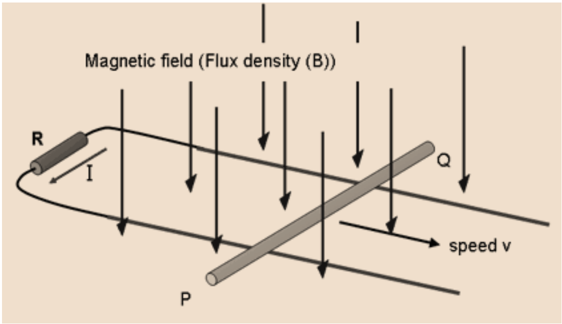
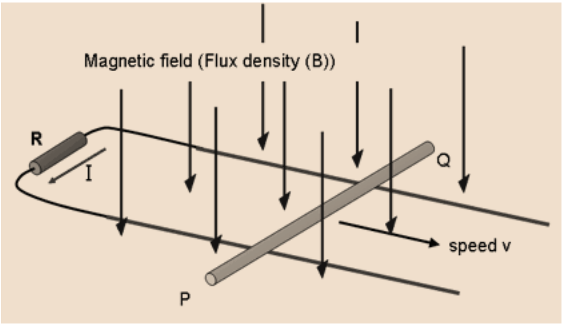
INDUCTIVE SENSORS

Part I – Self inductance
Part II – Mutual inductance
Part III – DC magnetic field blood flow meter
Part IV – AC magnetic field blood flow meter



Michael Fraraday
1791-1867

INDUCTIVE SENSORS



Part I – Self inductance

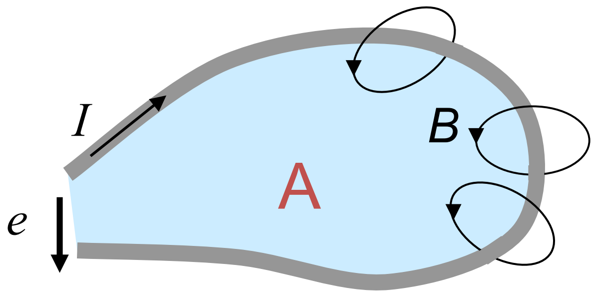
Respiratory inductive plethysmography

Michael Fraraday
1791-1867

Self inductance

- A conductor carrying a time-varying **current *I*** produces a magnetic **field *B*,** which generates a **magnetic flux Φ** across a **surface *A***:

$$\Phi = \int_S B \cdot dA$$



- **Faraday's laws of induction-** variation of Φ will induce an electromotive force (e); **Lenz' law:** the induced e will always have a direction such that the magnetic field it produces opposes the change in magnetic flux that caused it (indicated by sign -):

$$e = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

- ***L* is the (self)inductance of the conductor:** $L = \frac{\Phi}{I}$

Change of *L* when uniform field *B* across varying surface *A*

- Variations of A (frequency f_A) are negligible with respect frequency of magnetic field B: $f_A << f_B$

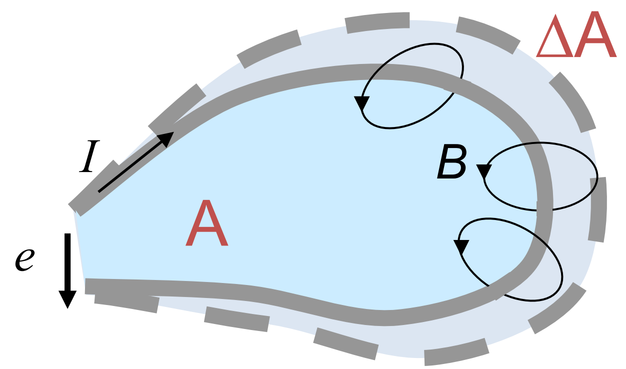
$$L = \frac{\Phi}{I}$$

$$\Phi = \int_S B \cdot dA$$

- Orientation(α) of A with respect to B is stable:

(1) $$\Phi = B \cdot A \cos\alpha, \quad L = \frac{B \cdot A}{I} \cos\alpha$$

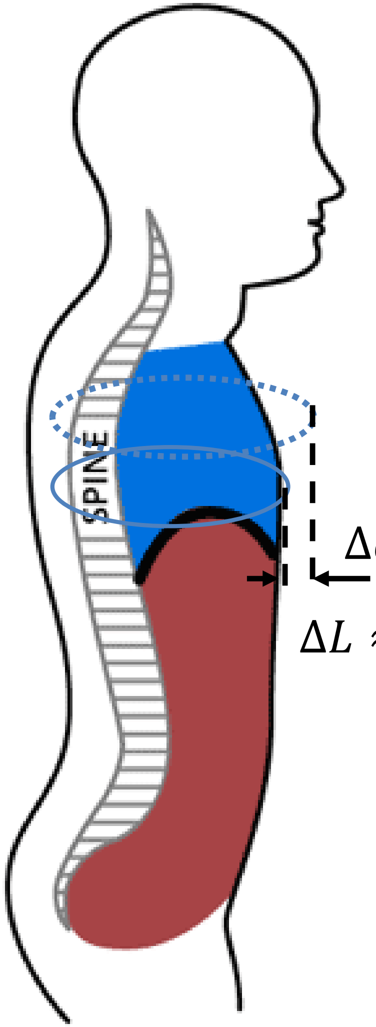
(2) $$L = Lo + \Delta L = \frac{B \cdot A}{I} \cos\alpha + \frac{B \cdot \Delta A}{I} \cos\alpha$$



- Measurement of *L* possible using the *resonant frequency* of an *oscillatory circuit including L*

Respiratory Inductive Plethysmography : RIP

- ***Tidal volume*** (ΔV) : volume of air moved into or out of the lungs in one breath calculated as the sum of the anteroposterior dimensions of the rib cage and abdomen

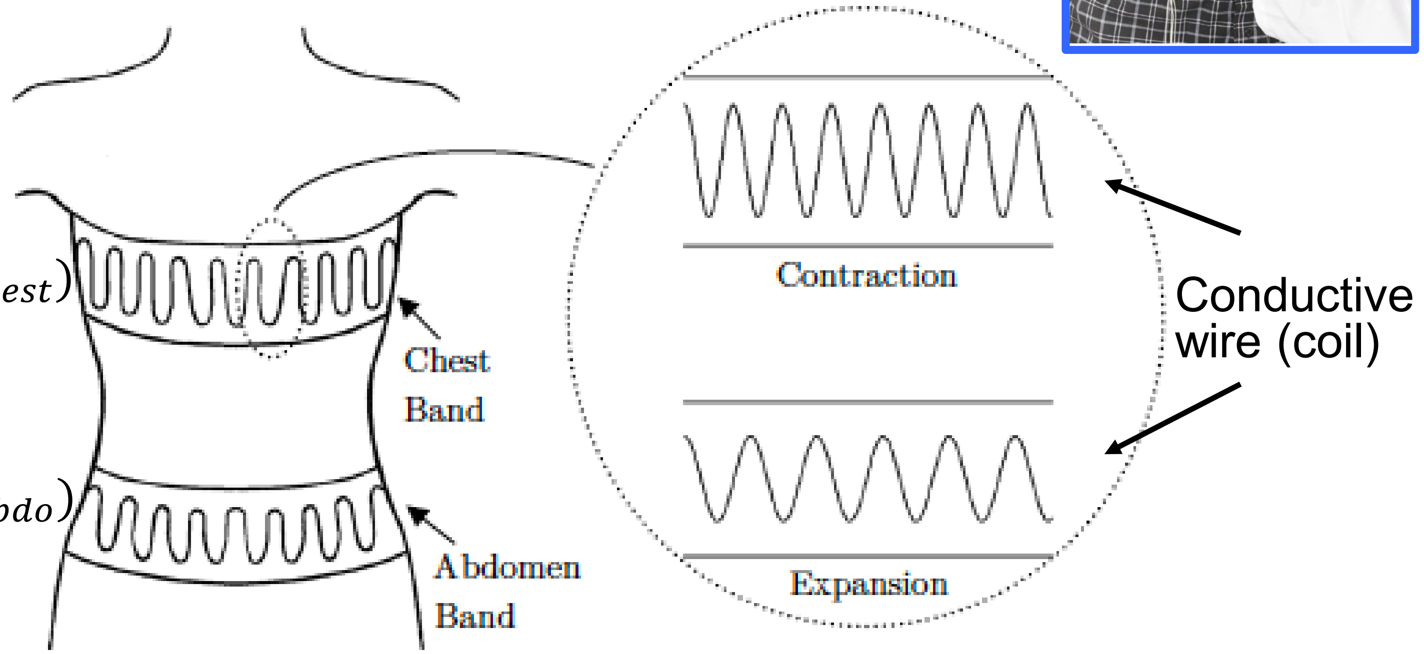


Δd

$$\Delta L \approx \alpha \cdot \Delta d + \beta$$

$\Delta V_{chest}(\Delta L_{chest})$

$\Delta V_{abdo}(\Delta L_{abdo})$



- $\Delta V = a \cdot \Delta V_{abdo} + b \cdot \Delta V_{chest}$
- *a* and *b* estimated by calibration

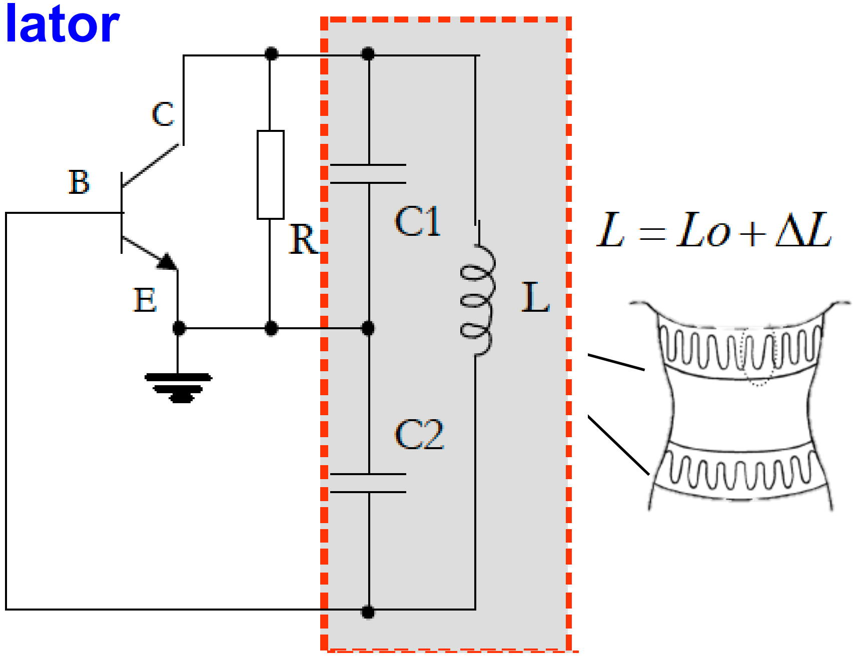
$\Delta L_{abdo}, \Delta L_{chest}$?

RIP conditioning circuit: Oscillator & PLL

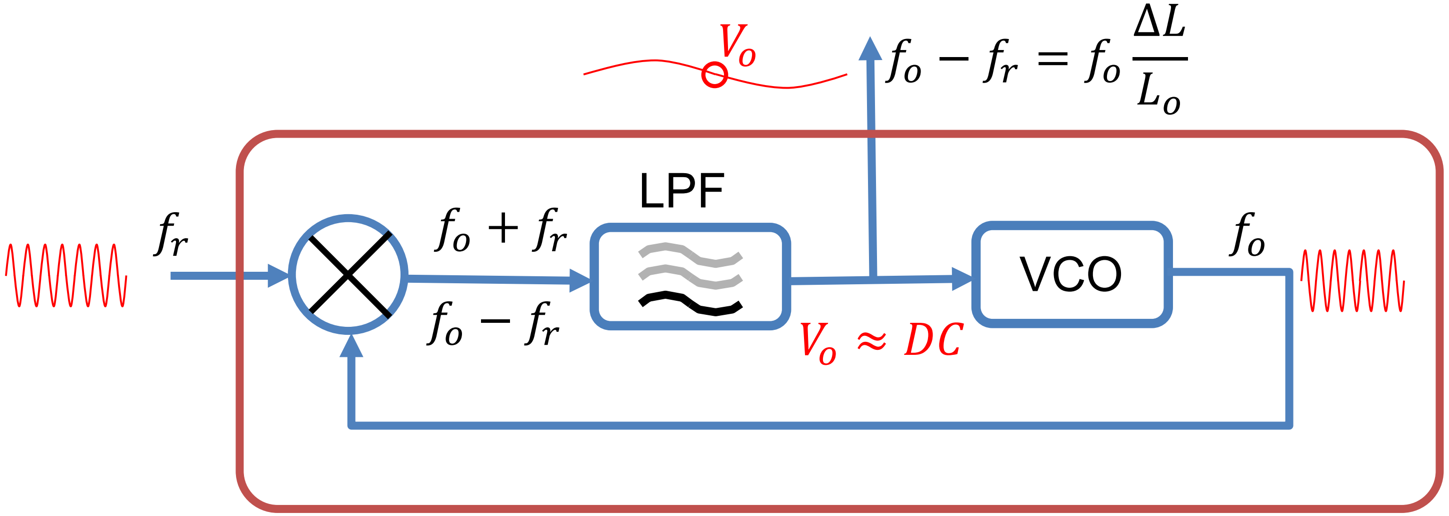
$\Delta L_{abdo}, \Delta L_{chest}$?

$$f_r = \frac{1}{2\pi\sqrt{\frac{LC_1C_2}{C_1+C_2}}} \approx f_o(1 - \frac{\Delta L}{L_o})$$

Oscillator



Hartley oscillator

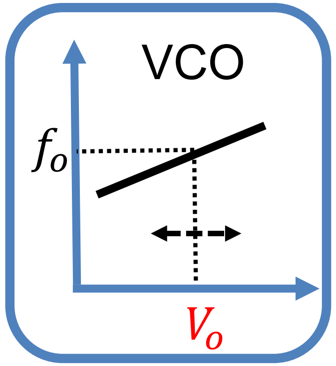


Phase Locked Loop (PLL)

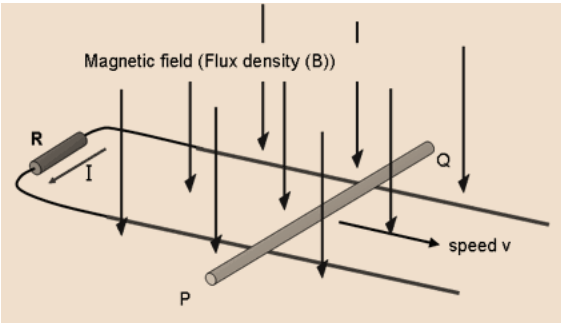
A Phase-Locked Loop (PLL) is a control circuit that continuously adjusts its output frequency to match and lock onto the frequency (and phase) of an input signal.

It includes:

- Analog multiplier (phase detector)
- VCO: Voltage Controlled Oscillator
- LPF: Low pass Filter



INDUCTIVE SENSORS



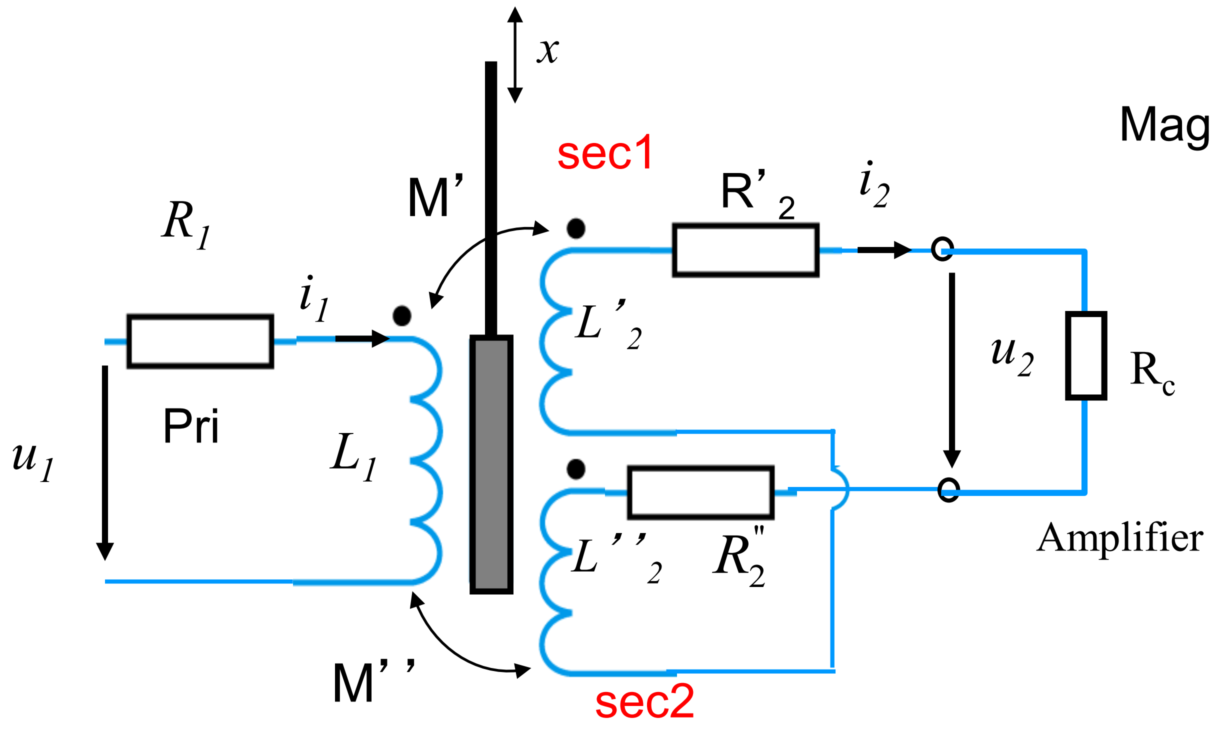
Part II a– Mutual inductance

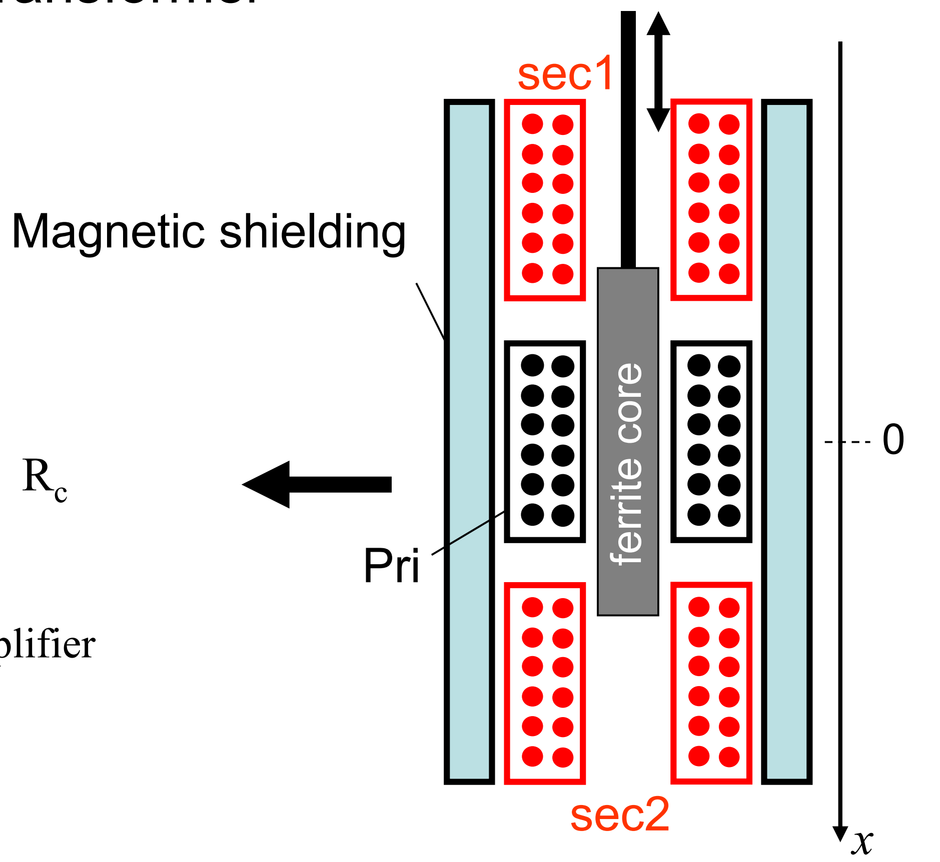
Linear Variable Differential Transformer (LVDT): principle

Michael Fraraday
1791-1867

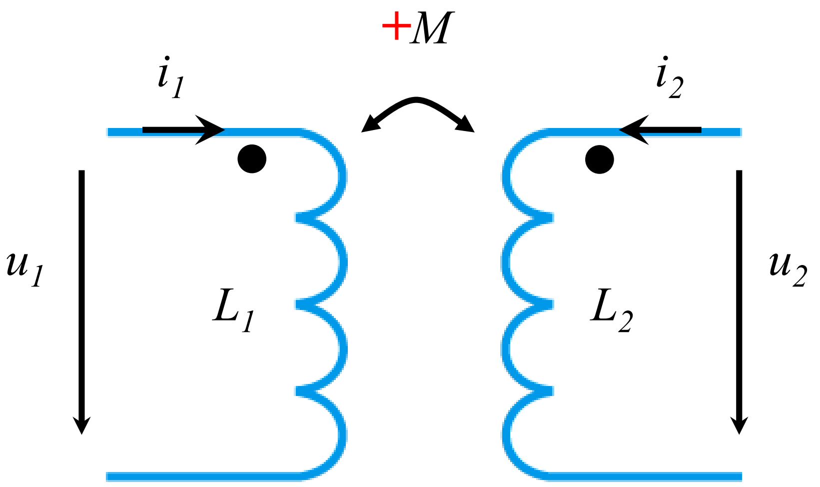
Mutual inductance :differential transformer

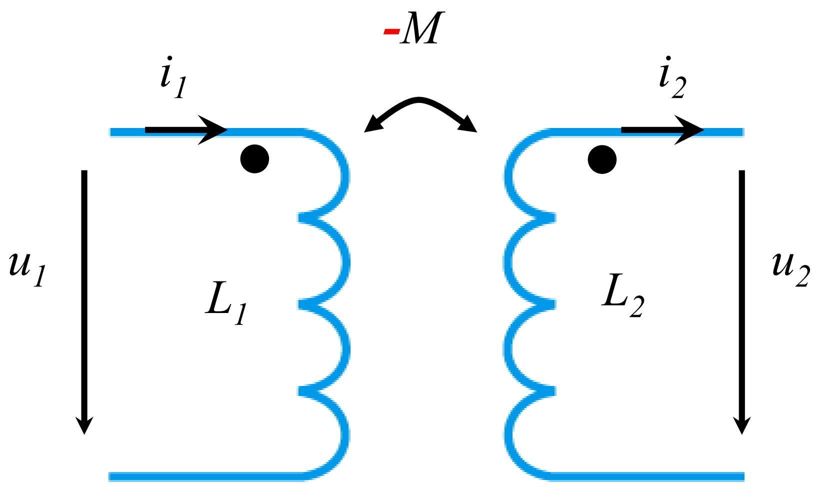
LVDT: **L**inear **V**ariable **D**ifferential **T**ransformer





Convention: mutual inductance

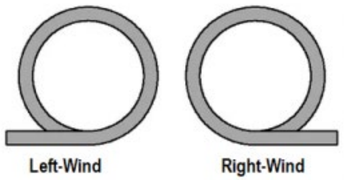




$$u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad u_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$u_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad u_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

●: indicate the polarity of coil winding,
The currents flowing into each winding at the connection indicated by the dot produce induced voltages of the same sign

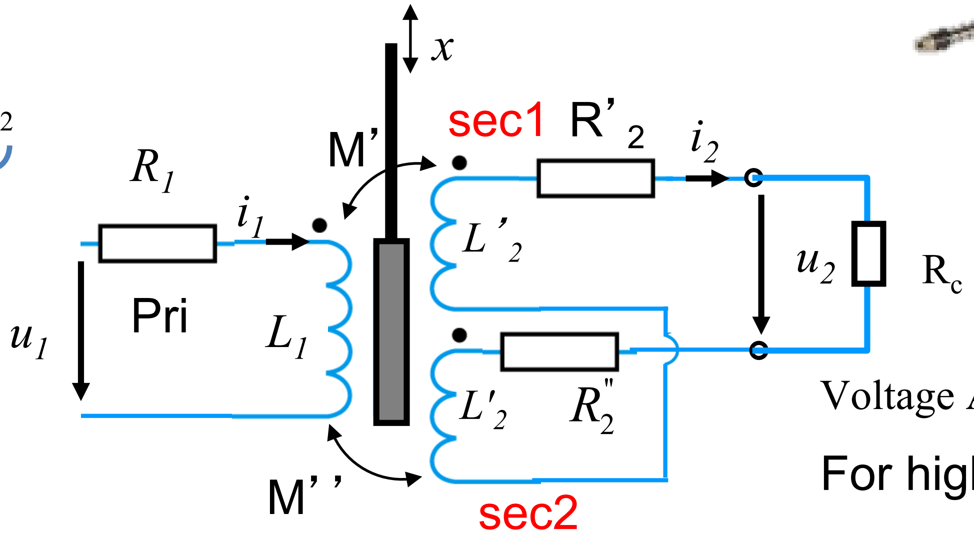


Mutual inductance: differential transformer

LVDT: **L**inear **V**ariable **D**ifferential **T**ransformer

$$u_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + (M''-M')\frac{di_2}{dt}$$

$$\underline{U}_1 = (R_1 + j\omega L_1)\underline{I}_1 + \underbrace{j\omega(M''-M')\underline{I}_2}_{\approx 0}$$



Voltage **Amplifier**

For high $R_c \rightarrow i_2 \approx 0$:

$$\underline{U}_2 = \frac{j\omega[M''(x)-M'(x)]}{R_1 + j\omega L_1}\underline{U}_1 \quad (1)$$

$$u_2 = -(R_2' + R_2'')i_2 - (L_2' + L_2'')\frac{di_2}{dt} + (M''-M')\frac{di_1}{dt}$$

$$\underline{U}_2 = \underbrace{-(R'_2 + R''_2 + j\omega L'_2 + j\omega L''_2)\underline{I}_2}_{\approx 0} + j\omega(M''-M')\underline{I}_1$$

LVDT

$$\underline{U}_2 = \frac{j\omega[M''(x) - M'(x)]}{R_1 + j\omega L_1}\underline{U}_1$$

McLaurin series

$$M'(x) = M(0) + ax + bx^2 + \ldots \quad \text{for } x > 0$$
$$M''(x) = M(0) - ax + bx^2 + \ldots \quad \text{for } x < 0$$

2nd order approximation:

$$M''(x) - M'(x) = -2ax$$

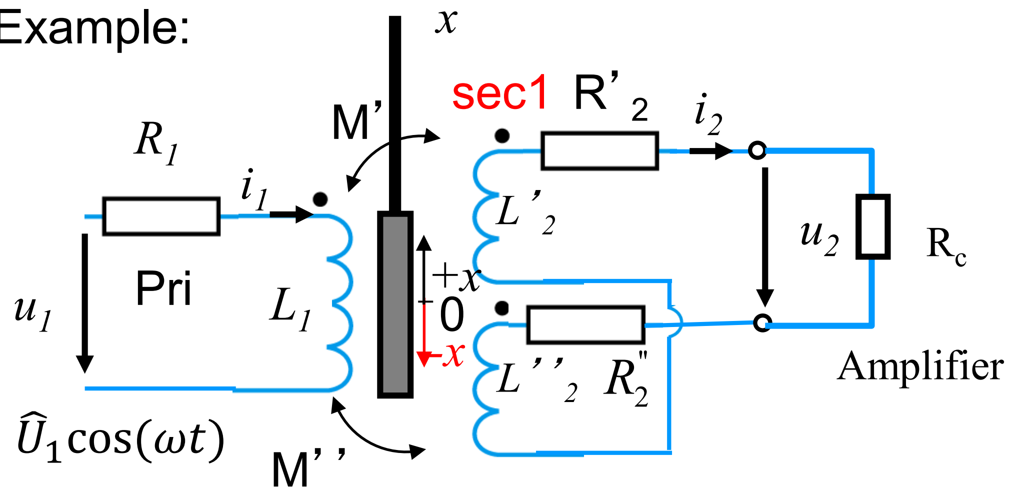
The relation become linear:

$$\underline{U}_2 = \frac{-2j\omega \cdot a\underline{U}_1}{R_1 + j\omega L_1}\underline{X}$$

(1)

$$\underline{U}_2 = \underline{G}(j\omega) \cdot \underline{X}$$

- Example:



$$u_1 = \hat{U}_1\cos(\omega t)$$

$$u_2 = \frac{2\omega \cdot a\hat{U}_1}{\sqrt{R_1^2 + \omega^2 L_1^2}} x(t)\cos(\omega t - \varphi)$$

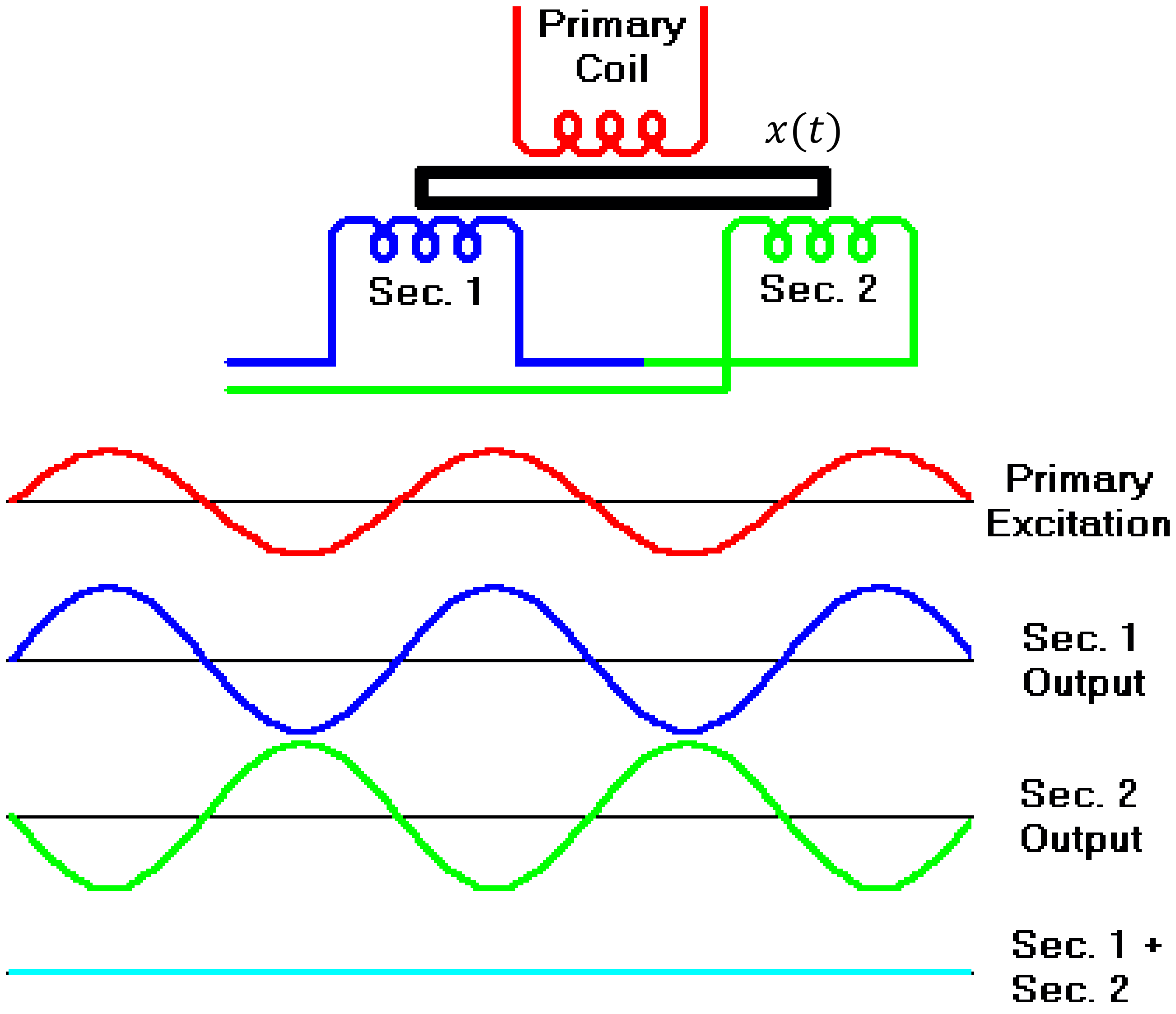
$$u_2 = A \cdot x(t) \cdot \cos(\omega t - \varphi)$$

|G(jω)|

$$\varphi = -90 - arctg\left(\frac{\omega L_1}{R_1}\right)$$

Phase shift at the output of LVDT, inherent to the device's design and operation

LVDT



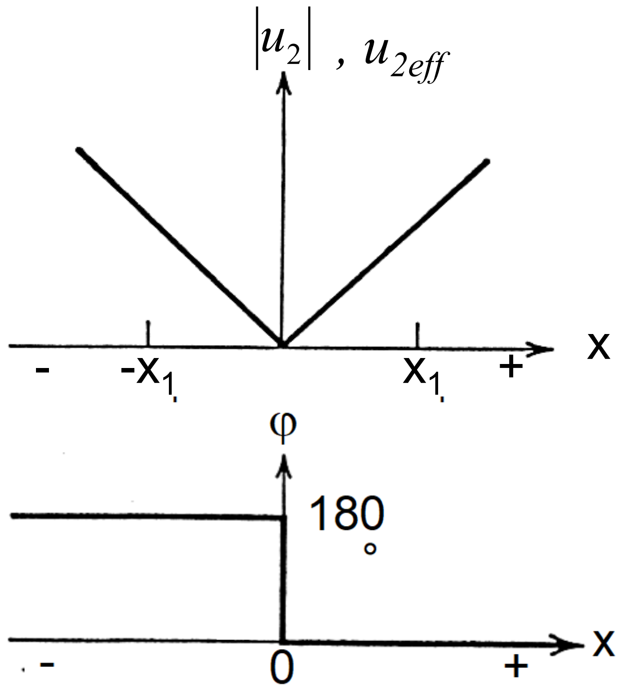
$$u_1 = \hat{U}_1 \cos(\omega t)$$

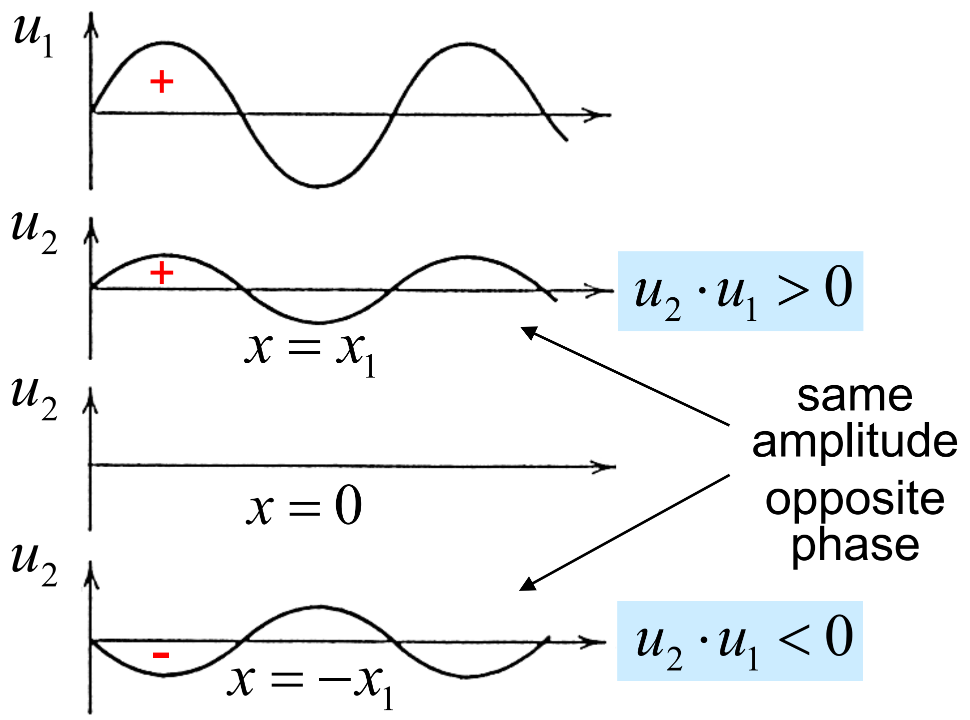
$$u_2 = A \cdot x(t) cos(\omega t - \varphi)$$

LVDT

$$u_2 = A \cdot x(t) \cdot \cos(\omega t - \varphi)$$

$$|\underline{U}_2| = |\underline{G}(\omega)| \cdot |\underline{X}|$$

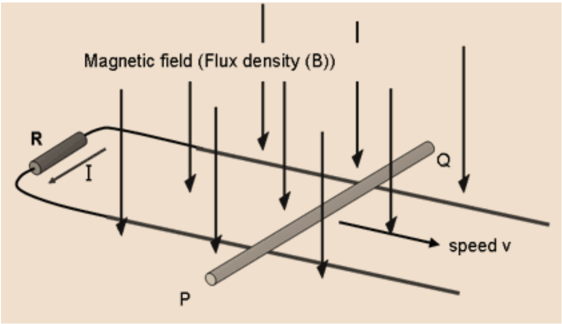




- Conditioning with a phase detection circuit :

  Detection of the **direction** of movement

INDUCTIVE SENSORS

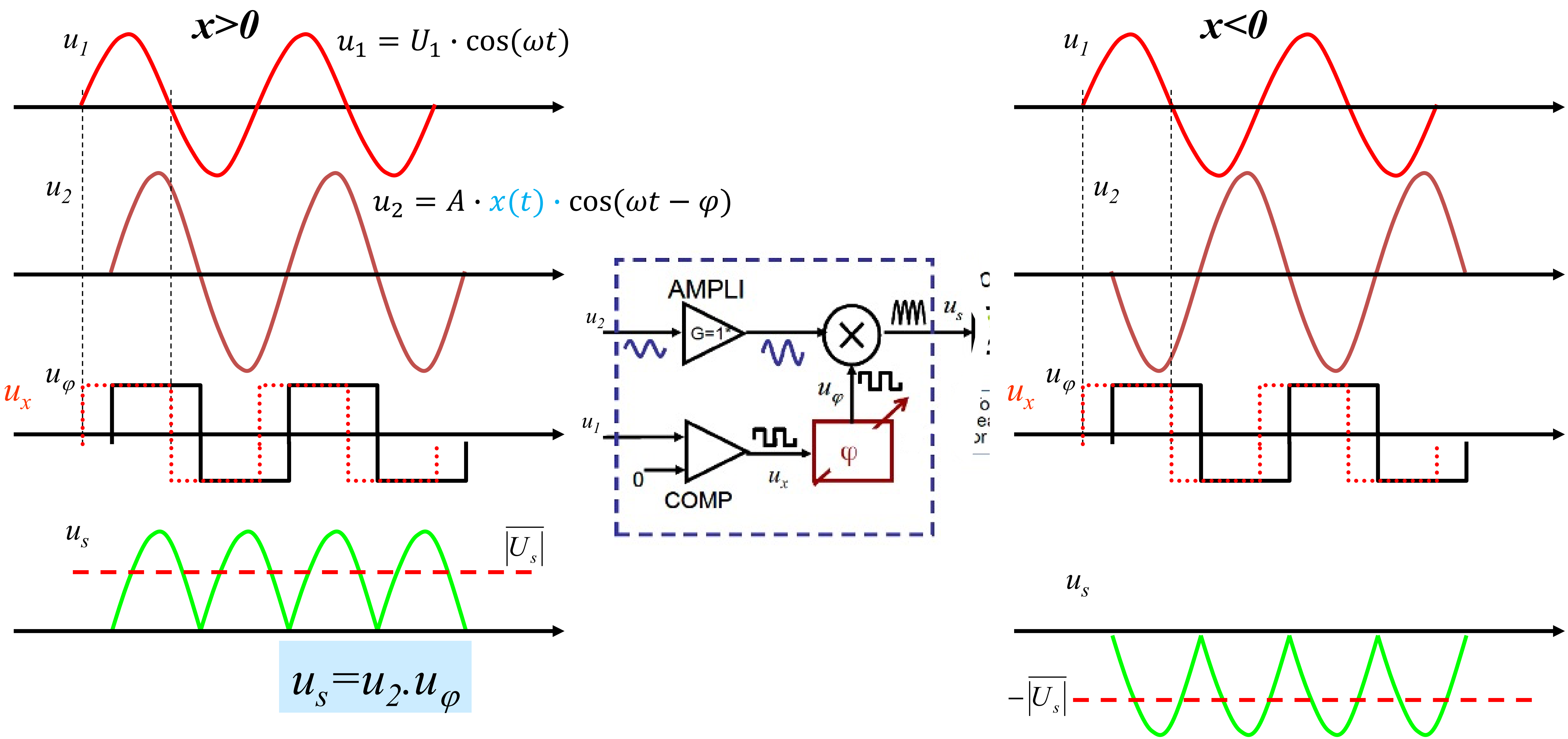


Part II b– Mutual inductance

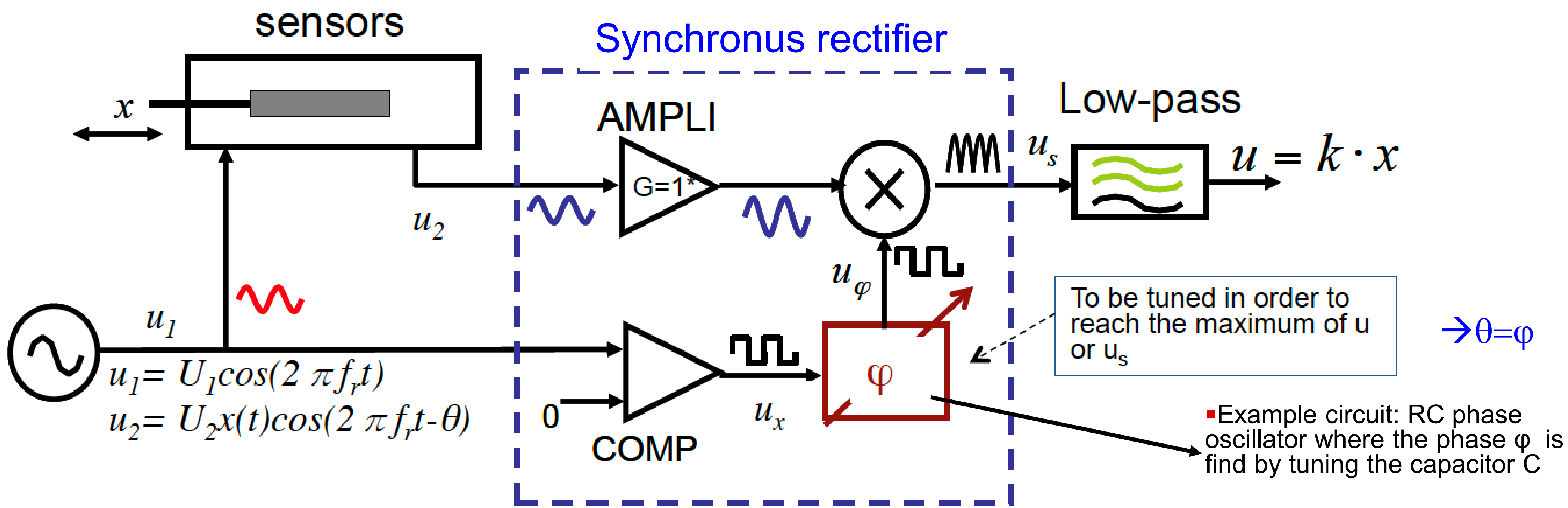
LVDT: Synchronous detection

Michael Fraraday
1791-1867

Synchronous rectifier



Synchronous detection: bloc diagram



$$u_\varphi = U\cos(2\pi f_r t - \varphi) + \frac{1}{3} U\cos(6\pi f_r t - \varphi) + \frac{1}{5} U\cos(10\pi f_r t - \varphi) + \cdots$$

High frequency

$$u_s = u_2 \cdot u_\varphi = U_2.x(t)\cos(2\pi f_r t - \theta).U\cos(2\pi f_r t - \varphi) + \frac{1}{3} U_2.x(t)\cos(2\pi f_r t - \theta).U\cos(6\pi f_r t - \varphi) + \ldots$$

$$= \frac{1}{2} U_2.U.x(t)\cos(2\pi f_r t - \theta \square 2\pi f_r t + \varphi) + \frac{1}{2} U_2.U.x(t)\cos(2\pi f_r t - \theta + 2\pi f_r t - \varphi) + \frac{1}{3} U_2.x(t)\cos(2\pi f_r t - \theta).U\cos(6\pi f_r t - \varphi) + \cdots$$

After low pass: $u = \frac{1}{2} U_2.U.x(t)[\cos(-\theta + \varphi)] = \frac{1}{2} U_2.U.x(t) = k.x(t)$

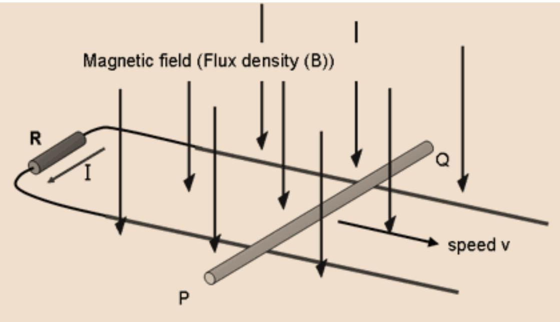
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

*G=1 for the sake of simplicity

LVDT characteristics

- Full scale (FS): ±1mm to 500mm
- Sensitivity: 1-500mV/V/mm
- Linearity: 0.05% to 1% of Range
- Precision: 0.002% to 0.5% of FS
- Excitation frequency: up to 50kHz
- Dynamic response up to 2kHz

INDUCTIVE SENSORS



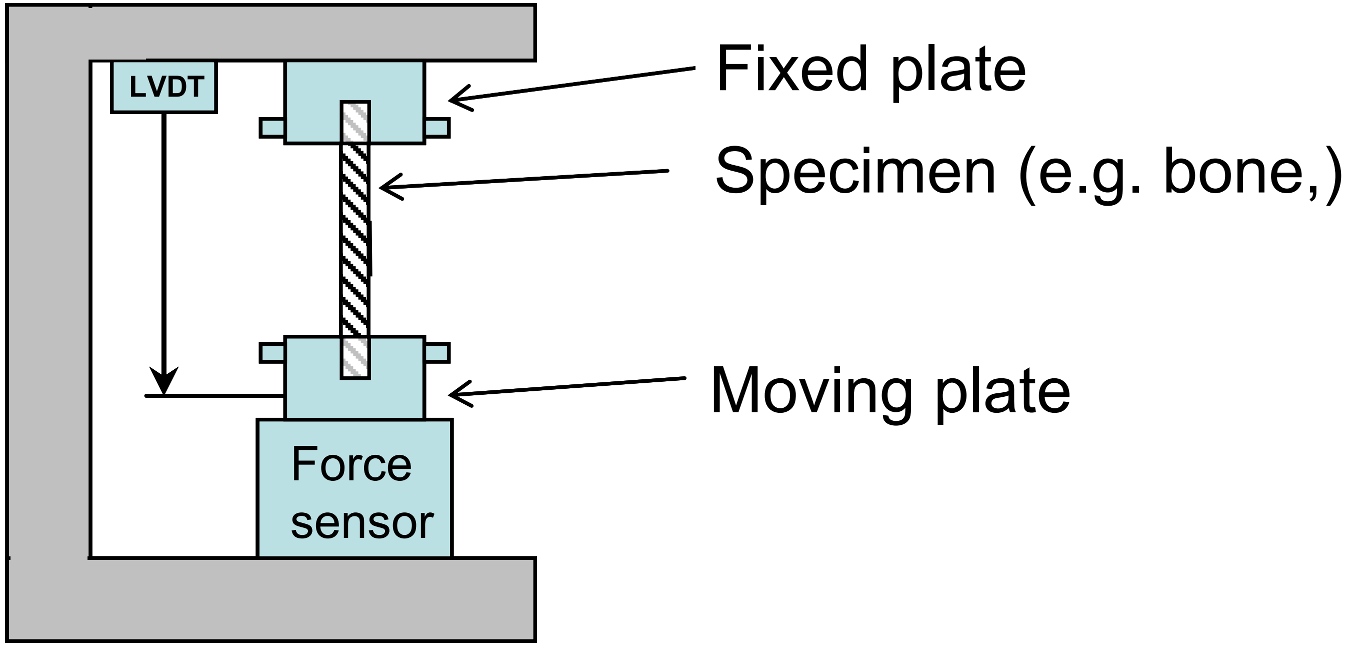
Part II c– Mutual inductance

LVDT: Applications

Michael Fraraday
1791-1867

Applications: tissue mechanical property

- Measure the the deformation of a bone by applying a know force: Young modulus estimation

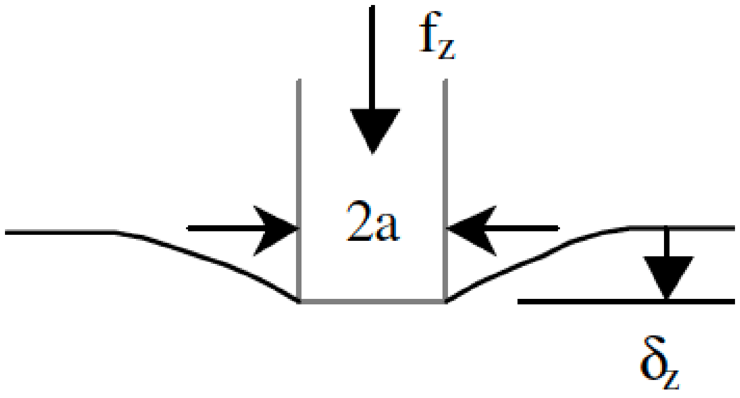


$$\sigma = \frac{F}{A}$$

$$\varepsilon = \frac{\Delta l}{l}$$

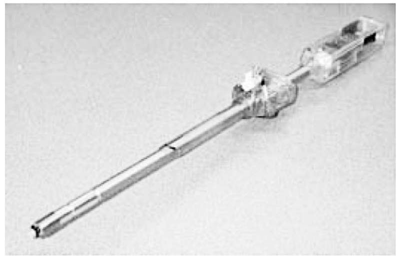
$$\sigma = \varepsilon Y$$

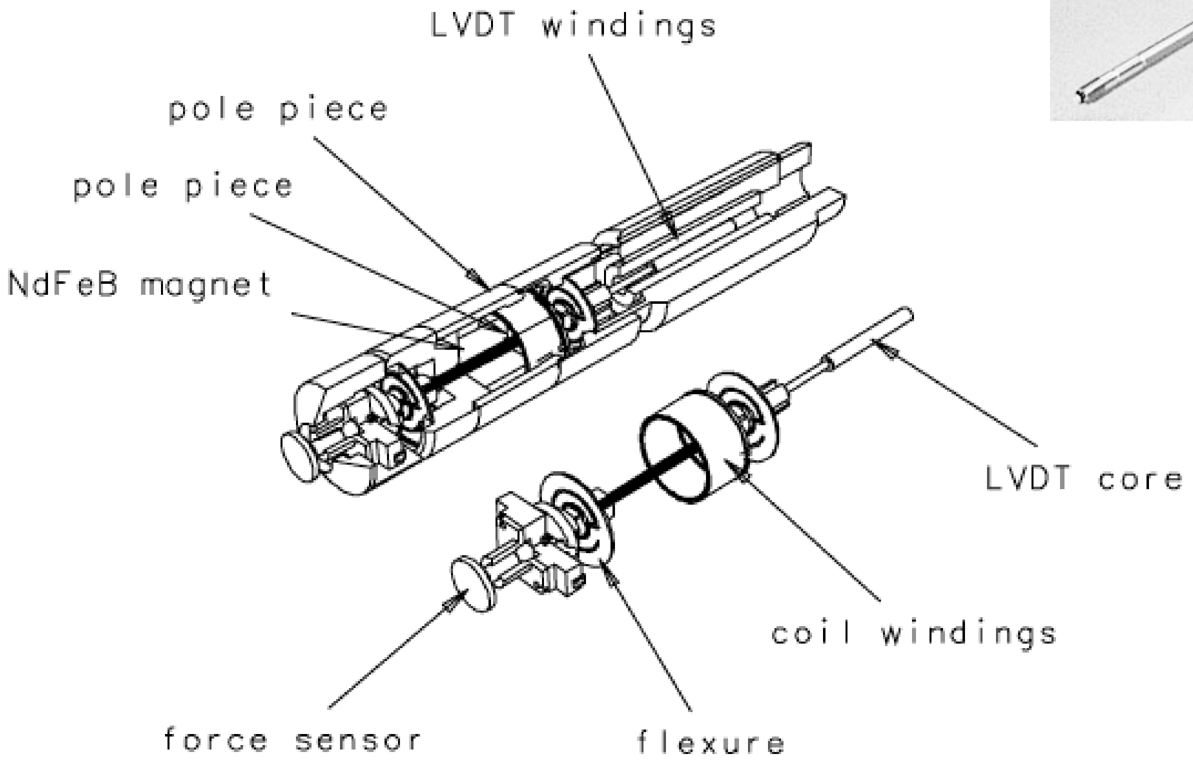
Mechanical property of living tissue



$$Young\ modulus:\ Y = K\frac{3f_z}{8a\delta_z}$$

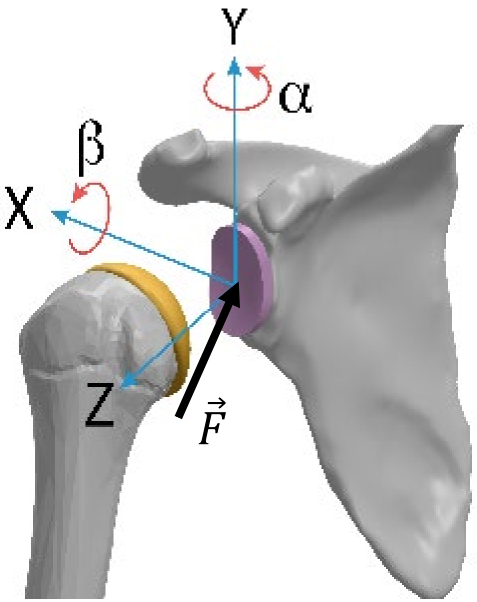
K: experimental constant depending on the size



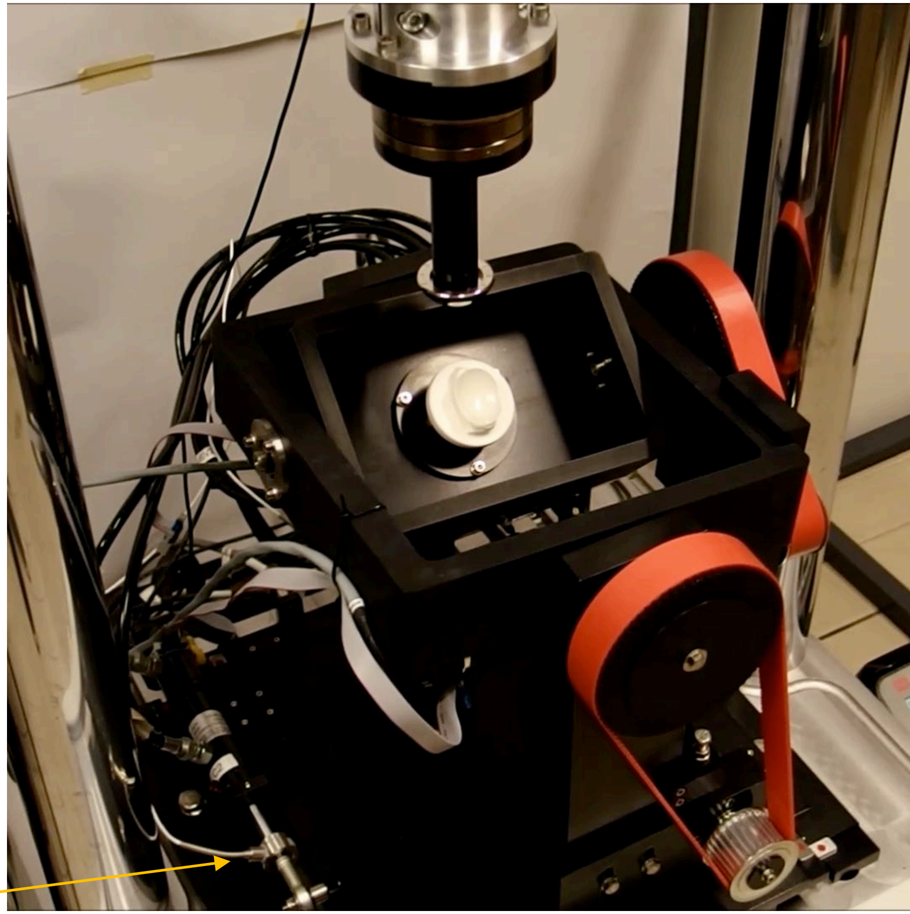


W. Niessen and M. Viergever (Eds.): MICCAI 2001, LNCS 2208, pp. 975–982, 2001. Springer-Verlag Berlin Heidelberg 2001

Robotic shoulder simulation of instability

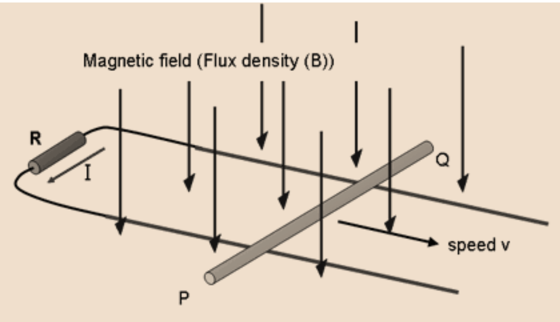


- Estimation of x and y displacement by LVDT when F is applied
- F can reach more than twice the body weight!



LVDT

INDUCTIVE SENSORS



Part III – DC magnetic field blood flow meter

Michael Fraraday
1791-1867

Measuring blood flow

***Faraday Law**: $e \propto \frac{d\Phi}{dt}$; The voltage can be induced by:*

- *A fixed conductor (fixed area A) in a variable magnetic field B*

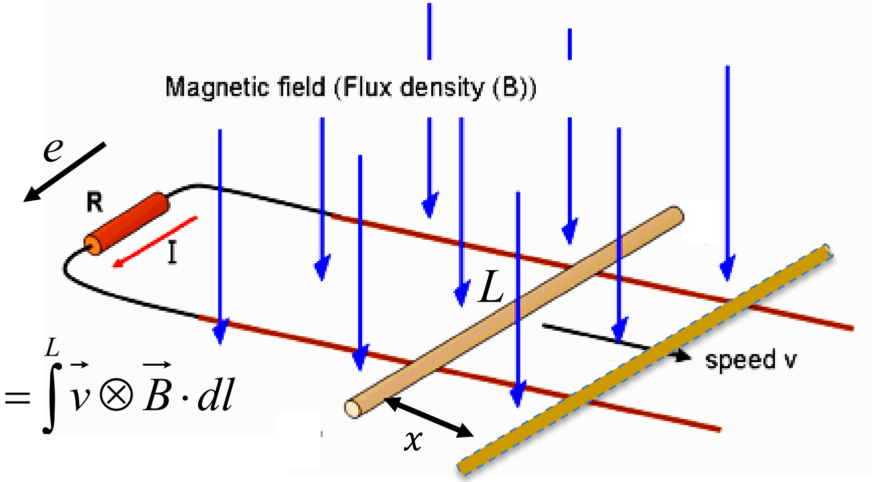
$$\frac{d\Phi}{dt} = \frac{dB}{dt} A$$

- ***A conductor moving in a uniform magnetic field***

$$\frac{d\Phi}{dt} = B\frac{dA}{dt} = B \cdot L\frac{dx}{dt} = B \cdot L \cdot v$$

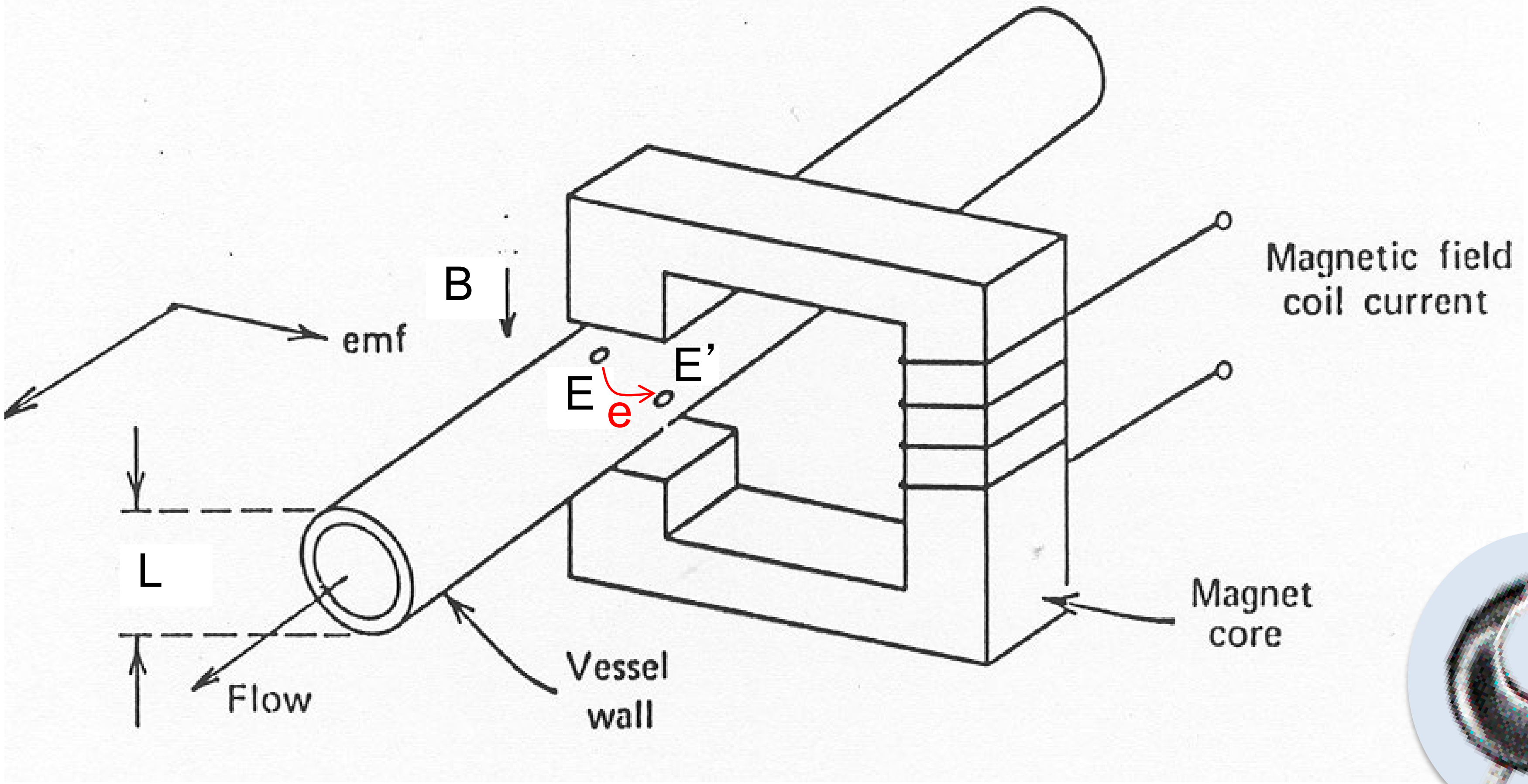
  - $L = length\ of\ the\ conductor$

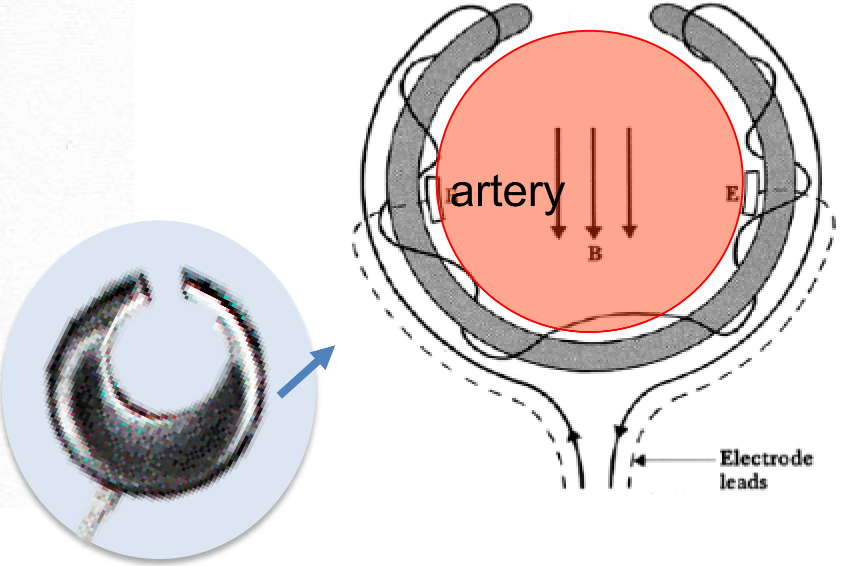
$$e = \int^{L} \vec{v} \otimes \vec{B} \cdot dl$$



- *electricity is only produced while something is varying/moving*
- *the faster the variation/movement, the more electricity we get*

Measuring blood flow





$$e = \int_0^L \vec{v} \otimes \vec{B} \cdot dl$$

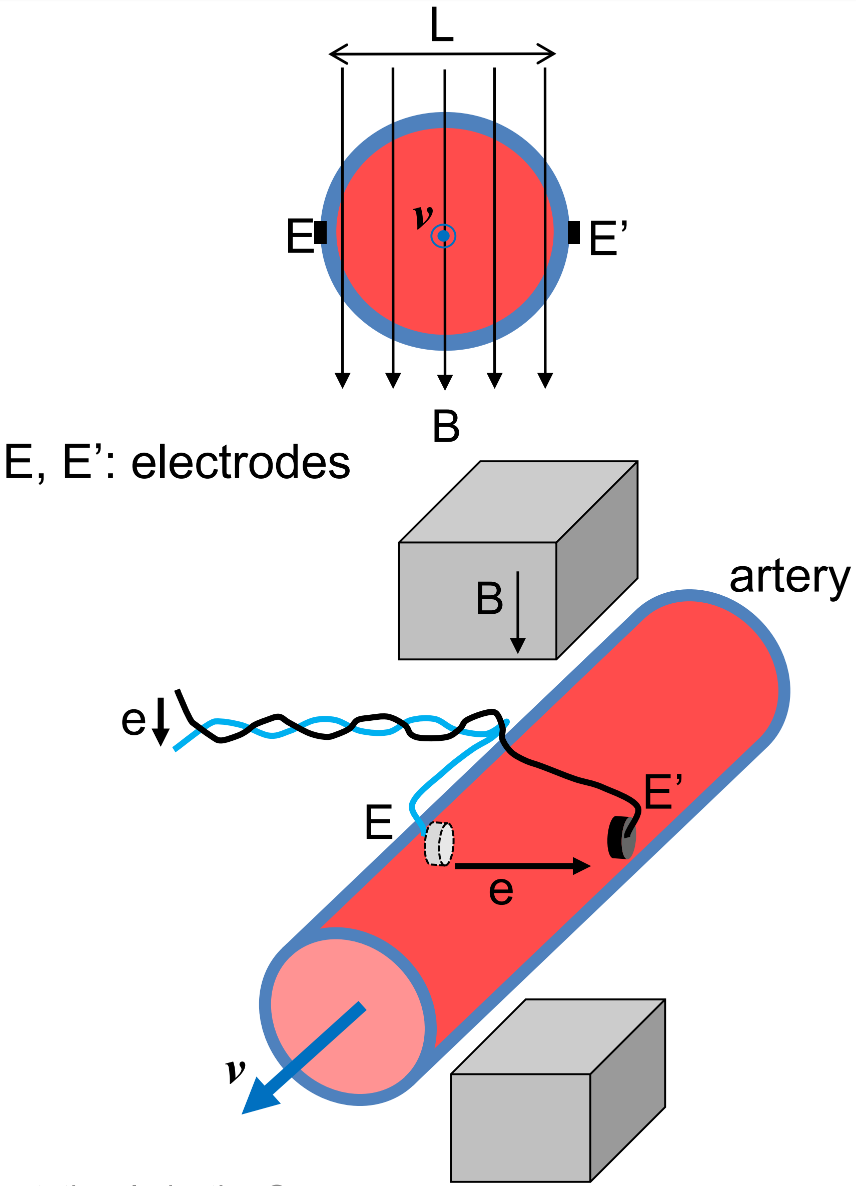
Probe (invasive measurement)

Measuring blood flow

- Faraday's law:

$$e = \int_0^L \vec{v} \otimes \vec{B} \cdot dl$$

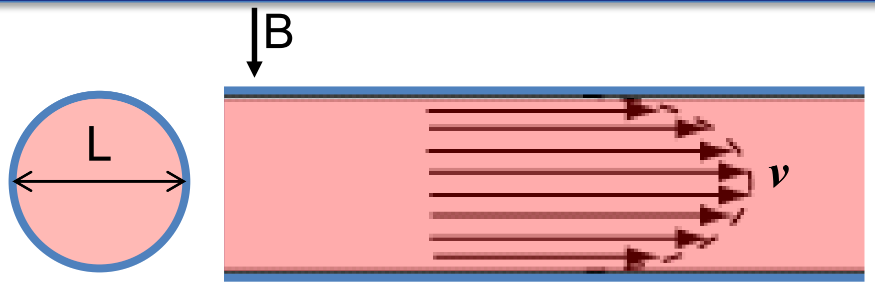
  - L: diameter of artery, m
  - $\boldsymbol{v}$: blood velocity, m/s
  - $\boldsymbol{B}$: magnetic field applied to vessel, T
  - $\boldsymbol{e}$: induced voltage perpendicular to the flow direction, V



Measuring blood flow

$$e = \int_0^L \vec{v} \otimes \vec{B} \cdot dl$$

$$\text{if } B \perp v \; : \; e = B \cdot L \cdot v$$



- The flow in m³/s is obtained by:

$$Q = v \cdot \frac{\pi}{4} L^2 \quad \rightarrow \quad v = \frac{4Q}{\pi L^2}$$

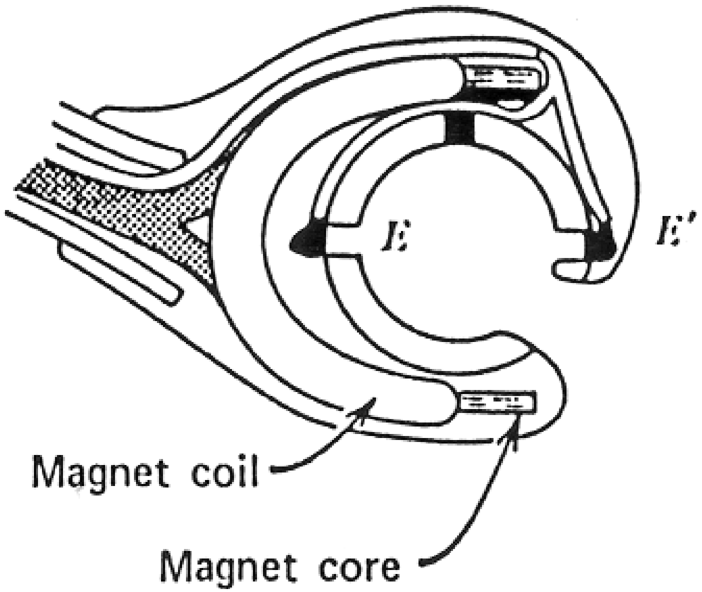
$$e = \frac{4B}{\pi L} Q = k \cdot Q$$

k: sensitivity

- **Measures instantaneous flow**: corresponds to the mean fluid velocity in the cross-section
- flows close to the electrodes contribute more than flows that are further away
- depends on conductivity of the vessel wall and the neighbouring tissues
- measurement accuracy affected by potential non-uniformity of magnetic field B
- the system needs calibration before measurements

DC flow meter

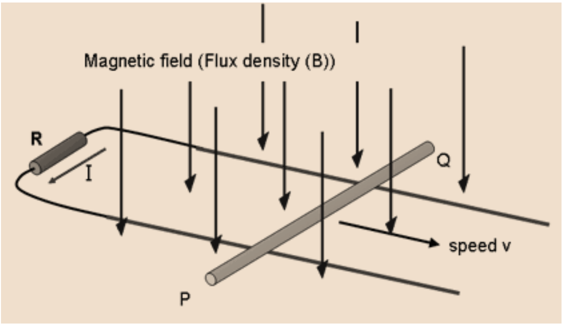
- B : constant
- $e = K \cdot v$
- Drawbacks:
  - Half-cell electrode voltage
  - ECG interference
  - Presence of 1/f noise



➔ Use an induction of higher frequency (by about 400Hz) than that of the flow

INDUCTIVE SENSORS

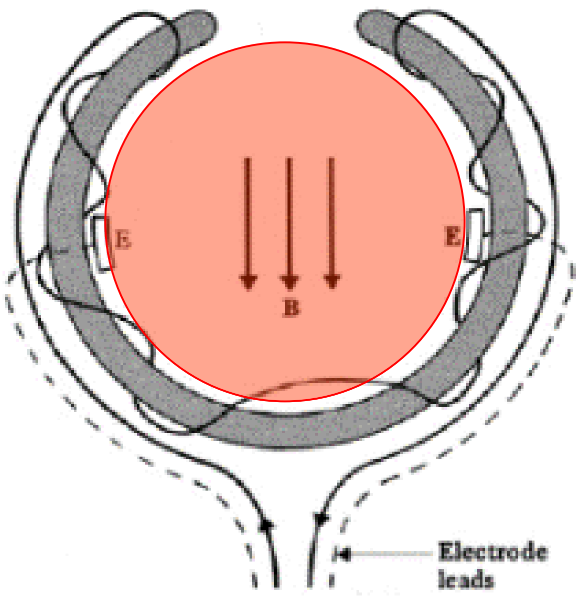
Part IV - AC magnetic field blood flow meter

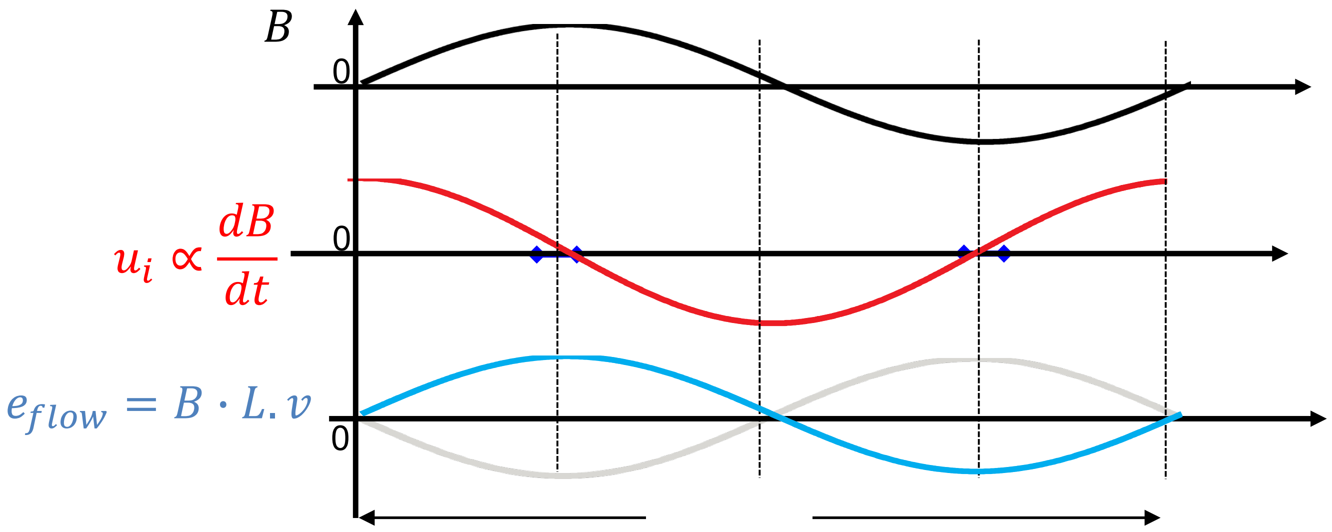


Michael Fraraday
1791-1867

AC flow meter: sine wave

- Measured voltage: $e \propto \frac{d\Phi}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt} = B \cdot L \cdot v + u_i$
- Transformer emf (induced voltage): $u_i \propto \frac{dB}{dt}$

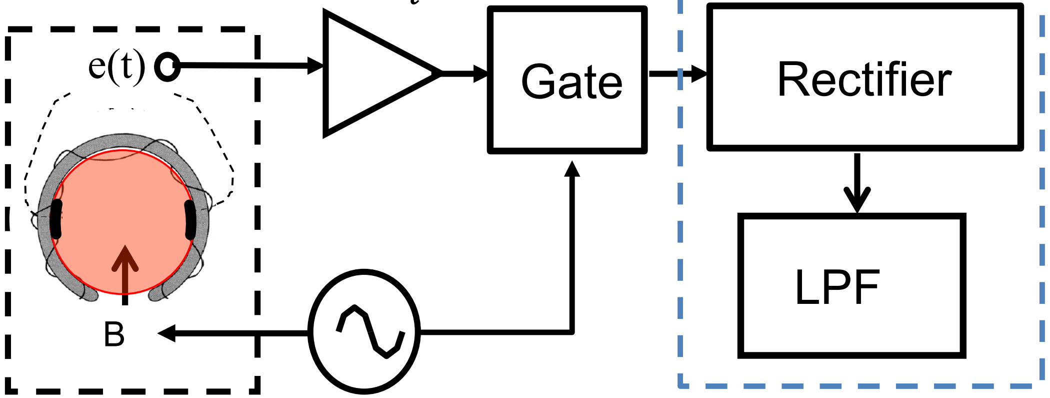




➔ measure when $u_i \approx 0$

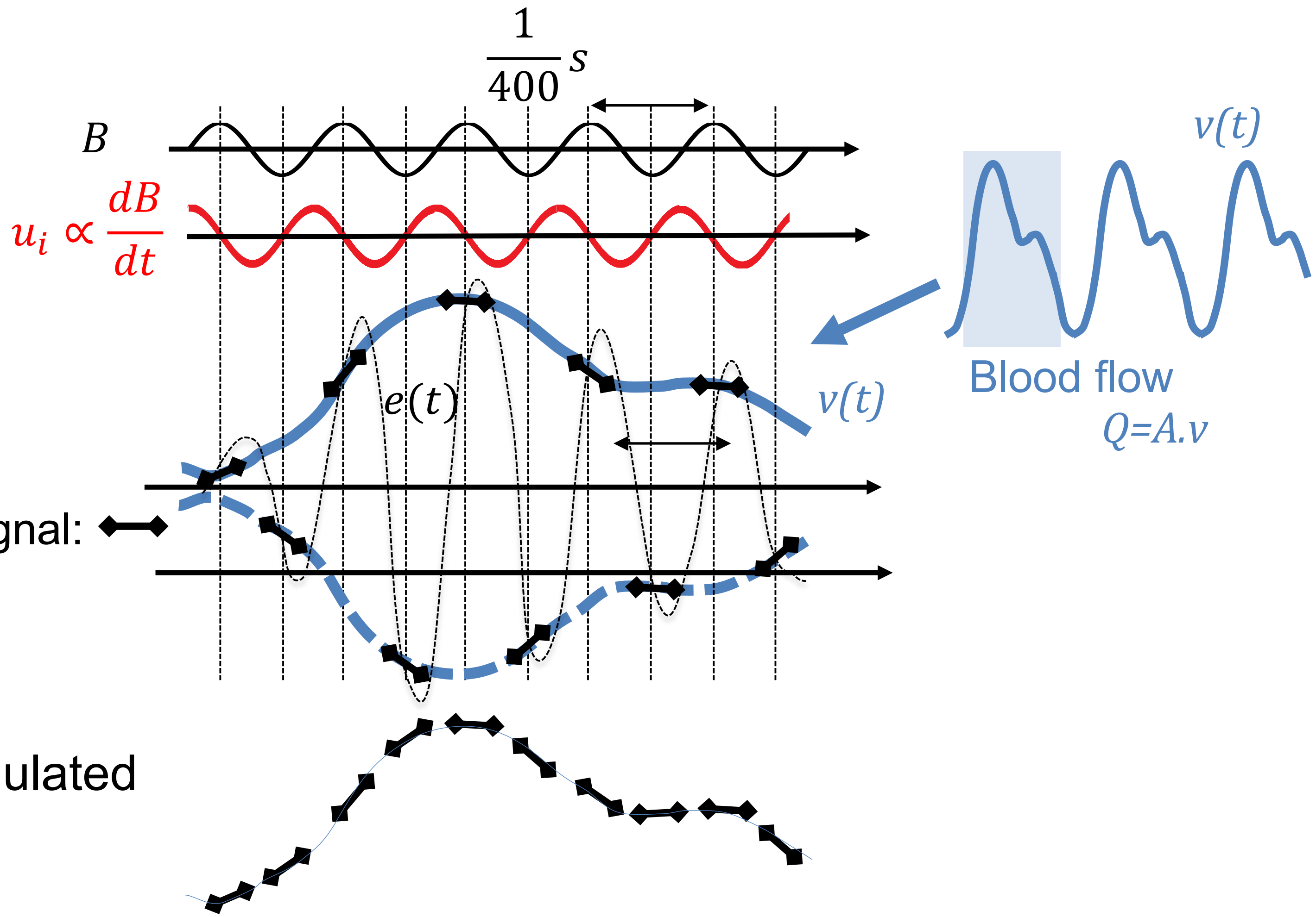
AC flow meter: sine wave

$e = B \cdot L \cdot v + u_i$



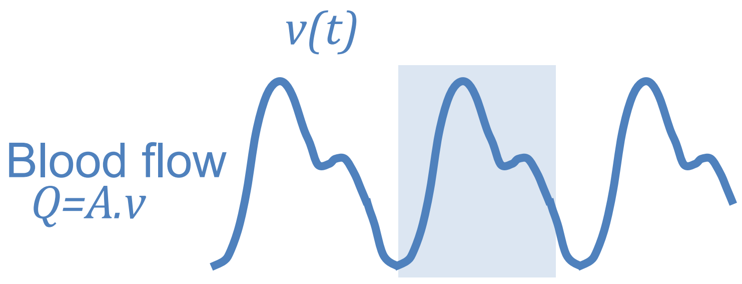
Peak detection in *B* for measurement when $\dot{B}$=0

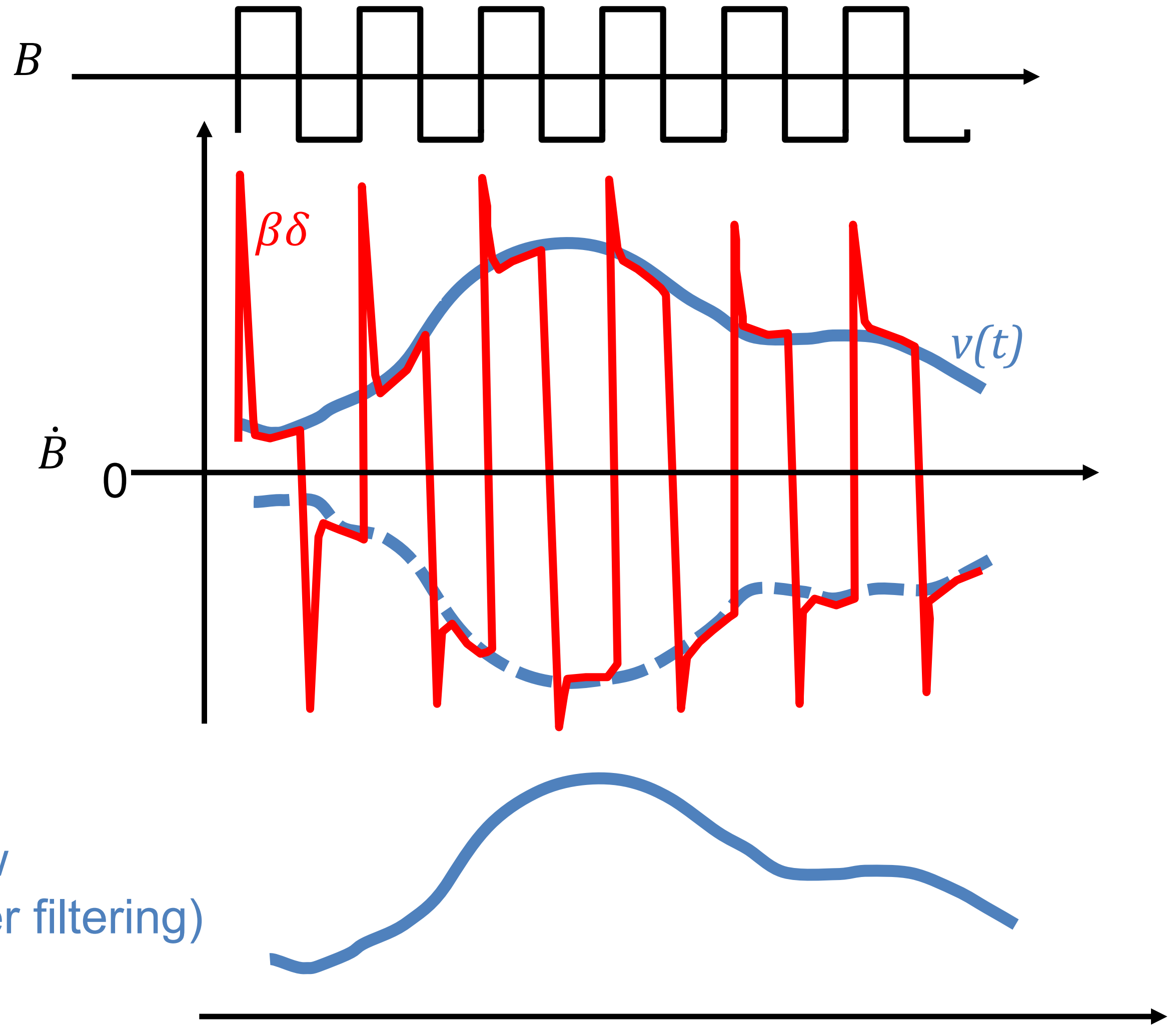
$e \propto B \cdot L \cdot v + u_i$



! Error on position of maximum

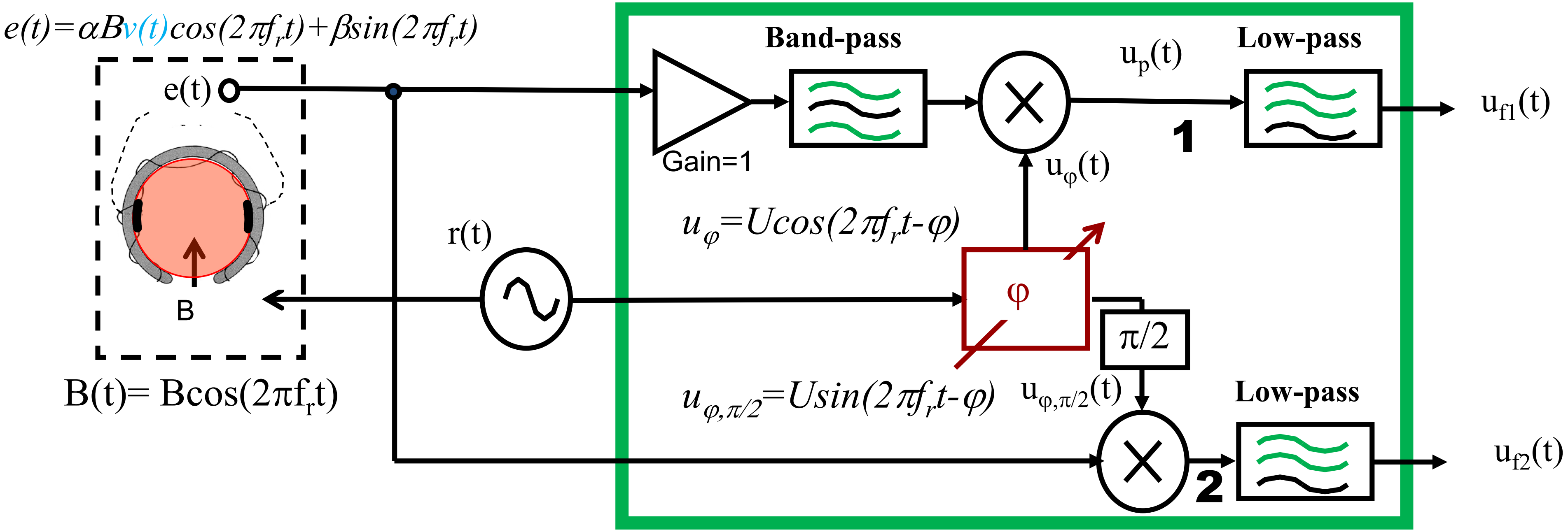
AC flow meter – square wave





$$e(t) = \alpha \cdot B \cdot v(t) + \beta \cdot \dot{B}$$
$$= \alpha' \cdot \varepsilon \cdot v(t) + \beta \cdot \delta$$
$$\varepsilon = \pm 1$$

Double phase Lock-in amplifier

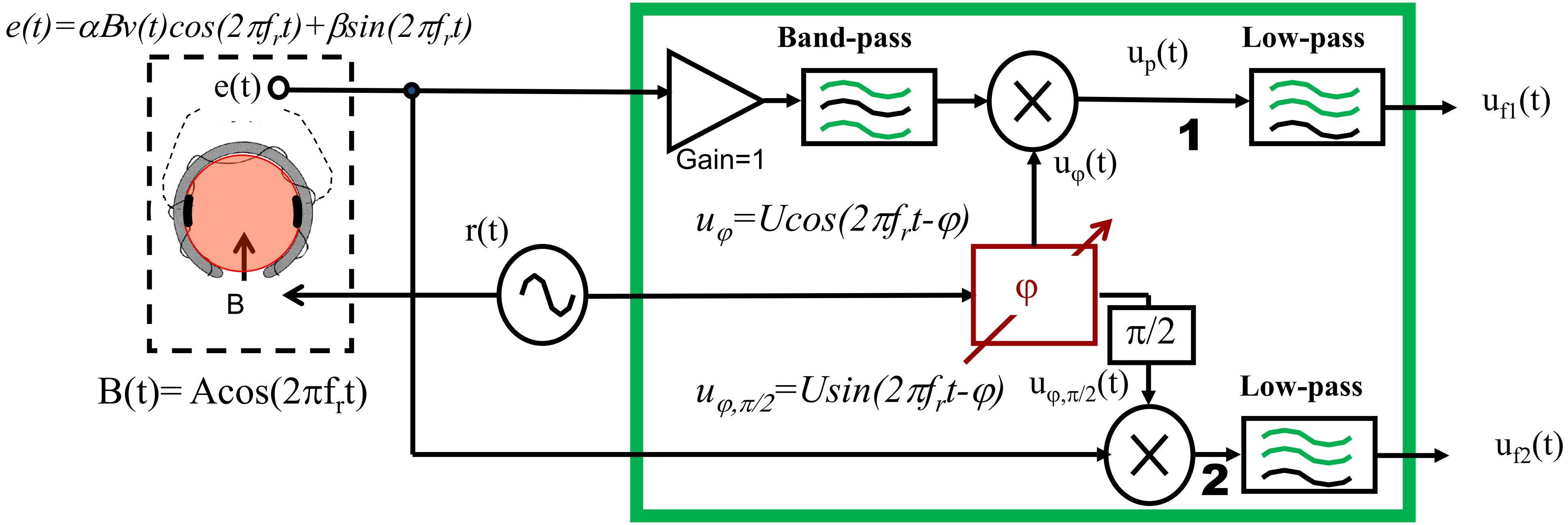


1: $e(t).u_{\varphi} = \alpha B U v(t)\cos(2\pi f_r t)\cos(2\pi f_r t-\varphi) + \beta U \sin(2\pi f_r t)\cos(2\pi f_r t-\varphi)$

$$= \frac{\alpha B U v(t)}{2}(\cos(4\pi f_r t+\varphi)+\cos(\varphi)) + \frac{\beta U}{2}(\sin(4\pi f_r t+\varphi)+\sin(\varphi))$$

After low pass filtering: $u_{f1}(t) = \frac{1}{2}\alpha B U v(t)\cos(\varphi) + \frac{\beta U}{2} sin(\varphi)$

Double phase Lock-in amplifier



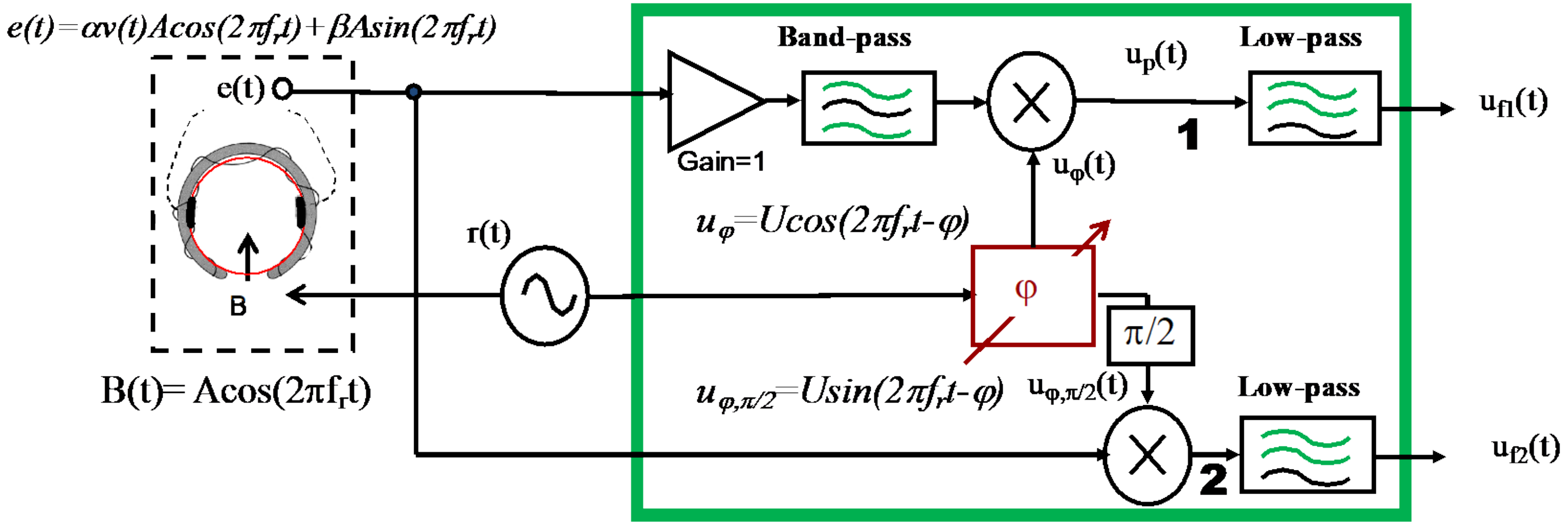
$$u_{f1}(t) = \frac{1}{2}\alpha B U v(t)\cos(\varphi) + \frac{\beta U}{2} sin(\varphi)$$

2: $e(t).\mathrm{u}_{\varphi,\pi/2} = \alpha B U v(t)\cos(2\pi f_r t)\sin(2\pi f_r t\text{- }\varphi) + \beta U \sin(2\pi f_r t)\sin(2\pi f_r t - \varphi)$

$$= \frac{\alpha B U v(t)}{2}(\sin(4\pi f_r t + \varphi) - \sin(\varphi)) + \frac{\beta U}{2}(-\cos(4\pi f_r t + \varphi) + \cos(\varphi))$$

After low pass filtering: $u_{f2}(t) = -\frac{1}{2}\alpha B U v(t)\sin(\varphi) + \frac{\beta U}{2}\cos(\varphi)$

Double phase Lock-in amplifier



$$u_{f1}(t) = \frac{1}{2}\alpha B U v(t)\cos(\varphi) + \frac{\beta U}{2} sin(\varphi)$$

$$u_{f2}(t) = -\frac{1}{2}\alpha B U v(t)\sin(\varphi) + \frac{\beta U}{2}\cos(\varphi)$$

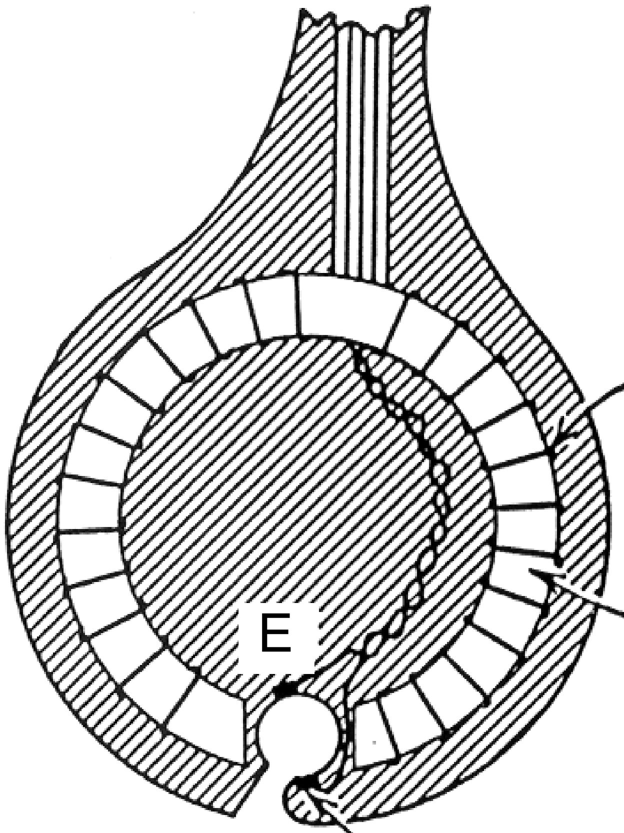
- We adjust the phase shifter in order to have the maximum amplitude for $u_{f1}(t)$.

$$\boldsymbol{u_{f1}(t) = \frac{1}{2}\alpha B U v(t)}$$

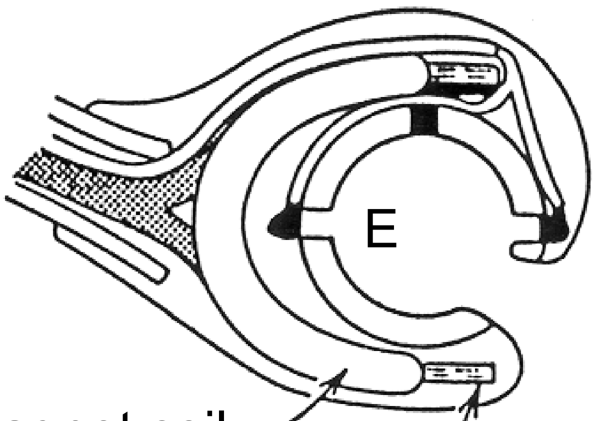
Noise→ $$u_{f2}(t) = \frac{\beta U}{2}$$

EPFL

Probes - Example

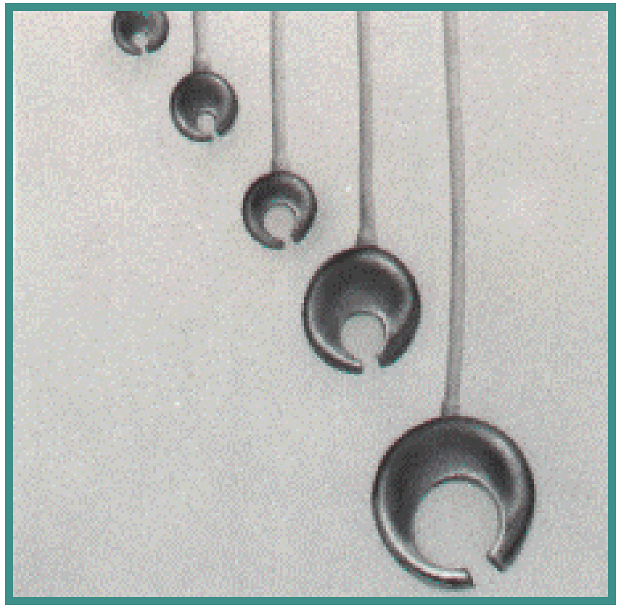


C-core



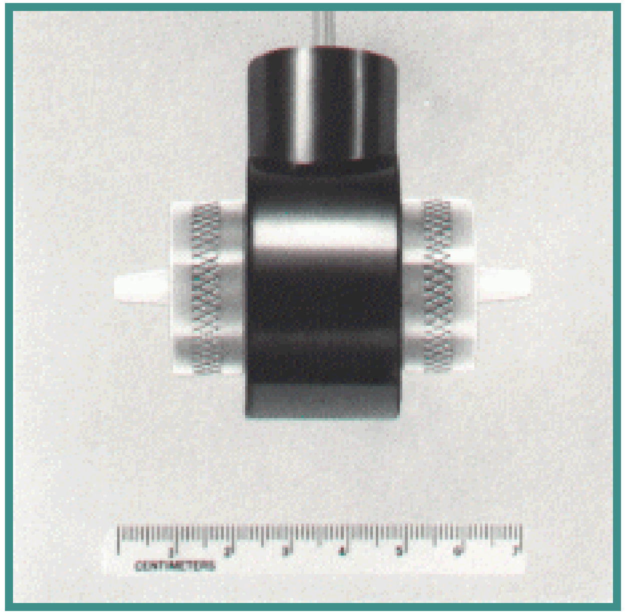
U-core

Probes - Example



Intracorporeal sensor

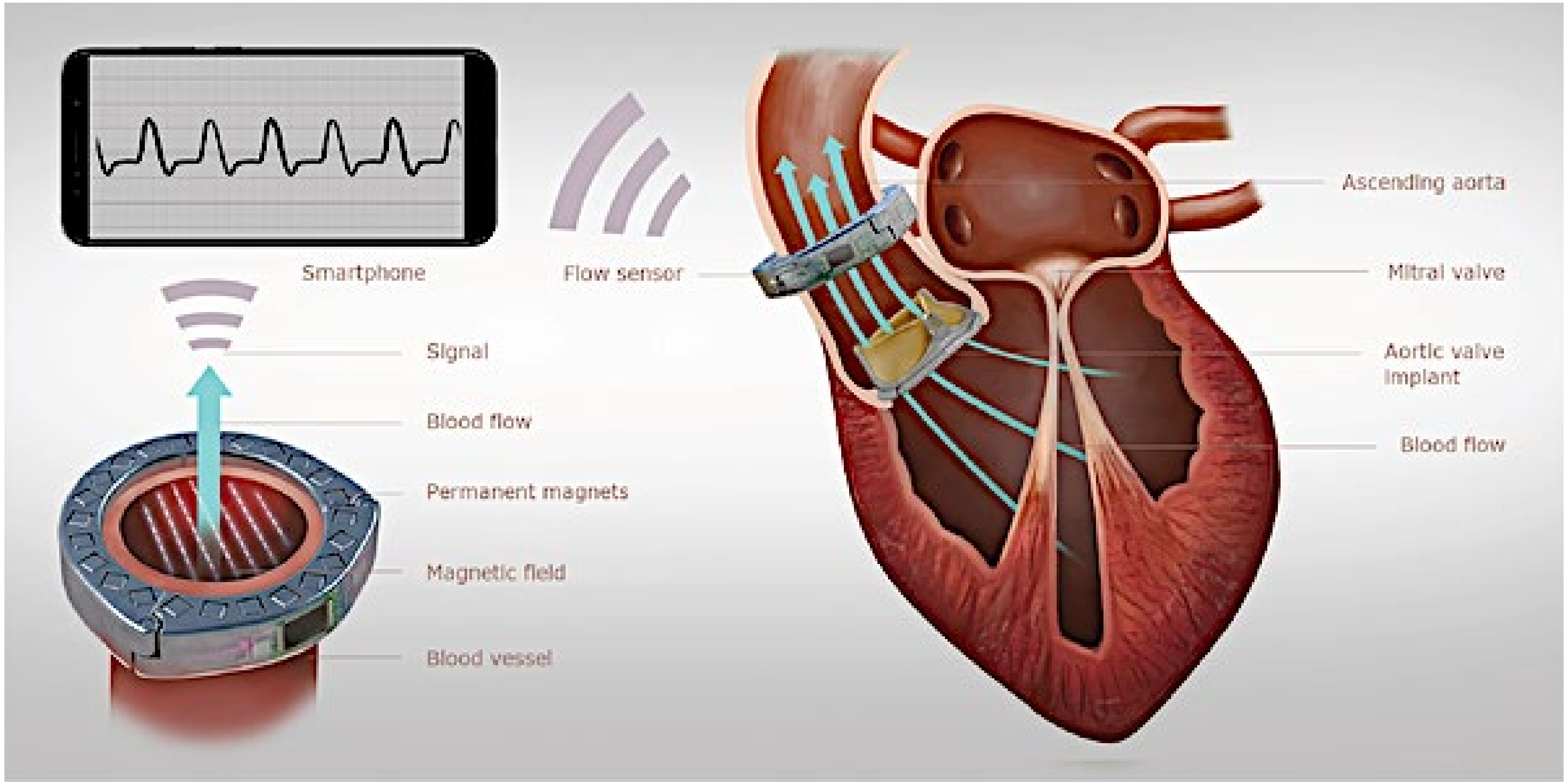
Internal dimension : 4 to 100 mm



Extracorporeal sensor

Measure in "bypass" or
by making perfusions
Dimension: 1/16" to 1/2"

A smartphone-enabled wireless and batteryless implantable blood flow sensor for remote monitoring of prosthetic heart valve function



Vennemann B, Obrist D, Rösgen T (2020). PLOS ONE 15(1): e0227372. https://doi.org/10.1371/journal.pone.0227372
https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0227372

EPFL