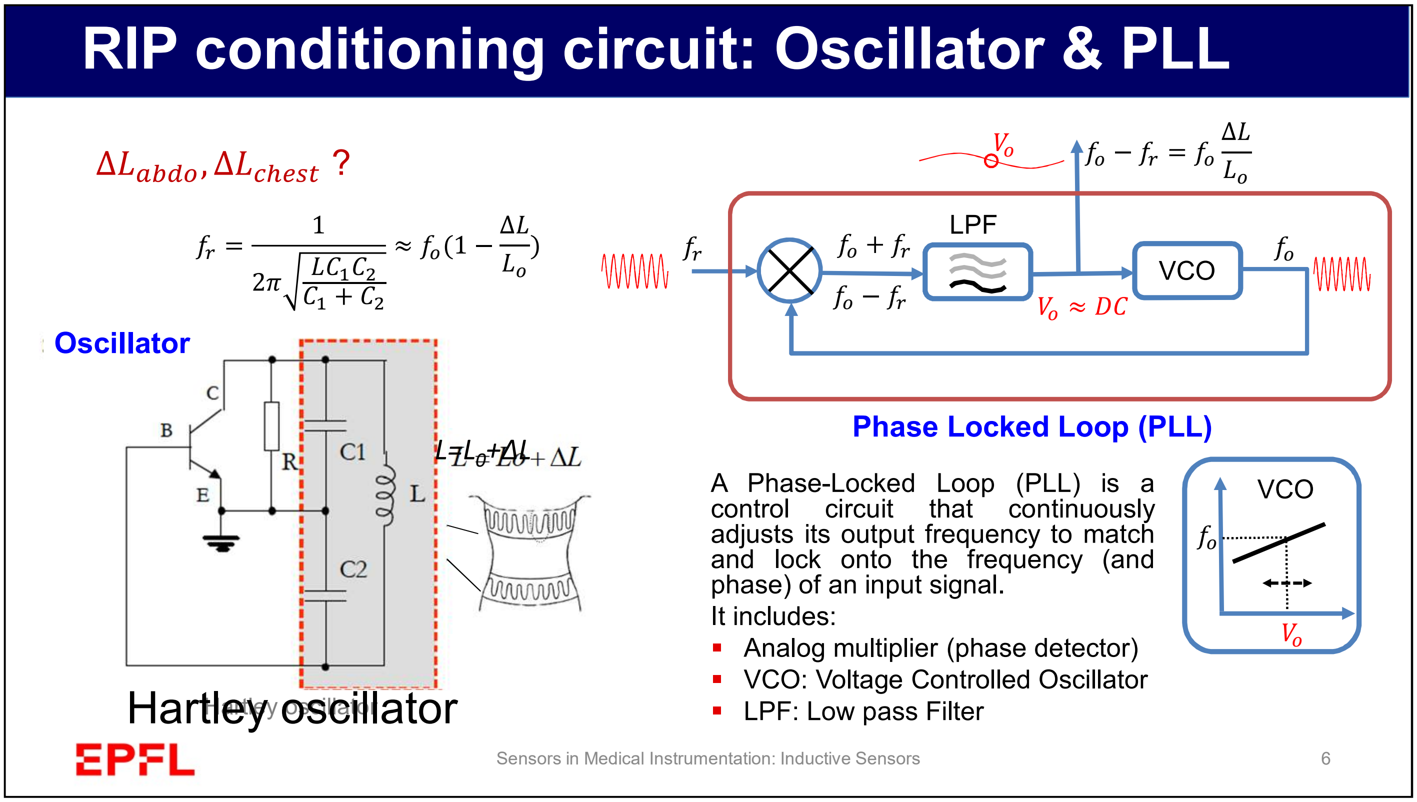
**Continuous monitoring** (long periods, even during sleep or exercise).

**Limitations**:

Very big posture changes (like lying down) can affect the signal and need adjustment.

Calibration drift can happen over time.

**The main part of the device is its conditioning electronics, which derive the change in volume from the change in inductance (see next slide).**




<u>**Principle**</u>: measure variation of ΔL by measuring the variation of resonant freq. of an oscillator including *L.* The measuring (conditioning) circuit includes two main parts:

- **A resonant circuit** (one for each band) where the inductive sensor is part of the LC oscillator (C fixed) with the resonant freq. *fr.* Since the resonant frequency depends on $L$, breathing causes a shift in the oscillator's frequency. For each coil placed on chest or abdomen, the oscillator converts the change of volume (ΔL) to the frequency modulation (f*o*-*fr*). The output voltage of each oscillator is then applied to a PLL.
- **A Phase-Locked Loop (PLL)** circuit is used to track the changes in the oscillator's resonant frequency precisely**. As $L$ changes (with breathing), the PLL locks onto the new frequency. The PLL output gives a signal proportional to the breathing pattern — tracking both rate and volume change.**

**In short: Breathing → changes L → shifts oscillator frequency → PLL tracks frequency shifts → output signal matches breathing cycles.**

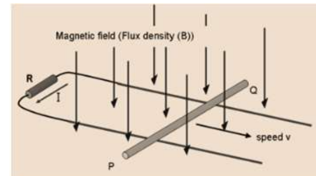
**How the PLL internally works (step-by-step):**

- **Two Frequencies to Compare**:
  One frequency comes from the **LC oscillator** (the breathing signal).
  Another comes from a **local adjustable voltage-controlled oscillator (VCO)** inside the PLL.
- **Phase Detector**:
  The PLL has a **comparator** (phase detector) that **compares the two frequencies.**
  It checks: "Are they at the same speed? Are they in phase?"
- **Error Signal**:
  If the two frequencies are **different**, the phase detector creates an **error signal.**
  This error tells **how much** and **which way** the local VCO must adjust.
- **Control Loop**:
  The PLL uses the error signal to **correct** its internal VCO **continuously**.
  It **pulls** the VCO's frequency **up or down** until it **matches** the LC oscillator's breathing frequency.
- **Result**:
  The PLL output **tracks** the breathing-related frequency changes.
  The system provides a clean signal that **follows the person's breathing** in real time

The PLL circuit offers the advantage of accurately and automatically tracking small frequency changes in real time, providing a stable and precise breathing signal

To note that here only the main parts of conditioning are described, the measurement device includes also additional electronics, for example to convert the DC voltage at freq fo-fr into tidal volume, ΔV.

# INDUCTIVE SENSORS




**Part II a– Mutual inductance**

**Linear Variable Differential Transformer (LVDT): principle**

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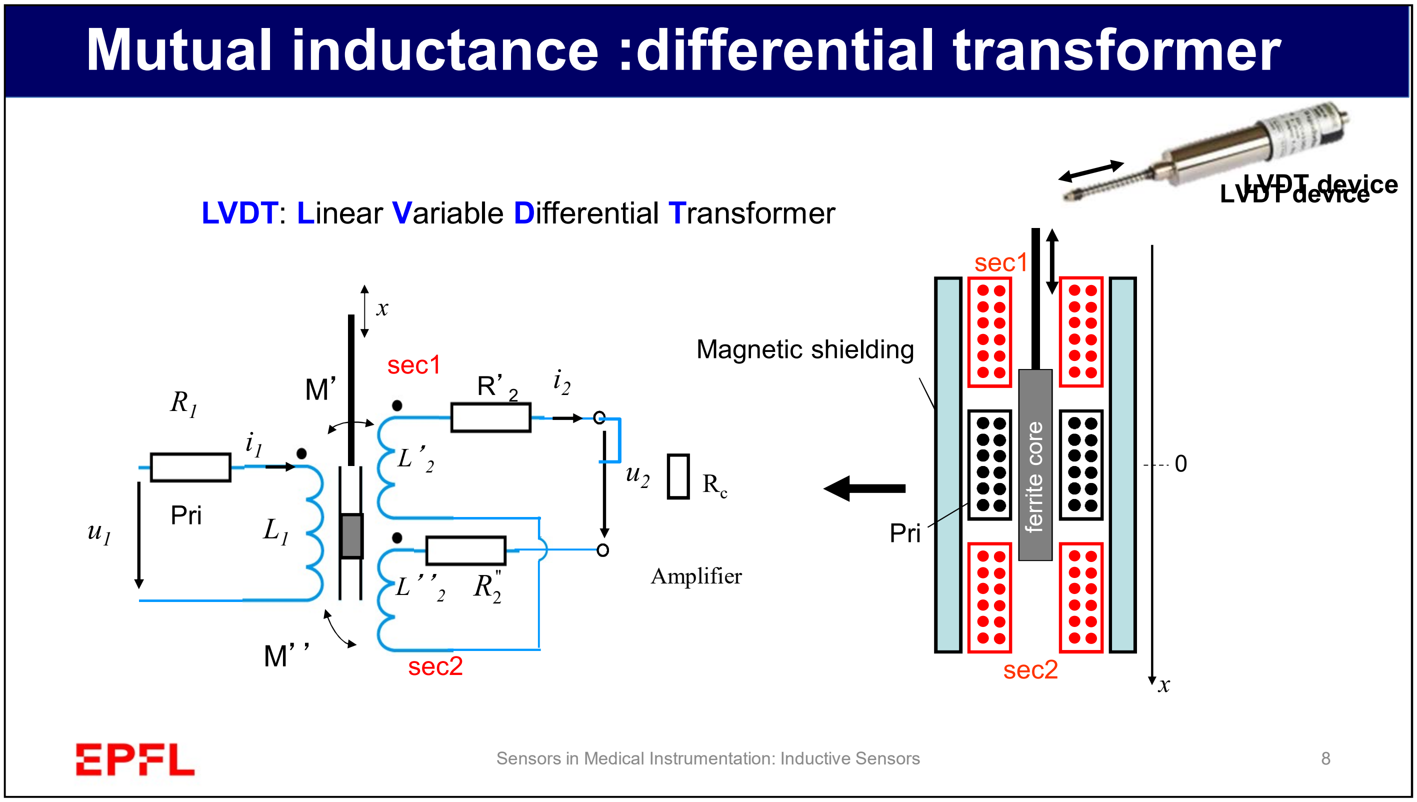
This part describes the **mutual inductance** and as application the **LVDT** (Linear Variable Differential Transformer) sensor, that can be **used as displacement or position sensor**. It can be **used for evaluation of the mechanical properties of the tissues, and in robotics (orthopedics).**

**Mutual inductance** is the property by which a change in current in one coil induces a voltage in a nearby coil due to the shared magnetic field.



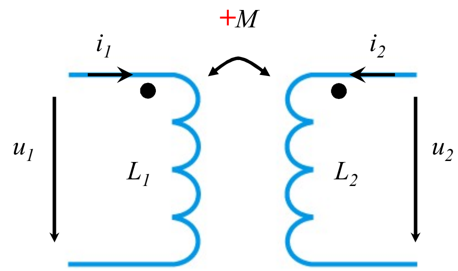

**The LVDT is an inductive sensor commonly used to convert linear motion into an electrical signal**. It consists of a movable magnetic core and three coils: a primary coil and two secondary coils placed opposite each other (see fig on right). The displacement being measured is attached to the core, and an AC current passing through the primary coil induces an AC voltage in the secondary coils. Magnetic shielding is necessary to prevent external interference.

The movement of the core changes the mutual inductances between the primary and secondary coils, which in turn induces a voltage in one or the other secondary coil.

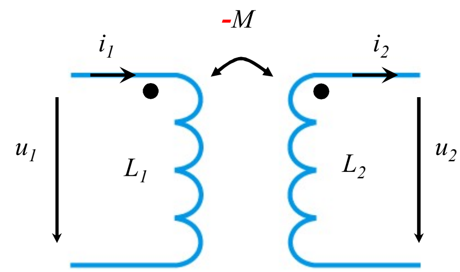
The LVDT equivalent circuit (see fig on left) consists of the primary coil and the two secondary coils, with respective resistance and inductance values. **The displacement x can be evaluated in terms of output voltage u2 by solving the equations derived from the circuit components. The secondary coils are connected in a way that the output voltage, u2, is the difference between the voltages on them.**

Voltage u1 is the input AC voltage (supply), and M', M'' are the mutual inductances between the primary and secondary coils.

# Convention: mutual inductance

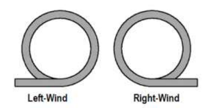



$$u_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad u_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$




$$u_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt} \qquad u_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

●: indicate the polarity of coil winding,
The currents flowing into each winding at the connection indicated by the dot produce induced voltages of the same sign






The (polarity of) mutual inductance is defined based on the direction of currents i1 and i2, and on the polarity of coils windings (clockwise or counter-clockwise)

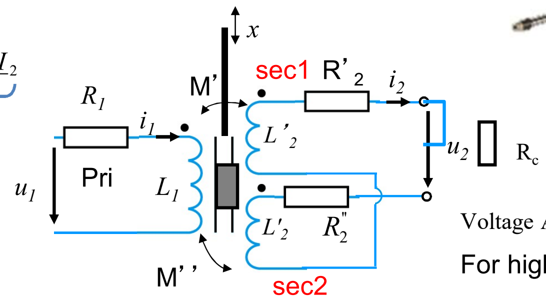
The dot expresses the sense of wire winding which induces opposite effect on M.

# Mutual inductance: differential transformer

**LVDT**: **L**inear **V**ariable **D**ifferential **T**ransformer

$$u_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + (M'' - M')\frac{di_2}{dt}$$

$$\underline{U}_1 = (R_1 + j\omega L_1)\underline{I}_1 + \underbrace{j\omega(M'' - M')\underline{I}_2}_{\approx 0}$$




Voltage **Amplifier**

For high $R_c \rightarrow i_2 \approx 0$:

$$\underline{U}_2 = \frac{j\omega[M''(x) - M'(x)]}{R_1 + j\omega L_1}\underline{U}_1 \quad (1)$$

$$u_2 = -(R_2' + R_2'')i_2 - (L_2' + L_2'')\frac{di_2}{dt} + (M'' - M')\frac{di_1}{dt}$$

$$\underline{U}_2 = \underbrace{-(R'_2 + R''_2 + j\omega L'_2 + j\omega L''_2)\underline{I}_2}_{\approx 0} + j\omega(M'' - M')\underline{I}_1$$



Computation steps:

- Write equations for u1 and u2 using the convention explain on the previous slide
- Apply Fourier transform
- The **input resistance (impedance)** of the amplifier is supposed to be **very high** (this is the requirement for an **ideal voltage amplifier**) **→ i2 ≈ 0**
- Write U2=f(U1), see eq. (1)
- In eq. (1), the values of primary coil are fixed (U1, R1, L1 and freq ω) → only the mutual inductances change with x
- Now, if we can express M'(x) and M''(x) we can express U2(x) which is the characteristic of our sensor ( because **the measurand is the displacement x**). To this end, we can consider that x has small values (e.g., applications in biomedical where x is related to measurement of orthopaedic prothesis vibration/displacement), and we can express variation of M'(x) and M''(x) using McLaurin series (see next slide)

# LVDT

McLaurin series

$$\underline{U}_2 = \frac{j\omega\left[M''(x) - M'(x)\right]}{R_1 + j\omega L_1}\underline{U}_1$$

$$M'(x) = M(0) + ax + bx^2 + \ldots \quad \text{for } x > 0$$

$$M''(x) = M(0) - ax + bx^2 + \ldots \quad \text{for } x < 0$$

2nd order approximation:

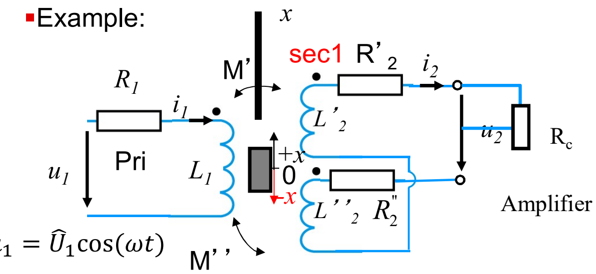
$$M''(x) - M'(x) = -2ax$$

The relation become linear:

$$\underline{U}_2 = \frac{-2j\omega \cdot a\underline{U}_1}{R_1 + j\omega L_1}\underline{X} \quad (1)$$

$$\underline{U}_2 = \underline{G}(j\omega) \cdot \underline{X}$$

▪Example:




$$u_1 = \hat{U}_1\cos(\omega t)$$

$$u_2 = \frac{2\omega \cdot a\hat{U}_1}{\sqrt{R_1^2 + \omega^2 L_1^2}} x(t)\cos(\omega t - \varphi)$$

$$u_2 = A \cdot x(t) \cdot \cos(\omega t - \varphi)$$

|G(jω)|

$$\varphi = -90 - arctg\left(\frac{\omega L_1}{R_1}\right)$$

Phase shift at the output of LVDT, inherent to the device's design and operation



- For low value of x, M can be expressed as McLaurin series and then approximated at second order in terms of x (neglecting x exponent 3 and higher).
- Eq. (1) is the final expression of LVDT device, u2=f(x) where we can identify the transfer function G(jω)

The example illustrates that based on the eq. (1) we can deduce the output u2 for given input voltage u1.

One important aspect is the fact that the output voltage u2 is phase shifted with respect to the input voltage u1. The phase shift φ indicates the direction of measured movement x

**Important remark**: **Is this value of φ we try to find by tuning the circuit (bloc φ) in the synchronous rectifier (slides 16, 17)**

**Detailed computation of u2 in Example, see next slide:**

- See detailed computation of u2(t)



$U_2(j\omega) = [j\omega(-2a\cdot x) / (R_1 + j\omega L_1)]\ U_1(j\omega)$ (1)

For the time-domain excitation, we assume the primary is driven by a sinusoidal voltage:

$u_1(t) = \hat{U}_1 \cos(\omega t)$ (2)

In phasor form, this corresponds to $U_1 = \hat{U}_1 \angle 0°$ . Plugging $U_1$ into (1) yields:

$U_2(j\omega) = [-2a\cdot j\omega\cdot x / (R_1 + j\omega L_1)]\ \hat{U}_1 \angle 0°$ (3)

Define the complex gain $G(j\omega)$ as:

$G(j\omega) = [-2a\cdot j\omega\cdot x / (R_1 + j\omega L_1)]$ (4)

The magnitude of $U_2$ is:

$|U_2| = (2a\omega|x| / \mathrm{sqrt}(R_1^2 + (\omega L_1)^2))\ \hat{U}_1$ (5)

Defining the amplitude gain A:

$A = 2a\omega\ \hat{U}_1 / \mathrm{sqrt}(R_1^2 + (\omega L_1)^2)$ (6)

Thus:

$|U_2| = A\ |x|$ (7)

The phase angle of $U_2$ is:

$\angle U_2 = (-90°) - \arctan(\omega L_1 / R_1)$ (8)

In radians:

$\angle U_2 = -\pi/2 - \arctan(\omega L_1 / R_1)$ (9)

Thus, the time-domain output voltage is:

$u_2(t) = |U_2| \cos(\omega t + \angle U_2)$ (10)

Rewriting in terms of A, x(t), and a positive phase shift φ:

$u_2(t) = A\ x(t) \cos(\omega t - \varphi)$ (11)

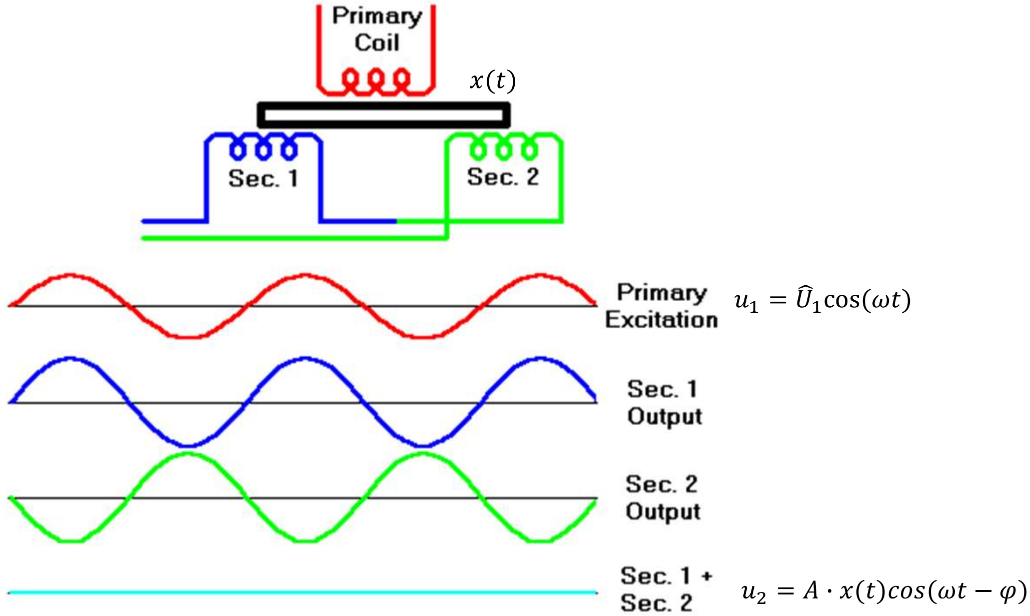
The phase shift φ is:

$\varphi = 90° + \arctan(\omega L_1 / R_1)$ (12)

Finally, the output voltage as a function of time:

$u_2(t) = (2a\omega\ \hat{U}_1 / \mathrm{sqrt}(R_1^2 + (\omega L_1)^2))\ x(t) \cos(\omega t - (\pi/2 + \arctan(\omega L_1 / R_1)))$ (13)

# LVDT



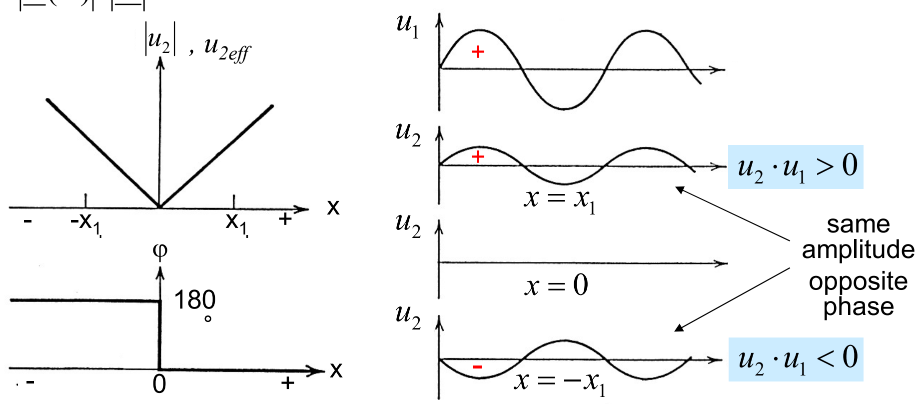



- This animation illustrates that as x increases or decreases (indicating the movement of the magnetic core to the right or left relative to the central position indicated in the figure, where x=0), the amplitude of *u*2 increases. Therefore, by measuring only the amplitude, one cannot determine the direction of movement.
- **The direction of movement can be detected by determining the phase φ**

# LVDT

$u_2 = A \cdot x(t) \cdot \cos(\omega t - \varphi)$

$|\underline{U}_2| = |\underline{G}(\omega)| \cdot |\underline{X}|$



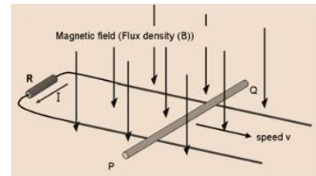

- Conditioning with a phase detection circuit :
  Detection of the **direction** of movement



For x =+x1 or x=-x1, u2 has same RMS value, but a shift of $\pi$ in phases.

To detect the direction, it is necessary to measure the phase φ

# INDUCTIVE SENSORS




**Part II b– Mutual inductance**

**LVDT: Synchronous detection**

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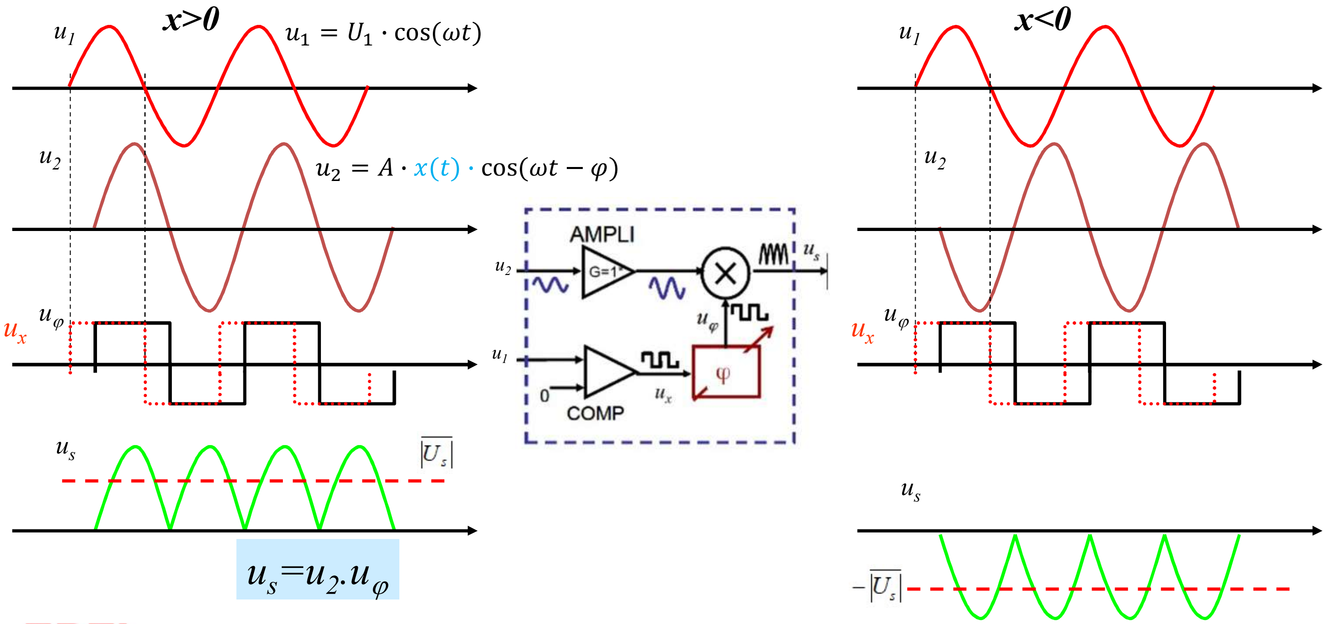
1791-1867



In this part we will see how we can make the conditioning of LVDT using a **synchronous detection** which allows us to find both, the amplitude (amount) of displacement and the direction (the sign)

# Synchronous rectifier

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The synchronous rectifier is a fundamental component of synchronous detection used to consider the phase of $u_2$.

The circuit contains:

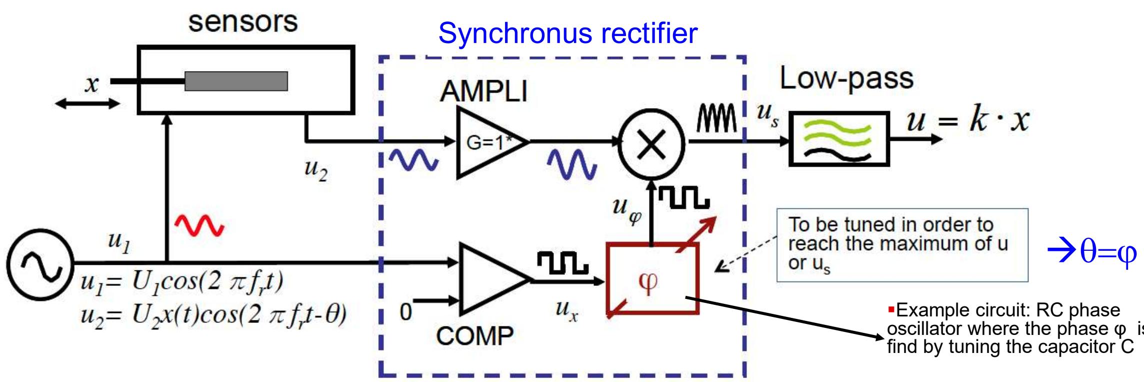
- An amplifier for signal $u_2$ and an analog multiplier
- A comparator to generate a square wave signal ($u_x$) in phase with the excitation signal ($u_1$).
- A phase shifter φ that can be realized with and RC circuit, where the capacitor is adjustable. The signal $u_x$ is phase shifted to generate the signal uφ .**The key idea is that this φ should be tuned to match the phase shift φ of $u_2$**

**How we can do this ? Based on which criteria ?**

- If you look to the signals illustrated, for example on the right side, you can observe that $u_2$ and uφ are in phase when the product us=$u_2$* uφ has a maximum magnitude. So, the RC circuit adjust φ until us reaches its maximum value

**So by multiplying the phase shifted $u_1$ signal and $u_2$ we can generate a rectified signal $u_s$ where its amplitude and sign correspond to x(t).**

# Synchronous detection: bloc diagram




$$u_\varphi = U\cos(2\pi f_r t-\varphi) + \frac{1}{3}U\cos(6\pi f_r t-\varphi) + \frac{1}{5}U\cos(10\pi f_r t-\varphi)+\cdots$$

High frequency

$$u_s = u_2 \cdot u_\varphi = U_2.x(t)\cos(2\pi f_r t-\theta).U\cos(2\pi f_r t-\varphi)+\frac{1}{3}U_2.x(t)\cos(2\pi f_r t-\theta).U\cos(6\pi f_r t-\varphi)+\ldots$$

$$= \frac{1}{2}U_2.U.x(t)\cos(2\pi f_r t-\theta\square 2\pi f_r t+\varphi)+\frac{1}{2}U_2.U.x(t)\cos(2\pi f_r t-\theta+2\pi f_r t-\varphi)+\frac{1}{3}U_2.x(t)\cos(2\pi f_r t-\theta).U\cos(6\pi f_r t-\varphi)+\cdots$$

After low pass: $u=\frac{1}{2}U_2.U.x(t)[\cos(-\theta+\varphi)]=\frac{1}{2}U_2.U.x(t)=k.x(t)$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

EPFL



*G=1 for the sake of simplicity

Here it is illustrated the LVDT with the conditioning circuit including the synchronous rectifier and a low pass filter.

The computation of $u_s$ is based on multiplication of cos signals according to the trigonometric formula

- High frequencies corresponding to the sum of frequencies
- Low frequencies corresponding to the difference of frequencies

The low-pass filter allows to estimate the low frequency signal closed to DC (average value of the rectified signal)

Remark:

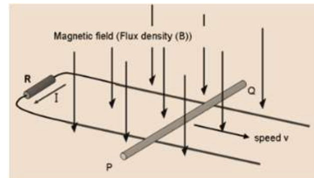
- the Fourier series expansion was used for the square wave signal uφ
- the RC circuit adjust φ to reach phase θ of u2 by looking when $u_s$ reach the maximum value

# LVDT characteristics

- Full scale (FS): ±1mm to 500mm
- Sensitivity: 1-500mV/V/mm
- Linearity: 0.05% to 1% of Range
- Precision: 0.002% to 0.5% of FS
- Excitation frequency: up to 50kHz
- Dynamic response up to 2kHz



- These are the usual LVDT specifications we can find in data sheets, useful to chose the optimal device for a given application




# INDUCTIVE SENSORS

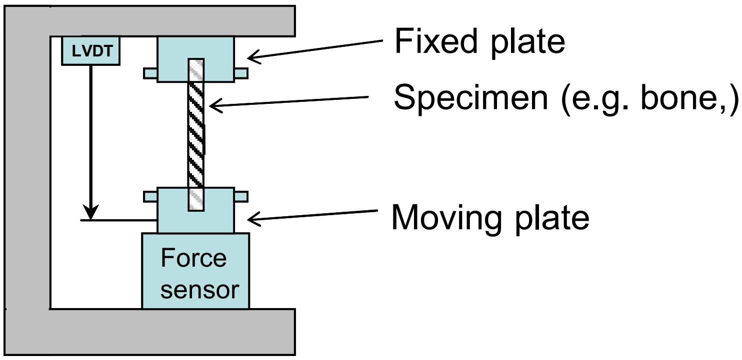
**Part II c– Mutual inductance**

**LVDT: Applications**

Michael Fraraday
1791-1867

# Applications: tissue mechanical property

- Measure the the deformation of a bone by applying a know force: Young modulus estimation




$$\sigma = \frac{F}{A}$$

$$\varepsilon = \frac{\Delta l}{l}$$

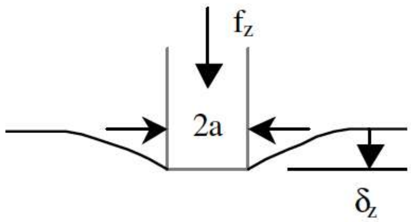
$$\sigma = \varepsilon Y$$



Here is a device for measuring the Young modulus of a bone, including a force sensor that applies a certain force to the bone, causing it to deform. The deformation is then measured using an LVDT.

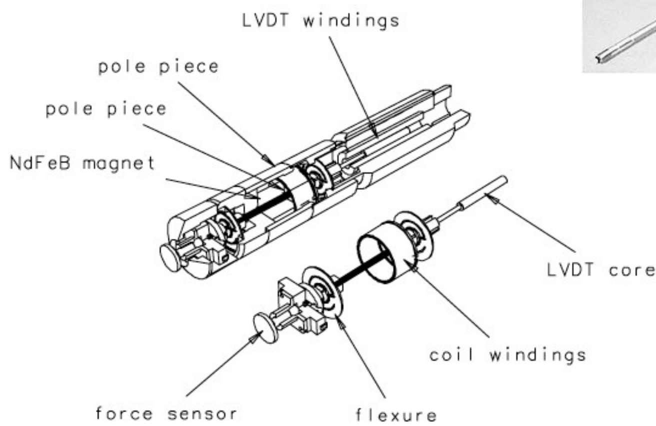
By knowing the applied force and the section, the stress σ can be estimated. The LVDT measures the deformation and estimates the strain ε, and by comparing the relationship stress and strain, the Young modulus can be calculated.

# Mechanical property of living tissue




$$Young\ modulus: Y = K\frac{3f_z}{8a\delta_z}$$

K: experimental constant depending on the size




W. Niessen and M. Viergever (Eds.): MICCAI 2001, LNCS 2208, pp. 975–982, 2001. Springer-Verlag Berlin Heidelberg 2001

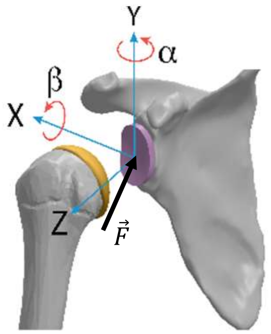


The device contains both an LVDT and a force sensor.

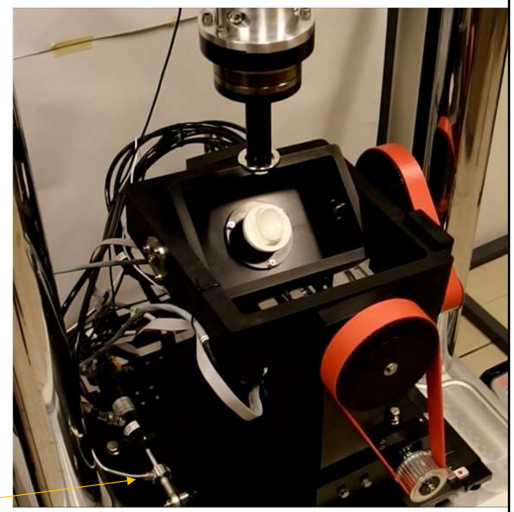
To illustrate the device's functionality, let's consider applying a force to skin or tissue. Depending on the magnitude of the force applied, the tissue will deform by a certain amount, which can then be measured by the LVDT.

The LVDT measures the value of $\delta_z$, while the force sensor measures the value of the applied force $f_z$. By using the experimentally determined constant K, we can express the tissue's elasticity in terms of the Young modulus.

# Robotic shoulder simulation of instability



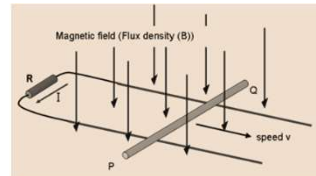

- Estimation of x and y displacement by LVDT when F is applied
- F can reach more than twice the body weight!






- Here is an example of a robot designed to estimated the potential displacement of a shoulder prothesis when various forces are applied (collaboration EPFL-Orthopaedics Hospital/CHUV)
- By varying the applied force on the gleno-humeral implant, the translation of the metallic implant relative to the glenoid polyethylene insert can be measured using LVDT sensors. This allows to evaluated the stability of the prostheses in presence of high forces (e.g. lifting weights in daily activities).
- The average contact forces evaluated with the simulator ranged from 1.3 to 2.4 times body weight (930-1720 N)

# INDUCTIVE SENSORS




**Part III – DC magnetic field blood flow meter**

Michael Fraraday
1791-1867



An inductive flowmeter measures blood velocity by detecting the voltage induced across the blood vessel when ion-rich blood flows through a magnetic field, following the principle of electromagnetic induction.

This type of blood flowmeter is widely used to measure **pulsatile blood flow (i.e., blood flow in arteries)**. It is suitable for determining instantaneous flow rates.

# Measuring blood flow

***Faraday Law***: $e \propto \frac{d\Phi}{dt}$ *; The voltage can be induced by:*

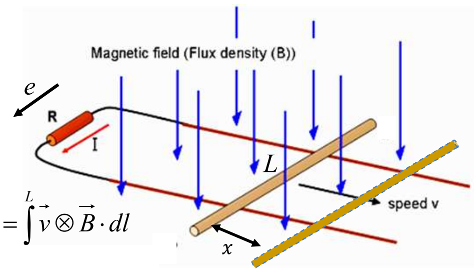
- *A fixed conductor (fixed area A) in a variable magnetic field B*

$$\frac{d\Phi}{dt} = \frac{dB}{dt}A$$

- ***A conductor moving in a uniform magnetic field***

$$\frac{d\Phi}{dt} = B\frac{dA}{dt} = B \cdot L\frac{dx}{dt} = B \cdot L \cdot v$$

- $L = length\ of\ the\ conductor$




$$e = \int^{L} \vec{v} \otimes \vec{B} \cdot dl$$

- *electricity is only produced while something is varying/moving*
- *the faster the variation/movement, the more electricity we get*



The Faraday law states that the induced voltage in a conductor depends on the derivative of the magnetic flux.

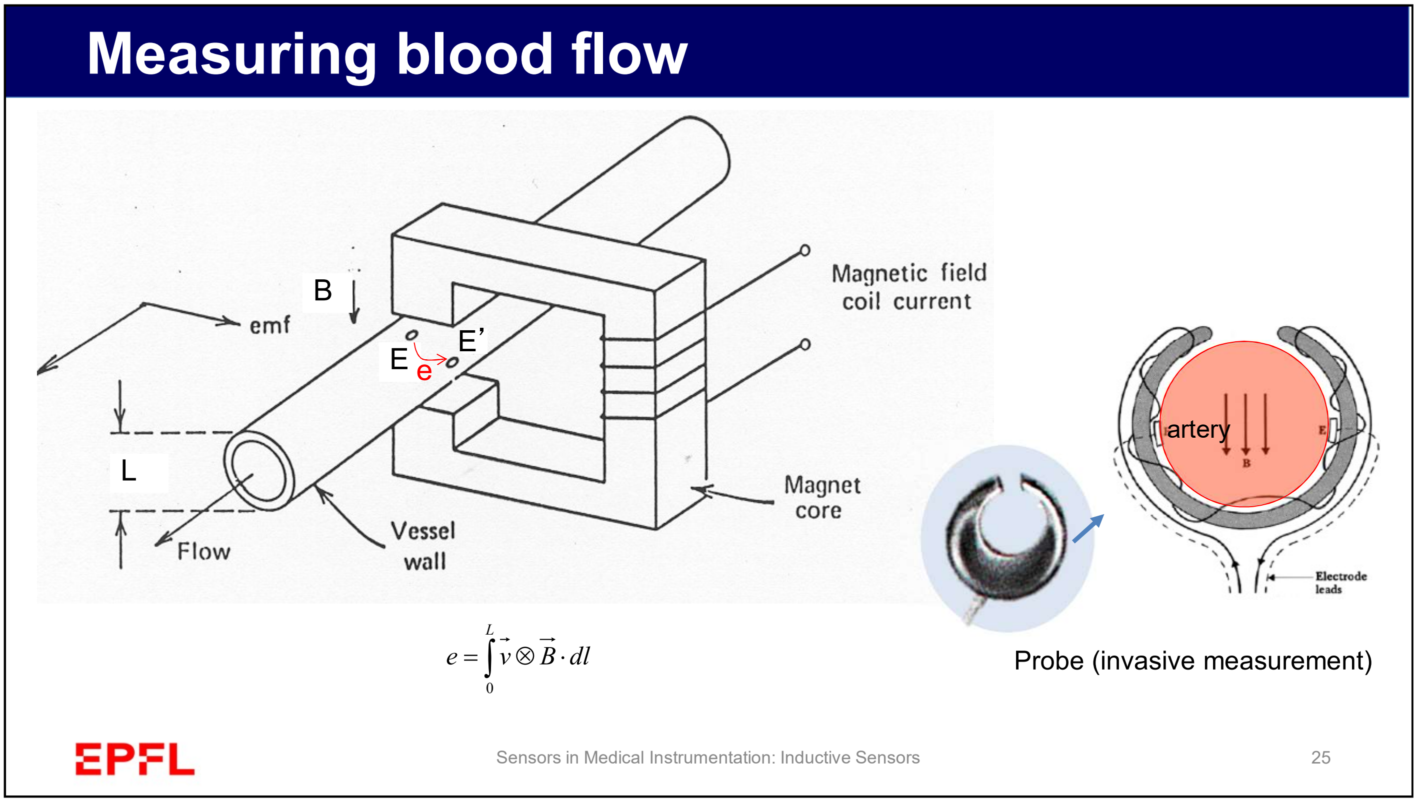
There are two ways in which the magnetic flux (Φ) can change:

(1) due to the variation of the magnetic field density (magnetic induction lines B) in a given area A

(2) due to the variation of the area A of the conductor. For instance, when a conductor moves in a magnetic field B, the area spanned by the conductor changes, causing the flux to vary.

To demonstrate the generation of induced voltage, we can perform the experiment shown in the figure. Take a conductor bar of length L and move it perpendicular to a magnetic field B. The moving bar is in contact with a conductor shown in red. As the bar moves with a speed v, the area spanned by the conductor (L*x(t)) changes, resulting in the generation of an induced voltage e. If the magnetic field is perpendicular to the velocity vector *v*, the induced voltage e is given by *e=BLv*.

"L" here represents the length of the conductor (to avoid confusion with inductance).




An **inductive flowmeter** (also called an **electromagnetic flowmeter**) measures **blood flow** based on the **principle of electromagnetic induction**, as described by **Faraday's Law**.

**Basic idea:** When a **conductor** moves through a **magnetic field**, it **induces a voltage** (an electromotive force, EMF) across the conductor. In blood flow measurement, **blood** acts as the **conductive fluid** because it contains ions (charged particles). When blood flows through a **magnetic field** applied across a blood vessel, a **voltage** is generated **perpendicular** to both the direction of blood flow and the magnetic field.

**How it works:**

1. A **magnetic field** B is applied **across** a blood vessel using external magnets.
2. As blood flows at a v, it cuts through the magnetic field.
3. According to **Faraday's Law**, a **voltage** e is induced
4. **Electrodes** placed on opposite sides of the vessel wall measure this induced voltage.
5. The **induced voltage** is **proportional** to the **blood flow velocity**.

The probe allows to:

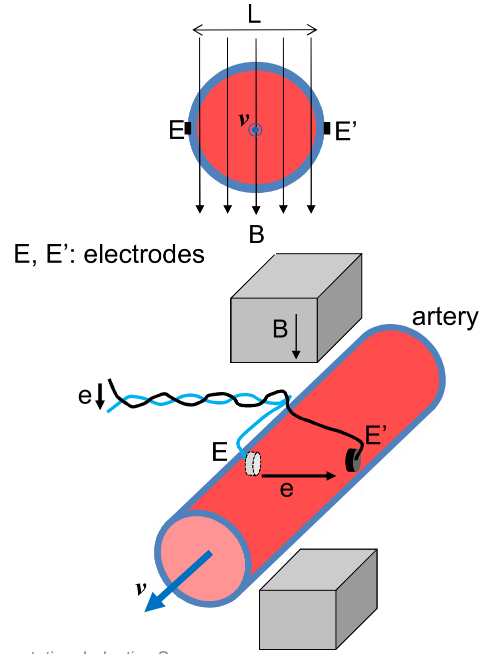
- Insert the vessel
- Guaranty a good contact with electrodes
- Generated the magnetic field

# Measuring blood flow

- Faraday's law:

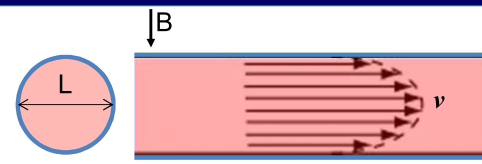
$$e = \int_0^L \vec{v} \otimes \vec{B} \cdot dl$$

  - $L$: diameter of artery, m
  - $v$: blood velocity, m/s
  - $B$: magnetic field applied to vessel, T
  - $e$: induced voltage perpendicular to the flow direction, V



# Measuring blood flow

$$e = \int_0^L \vec{v} \otimes \vec{B} \cdot dl \quad \longrightarrow \quad \text{if } B \perp v \; : \; e = B \cdot L \cdot v$$




- The flow in m³/s is obtained by:

$$Q = v \cdot \frac{\pi}{4} L^2 \quad \longrightarrow \quad v = \frac{4Q}{\pi L^2}$$

$$e = \frac{4B}{\pi L} Q = k \cdot Q \qquad k\text{: sensitivity}$$

- **Measures instantaneous flow**: corresponds to the mean fluid velocity in the cross-section
- flows close to the electrodes contribute more than flows that are further away
- depends on conductivity of the vessel wall and the neighbouring tissues
- measurement accuracy affected by potential non-uniformity of magnetic field B
- the system needs calibration before measurements



To measure the blood flow accurately, we need to express it in terms of velocity. The flow of blood in a vessel can be related to velocity using the following equation: Q (blood flow) = v • cross-sectional area of the artery.

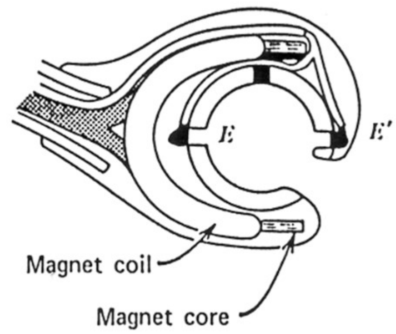
Assuming that the magnetic field is perpendicular to the velocity, we can use the equation e = B • L • v, where e is the voltage induced, B is the magnetic field, L is the diameter of the vessel, and v is the velocity. So, we can express the induced voltage as a function of the blood flow rate Q, using the constant of proportionality k, such that e = k • Q.

However, this method allows us to measure only the instantaneous blood flow rate. Since the velocity profile across the membrane is not uniform, what we measure is the mean velocity value across the membrane, which is a drawback of the system. Additionally, flows closer to the electrodes contribute more than those further away, and the assumption of a certain conductivity of the membrane may differ in neighboring tissues, potentially affecting the collected voltage.

Moreover, the magnetic field may not be uniform, which can affect the accuracy of the measurements. Therefore, the system needs to be calibrated using specific procedures to account for these factors.

# DC flow meter

- $B$ : constant
- $e = K \cdot v$
- Drawbacks:
  - Half-cell electrode voltage
  - ECG interference
  - Presence of 1/f noise




➔ Use an induction of higher frequency (by about 400Hz) than that of the flow

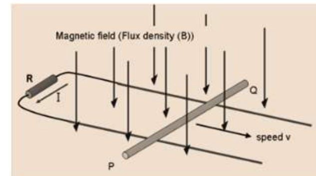


**Key limitations:**

- **Half-Cell Electrode Voltage (DC Offset):** Whenever electrodes contact an electrolyte, they develop a **half-cell potential** – a DC offset voltage due to electrochemical reactions at the electrode–electrolyte interface. This offset can be on the same order of magnitude as the true flow-induced signal (which is often only a few microvolts) . Because a steady magnet induces a steady (or slowly varying) voltage, the instrument cannot distinguish **true flow EMF** from these DC offsets
- **Interference from ECG (Bioelectric Signals):** The heart's electrical activity (the ECG) produces voltages throughout the body in the same low-frequency range as a DC flow signal. For example, pulsatile blood flow signals and ECG signals both have spectral components from ~1–30 Hz. If a DC magnetic flowmeter is used near the heart or major arteries, the small induced flow voltage can be overwhelmed by the much larger ECG potentials picked up by the sensing electrodes. In other words, the flowmeter's electrodes act like bio-potential electrodes and pick up cardiac signals (and even other slow bioelectric rhythms like respiration or EEG) that mask the flow measurement
- **Excess 1/f Noise at Low Frequencies:** The output signal for steady flow in a DC field is essentially a DC or very low-frequency voltage. Electronic amplifiers and sensors exhibit **1/f noise** (flicker noise) that dominates at low frequencies (near 0 Hz). This means a DC or slow signal is buried under a larger noise floor, yielding poor signal-to-noise ratio

**To overcome these limitations, we can use an AC magnetic flowmeter**

# INDUCTIVE SENSORS




**Part IV - AC magnetic field blood flow meter**

Michael Fraraday
1791-1867



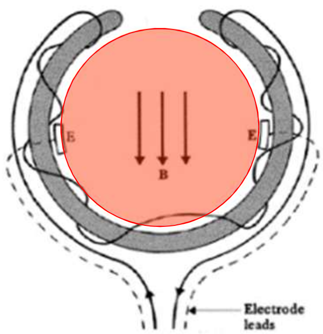
Given the above issues of DC flowmeters, modern electromagnetic blood flowmeters use an **AC or pulsed magnetic field** to achieve more reliable readings. In an AC-excited flowmeter, the induced voltage appears as an AC signal (often a sinusoid or square wave at the excitation frequency) instead of a DC level.

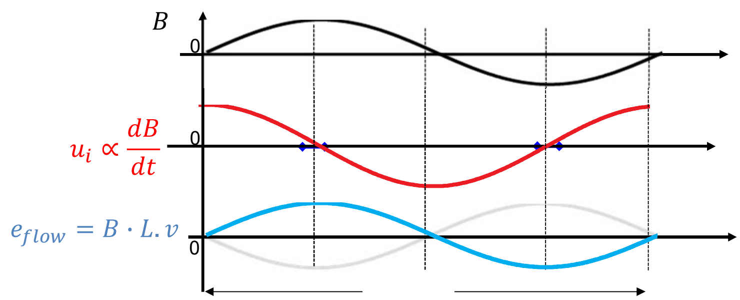
This offers several advantages critical for accuracy in medical measurements:

- **Improved Signal-to-Noise Ratio:** By up-shifting the signal to a higher frequency band, AC excitation avoids the heavy 1/f noise present at DC.
- **Elimination of DC Offset Errors:** Any constant electrode potentials or slow drift (from half-cell voltages or polarization) do not affect the AC measurement. They appear as DC offsets that can be blocked (e.g. via capacitive coupling or subtraction) before detection.
- **Reduced Interference from Biopotentials:** AC systems can discriminate against external low-frequency interferences like ECG. Since the flow signal is encoded at the excitation frequency (typically chosen in the range 50- 400Hz), the amplifier/detector can ignore out-of-band signals. As a result, alternating-field flowmeters are far less sensitive to patient bioelectric signals (or motion artifacts) compared to DC ones.
- **Minimized Electrode Polarization:** Using an alternating (bipolar) magnetic field means there is no continuous DC current path through the electrodes. This reduces electrode polarization and corrosion effects. The electrodes don't accumulate charge in one direction, so their characteristics remain stable over time, further enhancing measurement repeatability.

# AC flow meter: sine wave

- Measured voltage: $e \propto \frac{d\Phi}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt} = B \cdot L \cdot v + u_i$
- Transformer emf (induced voltage): $u_i \propto \frac{dB}{dt}$




➔ measure when $u_i \approx 0$

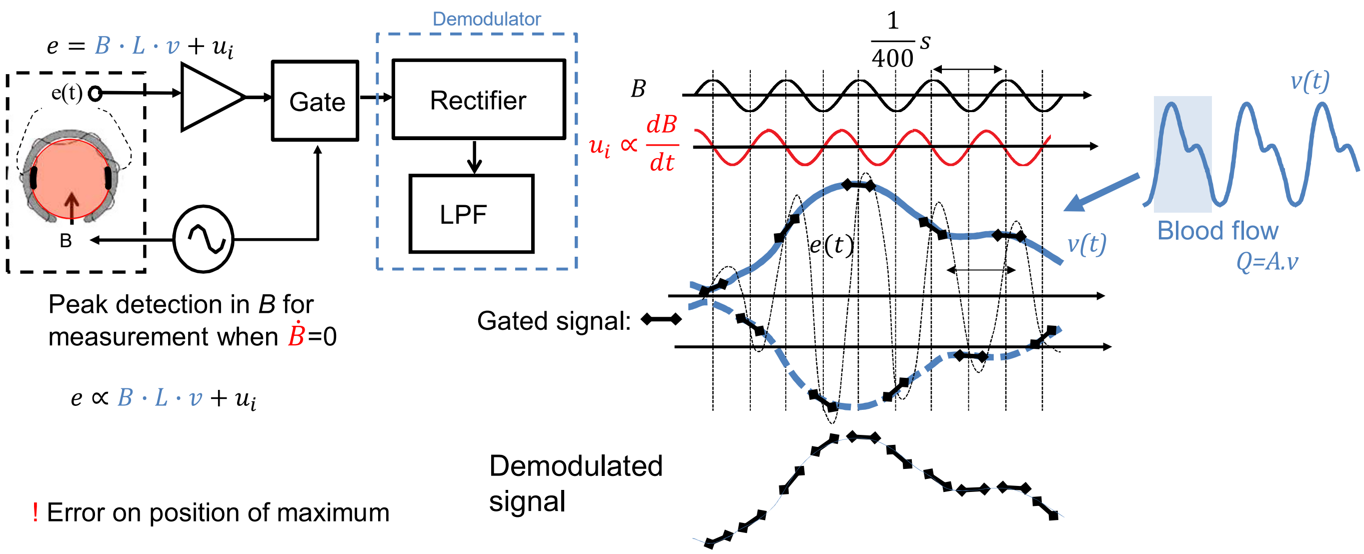


One important aspect to consider when using an AC magnetic field B(t), compared to the DC version, is that the measured voltage e depends not only on the velocity-related component (proportional to the blood flow velocity v, given by e=B·L·v, but also includes an additional unwanted noise component ui . This noise arises from the derivative of the magnetic field dB/dt , because B is no longer constant as in the DC case.

Since ui is directly related to dB/dt, it occurs at the same frequency as the desired measurement and thus cannot be removed simply by conventional filtering methods.

**To isolate the desired blood flow component in the voltage e and eliminate the noise ui, a practical solution is to measure e at moments when dB/dt=0, using a gating (sampling) circuit (illustrated in the following slide).**

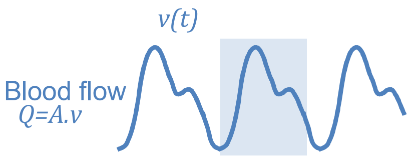
# AC flow meter: sine wave



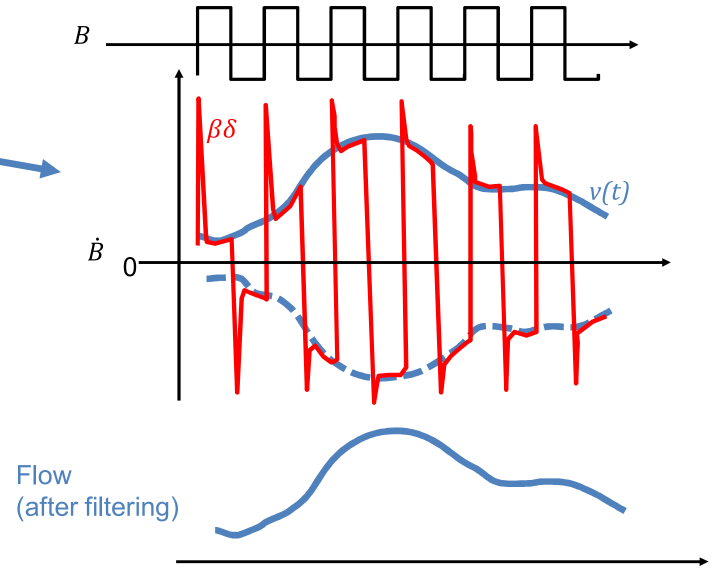



- The signal *e(t)* corresponds to amplitude modulation of B(t) by blood velocity v(t) therefore the conditioning circuit is a Demodulator
- To note that B(t) and *ui*(t) are centered around zero (AC signals)
- The error term can be observed as peaks surpassing the envelope, occurring close to instances when *ui* reaches minimum or maximum peaks.
- To eliminate this error component, the idea is to ‘gate’, i.e., to sample the signal e(t) at the instants when *ui≈0*, and then to reconstruct the envelope representing the profile of blood velocity v(t)
- The issue with this method is that for optimal results, it requires precise detection of periods when *ui*≈0, and these periods are very short to sample enough information for envelope reconstruction.

# AC flow meter – square wave




$$e(t) = \alpha \cdot B \cdot v(t) + \beta \cdot \dot{B}$$
$$= \alpha' \cdot \varepsilon \cdot v(t) + \beta \cdot \delta$$
$$\varepsilon = \pm 1$$




EPFL



Another method of measuring velocity is by using a square wave for B. The advantage of this method is that the dB/dt = 0 when B remains constant (+ or -). During the transition period, a peak is generated, and the interval where the derivative is zero increases compared to previous method (B sin wave).

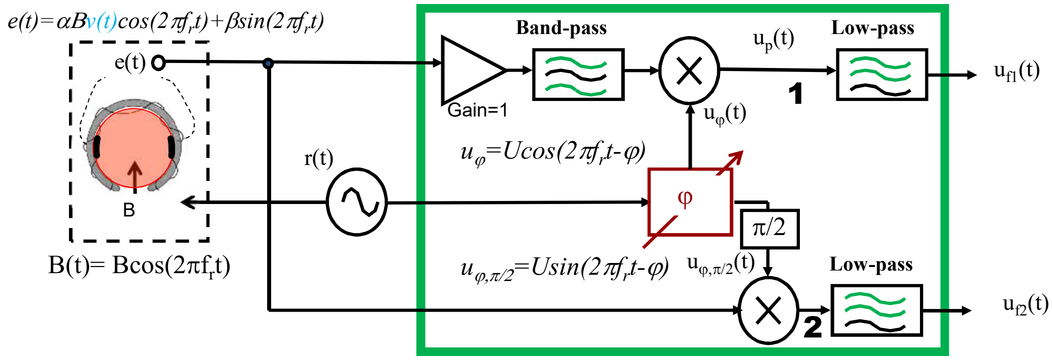
In this method, the signal comprises two components. The first component is modulated by induction and appears in blue, while the second component corresponds to the derivative of the magnetic field and is shown in red. Therefore, there are several periods in the signal where we can measure the velocity-related component of e.

The sign of the velocity-related component will be positive or negative, depending on the polarity of the magnetic field. An advantage of this method is that the term beta * amplitude of the peak (red spikes) can be removed by filtering low pass filtering, enabling to find the profile of blood velocity (and flow).

The ‘**gated’ sampling** is one method to eliminate the noise component from the measurement.

Another type of conditioning circuit is the **double phase lock-in amplifier** presented in the next slides

# Double phase Lock-in amplifier




$$\textbf{1}: \quad e(t).u_\varphi = \alpha BUv(t)\cos(2\pi f_r t)\cos(2\pi f_r t-\varphi) + \beta U\sin(2\pi f_r t)\cos(2\pi f_r t-\varphi)$$

$$= \frac{\alpha BUv(t)}{2}(\cos(4\pi f_r t+\varphi)+\cos(\varphi)) + \frac{\beta U}{2}(\sin(4\pi f_r t+\varphi)+\sin(\varphi))$$

After low pass filtering: $u_{f1}(t) = \frac{1}{2}\alpha BUv(t)\cos(\varphi) + \frac{\beta U}{2}sin(\varphi)$



A **Double Phase Lock-In Amplifier** is an advanced detection method to separate the **useful blood flow signal** from this unwanted **noise**:

**How Double Phase Lock-In Amplification works:**

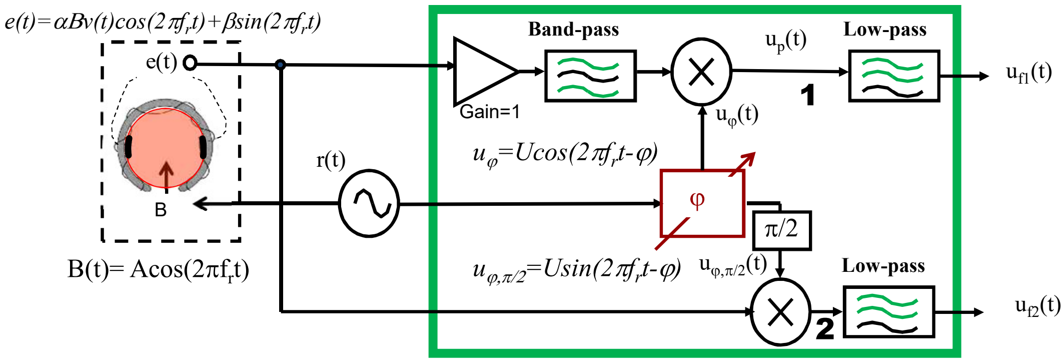
1. **Multiplication by Reference Signals (Phase Detection):**
   - The measured voltage signal e(t) is (amplified and filtered) is multiplied by a **reference signal** (synchronized with the excitation frequency of B(t)).
   - This multiplication converts the desired signal component (at excitation frequency) into a stable DC or low-frequency value, making it easy to filter.
2. **Quadrature (Double-Phase) Detection:**
   - The signal e(t) is simultaneously multiplied by two reference signals that differ in phase by 90° (cosine and sine reference signals).
   - Produces two outputs: one "in-phase" and one "quadrature-phase."

- **Extraction of True Signal (Phase Separation):**
   - The noise (induced directly by magnetic field changes) and the true blood-flow voltage differ in their phase relative to the excitation.
   - By analyzing both outputs, the lock-in amplifier clearly separates the desired flow-related signal (in-phase) from the noise component (quadrature or different phase).

We can demonstrate based on the equations expressing the voltages uf1(t) and uf2(t) that, with a particular value of φ (φ=0), the voltage uf1(t) will contain only the component of e(t) proportional with blood velocity, while the voltage uf2(t) will contain only the undesired component related to dB/dt

Calculation of expression for uf1(t) and uf2(t) is based on trigonometric formulas for multiplication of cos*cos and cos*sin (you can find them in tables…)

# Double phase Lock-in amplifier




$$u_{f1}(t) = \frac{1}{2}\alpha BUv(t)\cos(\varphi) + \frac{\beta U}{2} sin(\varphi)$$

**2**: $$e(t).\mathrm{u}_{\varphi,\pi/2} = \alpha BUv(t)\cos(2\pi f_r t)\sin(2\pi f_r t\text{- }\varphi)\text{+ }\beta U\sin(2\pi f_r t)\sin(2\pi f_r t - \varphi)$$
$$= \frac{\alpha BUv(t)}{2}(\sin(4\pi f_r t + \varphi) - \sin(\varphi)) + \frac{\beta U}{2}(-\cos(4\pi f_r t + \varphi) + \cos(\varphi))$$

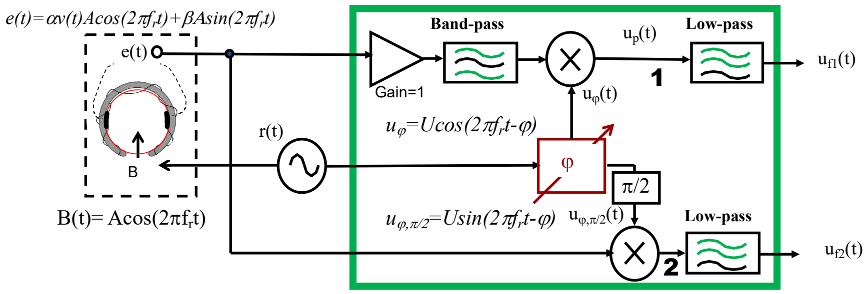
After low pass filtering: $$u_{f2}(t) = -\frac{1}{2}\alpha BUv(t)\sin(\varphi) + \frac{\beta U}{2}\cos(\varphi)$$

EPFL



Calcul based on trigonometric formulas….

# Double phase Lock-in amplifier




$$u_{f1}(t) = \frac{1}{2}\alpha B U v(t)\cos(\varphi) + \frac{\beta U}{2} sin(\varphi) \qquad u_{f2}(t) = -\frac{1}{2}\alpha B U v(t)\sin(\varphi) + \frac{\beta U}{2}\cos(\varphi)$$

- We adjust the phase shifter in order to have the maximum amplitude for $u_{f1}(t)$.

$$\boldsymbol{u_{f1}(t) = \frac{1}{2}\alpha B U v(t)}$$

Noise→ $u_{f2}(t) = \frac{\beta U}{2}$

35

If the value of φ is tuned correctly, for example φ =0, this is a robust method to separate the two signals (uf1 proportional to v(t) and uf2 the 'noise')

**Advantages over 'gated' sampling:**

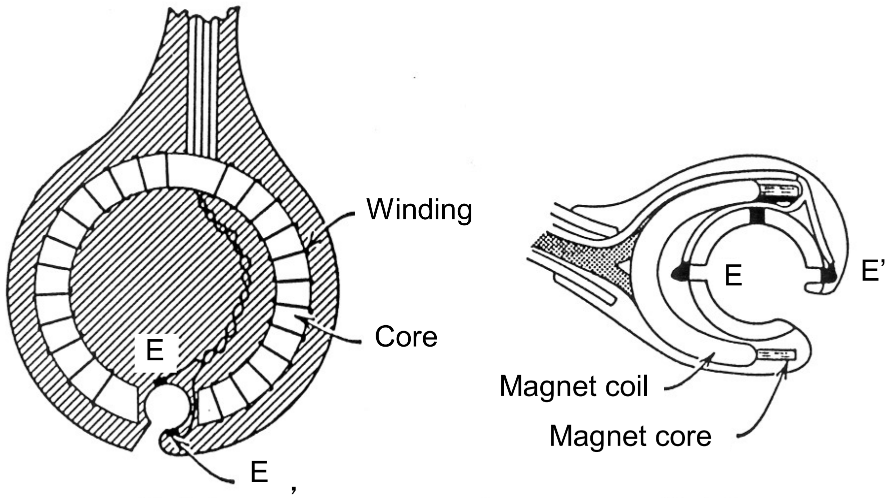
The simpler **'gated' sampling** method only measures voltage at specific moments (when dB/dt = 0) to minimize noise. However, it has limitations:

- **Lower noise rejection:** Any residual noise at the sampling instant remains.
- **Timing sensitivity:** Requires precise timing; small timing errors may introduce measurement inaccuracies.
- **Less robust:** Sensitive to minor changes in excitation frequency and phase shifts.

The **Double Phase Lock-In Amplifier** approach offers clear advantages:

- **Higher accuracy:** Continuous and precise separation of signal and noise.
- **Improved stability:** Less sensitive to phase and frequency drift.
- **Robust and reliable:** Provides consistently accurate flow measurements in clinical environments.

# Probes - Example




**C-core**

**U-core**

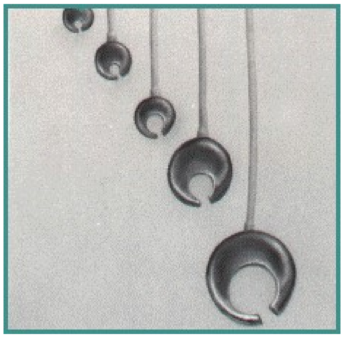


Some illustrations of the probes

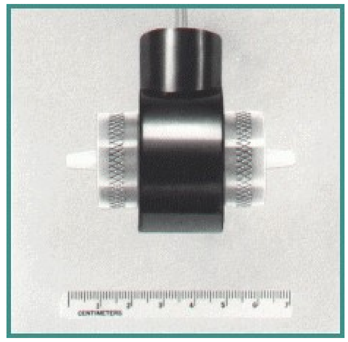
On right side a probe with a DC magnetic field generated by a magnet

On left side a probe with AC magnetic field generated by the core

# Probes - Example



**Intracorporeal sensor**
Internal dimension : 4 to 100 mm



**Extracorporeal sensor**
Measure in "bypass" or
by making perfusions
Dimension: 1/16" to 1/2"



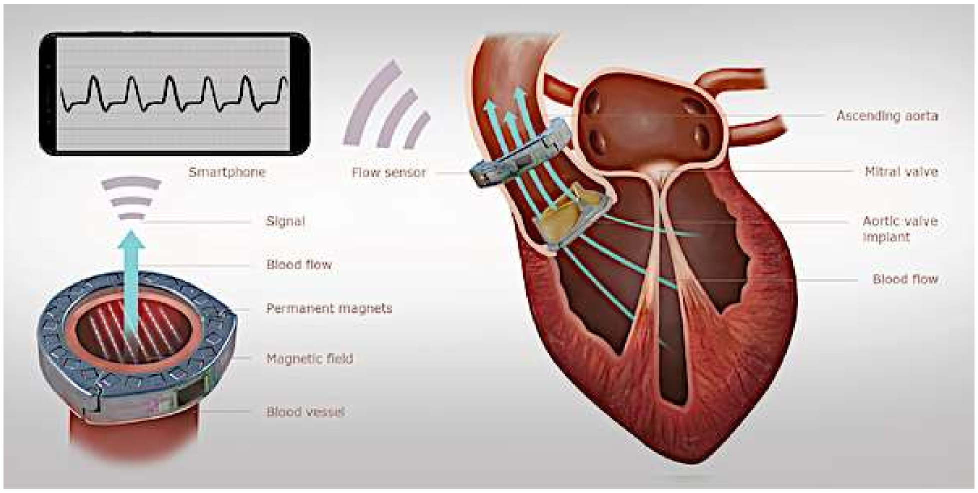
- Intracorporeal: invasive, probe around the blood vessel

- Extracorporeal: literally means "**outside the body**." Thus, an **extracorporeal sensor** measures blood flow from **outside** the patient's body. These sensors are widely used in clinical procedures involving extracorporeal circuits such as:
  - **Hemodialysis** (blood is circulated outside the body through a dialysis machine)
  - **Cardiopulmonary bypass (CPB)** during cardiac surgery

**Typical Setup of Extracorporeal Sensors:**

- Blood is temporarily diverted out of the patient's body into external **plastic tubing**.
- The sensor is placed **around the tubing**, **without direct contact with blood**.
- .An alternating magnetic field is applied externally around the tube.
- The induced voltage in blood flowing through the tubing is detected

## A smartphone-enabled wireless and batteryless implantable blood flow sensor for remote monitoring of prosthetic heart valve function




Vennemann B, Obrist D, Rösgen T (2020). PLOS ONE 15(1): e0227372. https://doi.org/10.1371/journal.pone.0227372
https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0227372

EPFL

This is an implantable magnetic flow sensor attaches to the ascending aorta to measure the characteristic flow profile downstream of the aortic valve. A magnet placed around the probe generates the induction B. The electrodes measure the induced voltage, A smartphone wirelessly receives the measurement data and inductively powers the implant.

Reference:

Vennemann B, Obrist D, Rösgen T. A smartphone-enabled wireless and batteryless implantable blood flow sensor for remote monitoring of prosthetic heart valve function. PLoS One. 2020 Jan 14;15(1):e0227372. doi: 10.1371/journal.pone.0227372. PMID: 31935231; PMCID: PMC6959614.