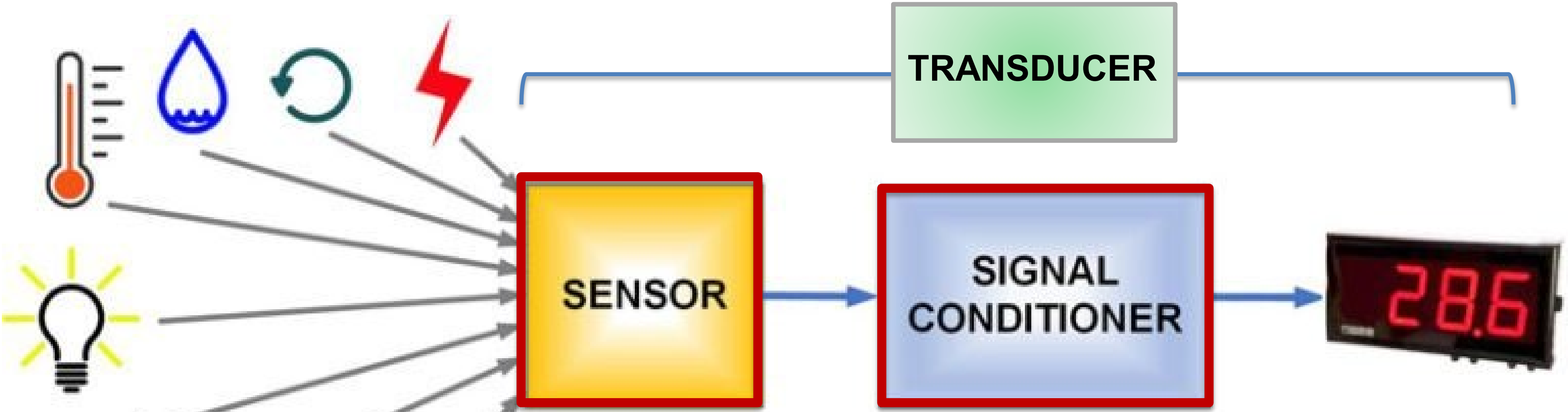
GENERAL INTRODUCTION

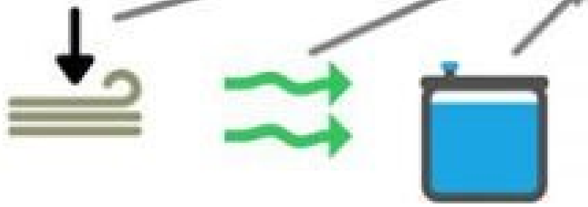
SENSORS

Definitions and Classification

EPFL

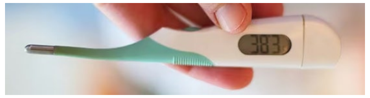
Definitions: Sensors & Transducers

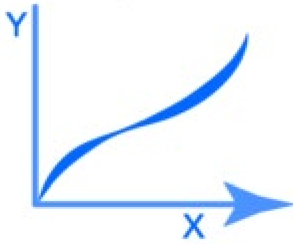


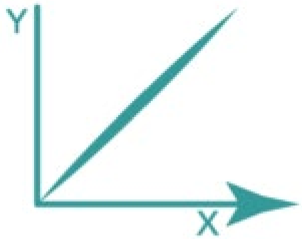


- voltage
  capacitance
  inductance
  resistance

- voltage







- measurand

Classification based on the power requirement

- **Active sensors:** Convert a form of energy into another form (electrical) **without using an external source of power.**

  Examples: Electrodes, Piezoelectric

- **Passive sensors:** Convert a form of energy into another form (electrical) by **making use of an external source of power**.

  Examples: Resistive, Inductive, Capacitive

Active sensors

Measured variable	Physical effect	Output
temperature	thermoelectricity (Peltier-Seebeck effect)	voltage
biopotential, pH	redox	voltage
force, pressure acceleration, vibrations, sound	piezoelectricity	charge
speed, flow	magnetic induction	voltage
optical radiation flux	photovoltaic	voltage

Passive sensors

Impedance

Measured variable	Sensitive characteristic	Comment
temperature	resistance	semiconductor: thermistor, metal: Pt, Ni
Strain- deformation	resistance	Strain gage: metal, semiconductor
force, pressure, acceleration, vibrations, sound, displacement	resistance, capacitance, inductance	potentiometer, microphone LVDT
humidity	resistance, capacitance	

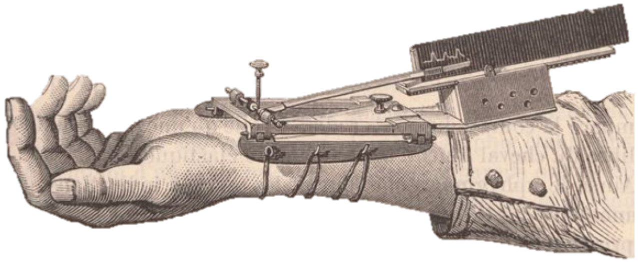
RESISTIVE SENSORS
& Applications

Introduction

Part I- Temperature sensors
Part II- Thermal mass flowmeter

Part III- Strain gage
Part IV- Direct measurement of arterial pressure
Part V- Indirect measurement of arterial pressure

Part VI- Force measurement: instrumented implant
Part VII- Force plate: application in biomechanics
Part VIII- Wearable force measurement: gait analysis



Sphygmograph de Marey, 1878

EPFL

Introduction: Theory of operation

The resistance of a material depends on four factors:

- Composition
- Temperature
- Length
- Cross-sectional Area

$$R = \rho \frac{l}{A}$$

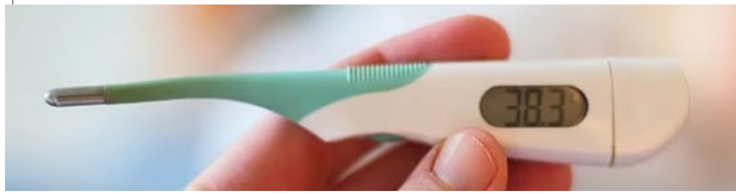
Possibility to measure:

- Temperature
- Deformation (due to Pressure, Force)

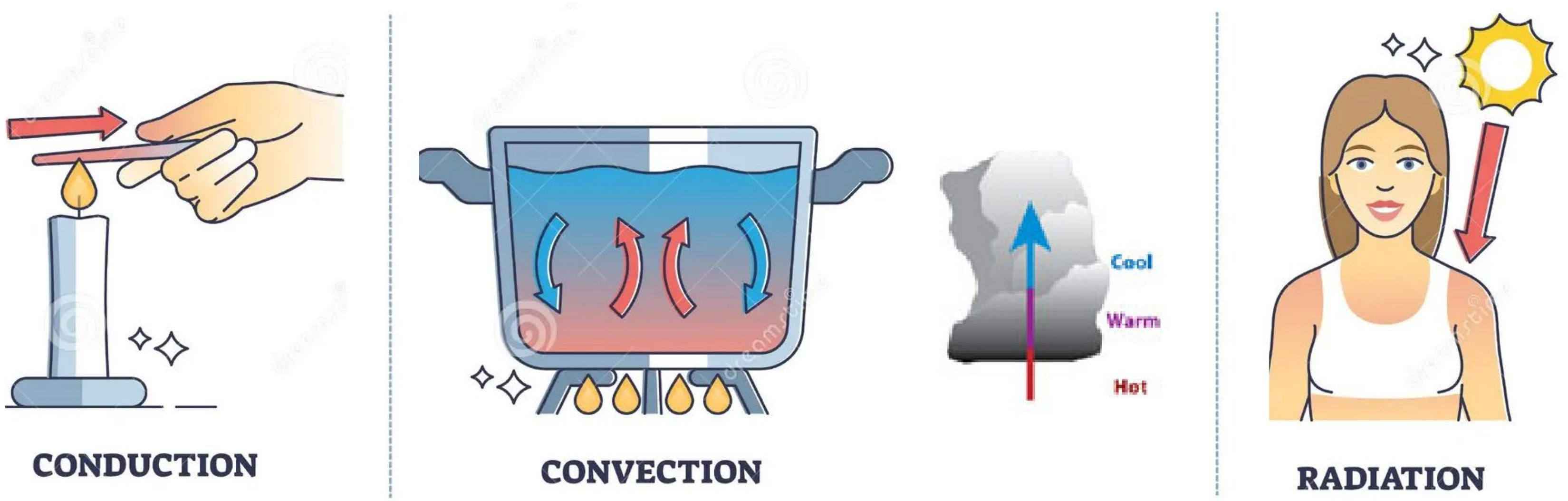
- When the length l of a conductor changes, the resistance varies directly. Conversely, modifying the cross-sectional area A leads to an inverse change in resistance.
- But, **alterations in material composition or temperature affect resistivity ρ in a more complex manner.**

RESISTIVE SENSORS

Part I- Temperature sensors



Heat transfer



- Heat is transferred via solid material contact (conduction), liquids and gases (convection), and electromagnetic waves (radiation)
- Heat will flow from one object (milieu) to the other if the two objects are at **different temperature** .

Temperature measurement

$R = \rho \frac{l}{A}$

- Relation resistance ← temperature

  $R(T) = R(T_0) f(T - T_0)$, with $f(T - T_o) = 1$ for $T = T_o$

  f is generally a non-linear function depending on the thermosensitive element

- **Metallic resistance** (RTD: resistance temperature detector)

  Platinum:

  Linear approximation

  $$\boxed{R(T) = R(T_o) \cdot (1 + a(T - T_o)} + b(T - T_o)^2 + c(T - T_o)^3)$$

- **Oxide semiconductor**

  Thermistor:

  $T : °K$

  $T_o = 273°K$

  $\beta \cong 4000°K$

  $$R(T) = R(T_0) e^{\beta\left(\frac{1}{T} - \frac{1}{T_o}\right)}$$

- *R(T)* is the resistance at temperature *T* (K)
- *R(T_0)* is the resistance at temperature T_0 (K)
- T_0 is the **reference** temperature (normally 273K or 25^{o}C, but can differ according to the application)
- β is a constant, its value depends on the characteristics of the material.

Metallic resistance (RTD)

$$R(T) = R(T_o)\cdot(1+\alpha_R \Delta T)$$

$$\alpha_R = \frac{1}{R(T_0)}\frac{\Delta R}{\Delta T}$$ Temperature coefficient

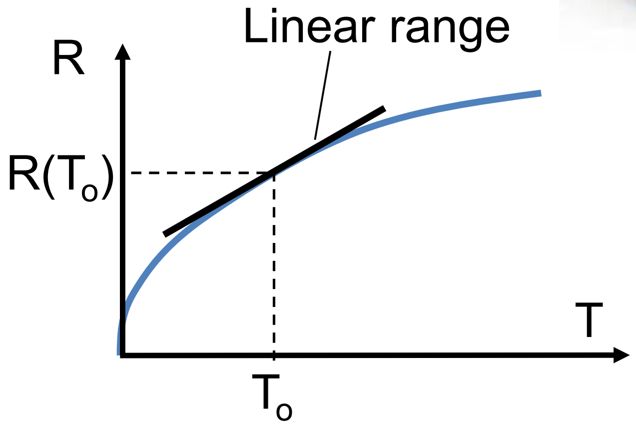
- Example
  - ***Platinum***

    $$\alpha_R = 3.9\cdot 10^{-3}\,/\,°C$$

    Pt100 sensor : $R(0°C) = 100\Omega$

    $R(37°C) = 114.4\Omega$

    - Linear
    - Accurate
    - Reproducible (precise)
    - Large measurement span



R(37°C)= R(0°C)(1+3.9x10^{-3}x37)=100(1.14)=114Ω

Oxide semiconductor

$$R(T) = R(T_0)e^{\beta\left(\frac{1}{T}-\frac{1}{T_o}\right)}$$

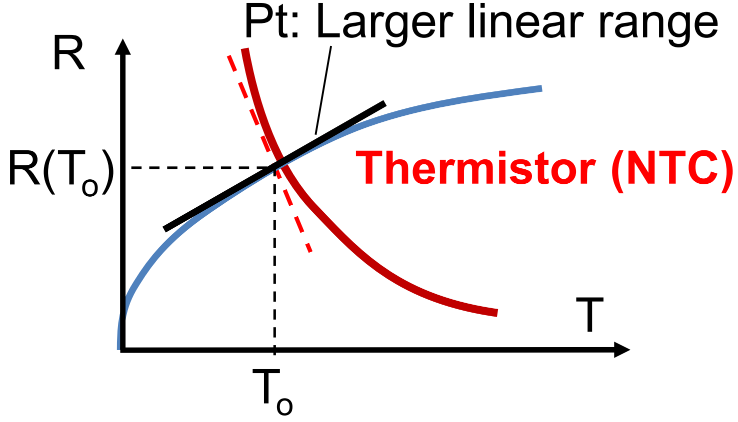
$$\alpha_R = \frac{1}{R}\frac{dR}{dT} = -\frac{\beta}{T^2}$$

- Example
  - ***Thermistor***

    For $T_o = 310°K$

    $\alpha_R = -0.03 \text{ to } -0.05/°C$ (negative)

    - Small volume
    - High sensitivity
    - Non-linear
    - Low measurement span



NTC: Negative Temperature Coefficient

For Positive temperature coefficient (**PTC**) thermistors when temperature increases, the resistance increases

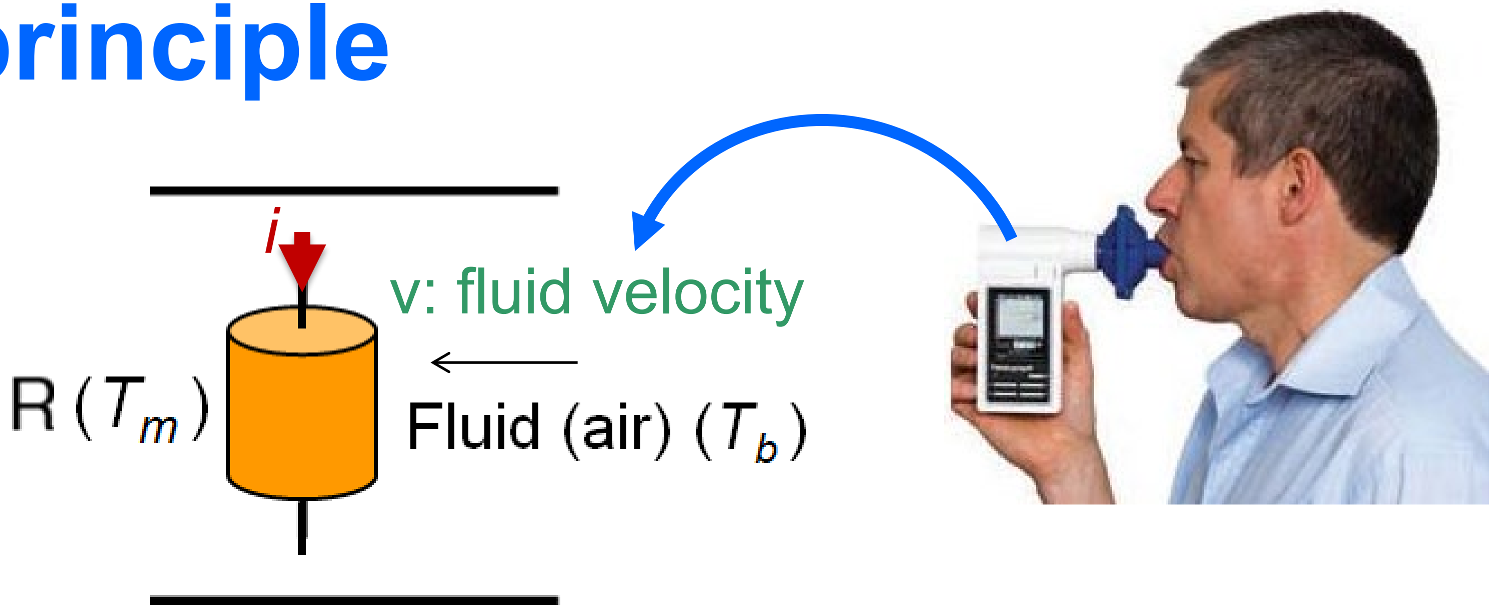
RESISTIVE SENSORS

Part IIa- Thermal mass flowmeter:

Constant current method

Measuring the flow

Basic principle



- R is heated to temperature T_m slightly *higher* than the temperature of the fluid T_b: $T_m > T_b$
- T_m varies (decreases) when the air is circulating into tube by breathing

Measuring the flow with *constant current supply*

Principle

1. The resistance is *heated* by *Joule effect* with constant current, *i* → thermal power:

$$P = R \cdot i^2$$

2. The resistance is *cooled* by the fluid circulation → heat transfer by *convection:*

$$P = A \cdot h \cdot (T_m - T_b)$$

A : sensor's lateral surface
h : heat transfer coefficient, *function of velocity v of fluid*
T_m: temperature of the resistance
T_b: temperature of the fluid

Measuring the flow with *constant current supply*

Principle (cont.)

- At equilibrium (i.e., no heat transfer):

$$A \cdot h \cdot (T_m - T_b) = R \cdot i^2$$

$$h = f(v) = \frac{R(T_m) \cdot i^2}{A \cdot (T_m - T_b)}$$

(T_m variable, T_b fixe)

$$T_m = f(v)$$

Temperature sensor

$$\Delta R = f(v)$$

- h varies mainly in a non-linear fashion as a function of velocity v (e.g. $h = a + b \cdot \log v$)

Measuring the flow with *constant current supply*

Signal conditioner

- If $v\uparrow \;\Rightarrow\; T_m\downarrow \;\Rightarrow\; R\uparrow$ (e.g. NTC thermistor)

- Dynamic situation (when *t* varies):

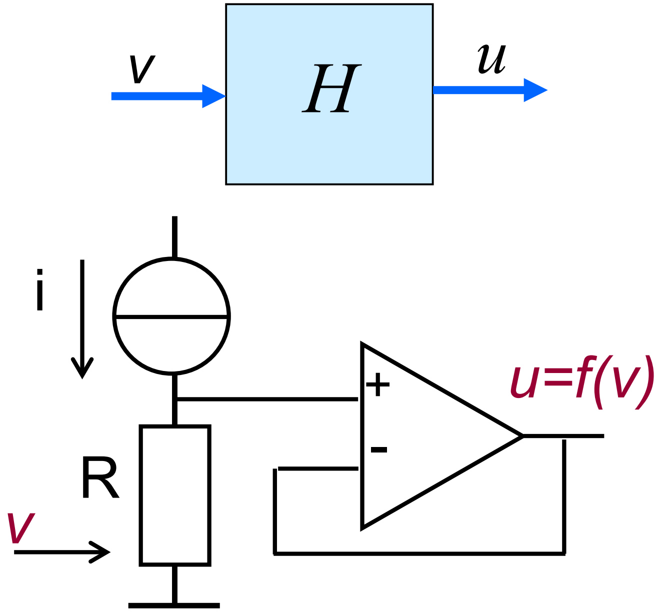
$$i^2(R+\Delta R) - Ah(T_m - T_b) = K\frac{dT_m}{dt}$$

K: thermal capacity

Assuming linear sensitivity around equilibrium:

$$h = h_o + K_v \cdot \Delta v \qquad K_v\text{: constant}$$

$$T_m = K_T(R+\Delta R) \qquad K_T\text{: constant}$$



Measuring the flow with *constant current supply*

Signal conditioner (cont.)

$$h = h_o + K_v \cdot \Delta v$$

$$T_m = K_T (R + \Delta R)$$

$$i^2 (R + \Delta R) - Ah(T_m - T_b) = K \frac{dT_m}{dt}$$

$$i^2 (R + \Delta R) - A(h_o + K_v \Delta v)(K_T R + K_T \Delta R - T_b) = K K_T \frac{d\Delta R}{dt}$$

$$\cancel{i^2 R} + i^2 \Delta R \cancel{- A h_o K_T R} - A h_o K_T \Delta R \cancel{+ A h_o T_b} - A K_v \Delta v K_T R - A K_v K_T \cancel{\Delta v \Delta R} + A K_v \Delta v T_b = K K_T \frac{d\Delta R}{dt}$$

$i^2 R - A h_o K_T R + A h_o T_b = 0$ (static condition)

$\Delta v \Delta R$: second order (can be negligible):

$$(i^2 - A h_o K_T)\Delta R - K \cdot K_T \frac{d\Delta R}{dt} = A K_v (K_T R - T_b)\Delta v$$

Measuring the flow with *constant current supply*

Signal conditioner (cont.)

- First order system:

$$(i^2 - Ah_o K_T)\Delta R - K \cdot K_T \frac{d\Delta R}{dt} = AK_v(K_T R - T_b)\Delta v$$

Laplace transform:

$$((Ah_o K_T - i^2) + sK \cdot K_T) \cdot \Delta R = AK_v(T_b - K_T R)\Delta v$$

$$\left(\left(Ah_o - \frac{i^2}{K_T}\right) + sK\right) \cdot \Delta R = AK_v(T_b - K_T R)\Delta v = k\Delta v$$

$$\frac{\Delta R}{\Delta v} = \frac{k}{1 + \tau_i s}$$

- Slow response !

$$\tau_i = \frac{K}{Ah_o - \dfrac{i^2}{K_T}}$$

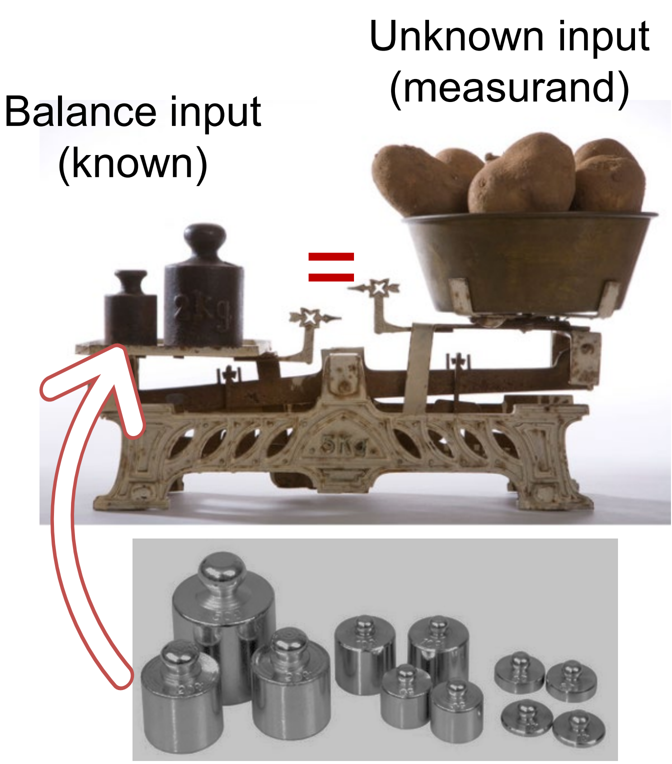
RESISTIVE SENSORS

Part IIb- Thermal mass flowmeter:

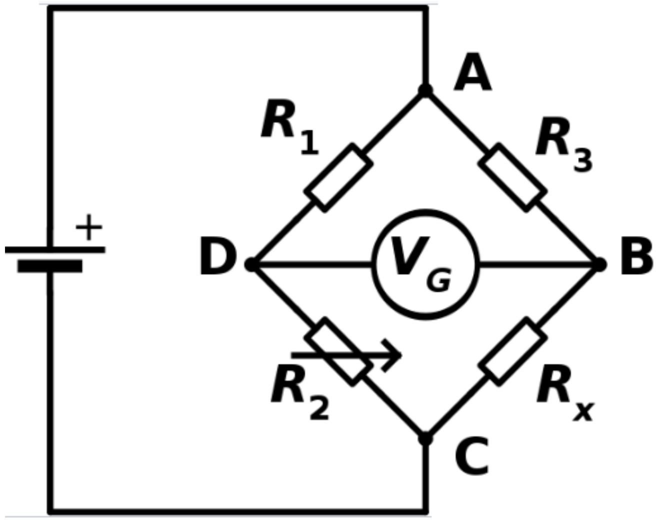
Constant temperature method

Null method:

- Exerts an influence on the measured system so as to oppose the effect of the measurand
- The influence and the measurand are balanced



Null method measurement circuit: Wheatstone bridge



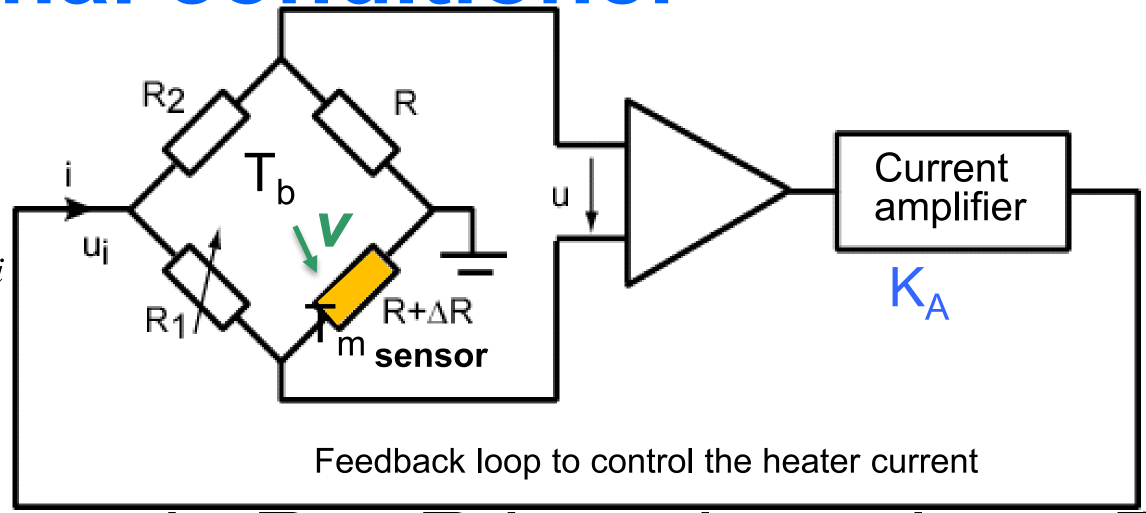
Example circuit:

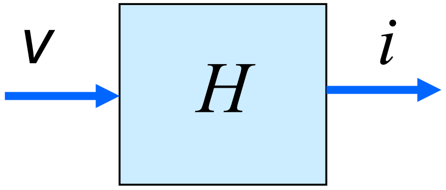
- Rx unknown (to be measured)
- R_1, R_2 and R_3 known, R_2 adjustable until . $V_G = 0$
- When the measured voltage $V_G = 0$, both legs have equal voltage ratios: $R_2/R_1=Rx/R_3$, → $Rx=R_3R_2/R_1$

Measuring the flow with T_m-T_b= constant

Principle & Signal conditioner

$$u = \frac{R \cdot R_1 - (R + \Delta R)R_2}{(R + R_2)(R + \Delta R + R_1)} u_i$$





- Adjusting the current in R+ΔR in order to have T_m-T_b=*constant* by Joule effect (T_m>T_b)
- $i = f(v)$ → measuring the current i
- if $v\uparrow$: R+ΔR is cooled→ increase i to heat R+ΔR $\Rightarrow i\uparrow$

$$v\uparrow \quad \Rightarrow \quad i\uparrow$$

Measuring the flow with T_m-T_b= constant

Principle (cont.)

- Balance the bridge in the absence of fluid circulation (Δv=0):
  - Slightly increase the potentiometer value R_1

$$\Rightarrow u\uparrow \Rightarrow i\uparrow \Rightarrow T_m\uparrow \Rightarrow R_c\uparrow \text{(Platinum)}$$

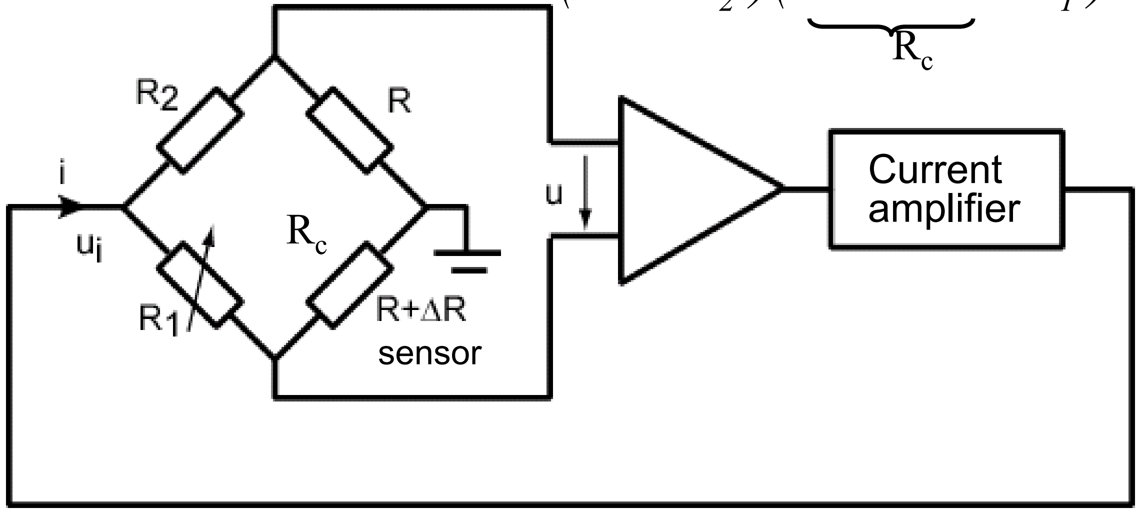
  - Bridge balanced again with $T_m = T_b + T_o$ and $u=0$

$$u = \frac{R\cdot R_1 - (R+\Delta R)R_2}{(R+R_2)(\underbrace{R+\Delta R}_{R_c}+R_1)}u_i$$

- $\Delta v \neq 0$

$$v\uparrow \Rightarrow T_m\downarrow \Rightarrow R_c\downarrow$$

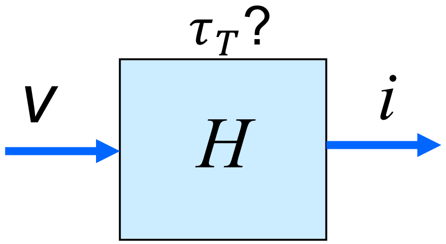
$$\Rightarrow u\uparrow \Rightarrow i\uparrow \Rightarrow T_m\uparrow$$



Measuring the flow with T_m-T_b= constant

Principle (cont.)

- The resistance varies with i and v : $\Delta R = f(\Delta i, \Delta v)$: $\Delta R = \frac{K_i}{1+\tau_i s}\Delta i - \frac{K_v}{1+\tau_i s}\Delta v$
- Wheatstone bridge of gain **K_B**: $\Delta R = -\frac{\Delta u}{K_B}$
- Current amplifier of gain (transconductance) **K_A**:$\Delta i = K_A \cdot \Delta u$
- $\Delta R = -\frac{\Delta i}{K_A \cdot K_B}$
- Time constant of the measurement system i=f(v):



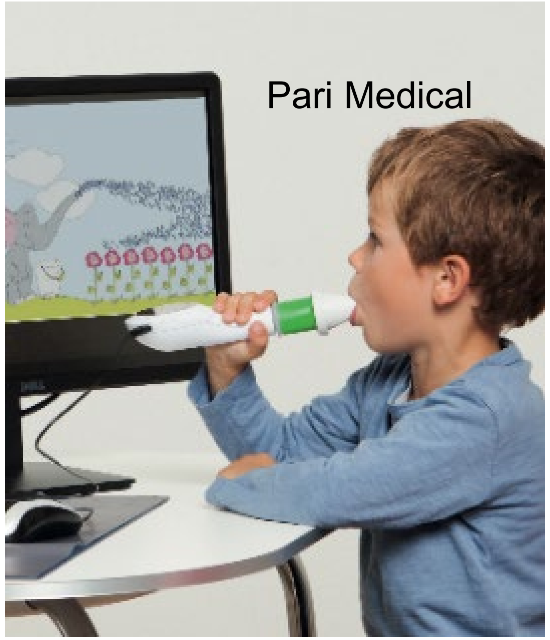
$$\left(\frac{1}{K_A \cdot K_B} + \frac{K_i}{1+\tau_i s}\right)\Delta i = \frac{K_v}{1+\tau_i s}\Delta v$$

$$\boxed{\tau_T = \frac{\tau_i}{1 + K_i \cdot K_A \cdot K_B}} \quad << \tau_i \;!$$

→**System is much faster than the circuit with constant current**

Measuring the flow

- Example: Flowmeters to measure respiration (spirometer)



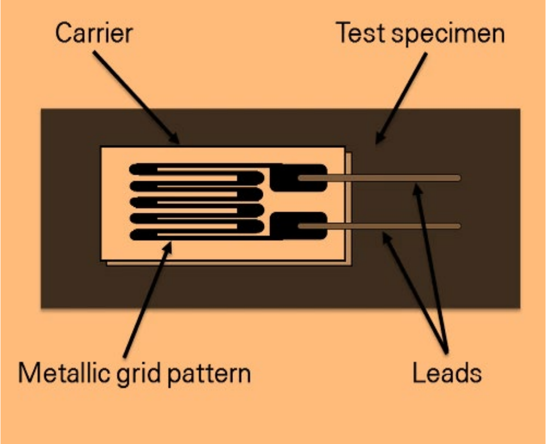
Spirometers are used to diagnose and assess a number of conditions and diseases, for example:

- **Asthma** – an obstructive lung disease in which the airways become periodically swollen and narrowed.
- **Chronic Obstructive Pulmonary Disease (COPD)-** lung conditions that narrow the airway and make it difficult to breathe.
- **Cystic fibrosis**– a degenerative condition in which the lungs and digestive system become clogged with thick, sticky mucus.
- **Pulmonary fibrosis**– scarring of the lungs caused by pollutants, medications and interstitial lung disease.

RESISTIVE SENSORS

Part III- Strain gage (gauge)

Strain gage



Strain gage: resistive property

- Converts a mechanical elongation/displacement produced by a force in its corresponding change in resistance

- Conductor of length *l* and cross-section surface *A*:

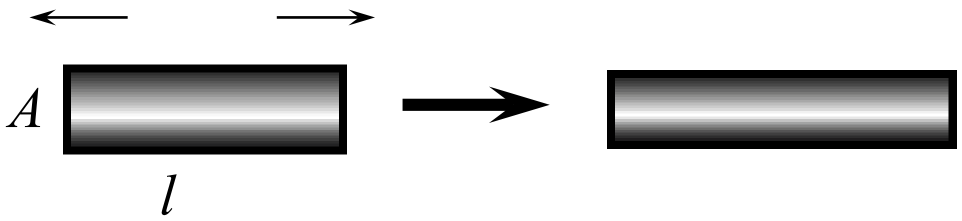
  R : resistance

  ρ : resistivity

  l : length

  A : section area

$$R = \rho \frac{l}{A}$$



$$R_2 = R_1 + \Delta R$$

$$R_1 < R_2$$

$$R_1 = \rho \frac{l}{A}$$

$$R_2 = \rho \frac{l + \Delta l}{A - \Delta A}$$

Strain gage: mechanical property

- Stress: $\sigma = \frac{F}{A}$

F: force
A: area

Stress measures the internal pressure experienced by a material when subjected to a force. A larger force or smaller area increases the likelihood of deformation (shape change)

- Strain: $\varepsilon = \frac{\Delta l}{l}$

Strain is the deformation resulting from stress. When a material experiences stress, it typically lengthens if pulled or shortens if compressed.

- Strain in direction of the stress:

$$\varepsilon_{\parallel} = \frac{\sigma}{Y}$$

Y: Young's modulus

- Strain perpendicular to the stress:

$$\varepsilon_{\perp} = -\nu\varepsilon_{\parallel}$$

ν: Poisson's ratio

Strain gage response

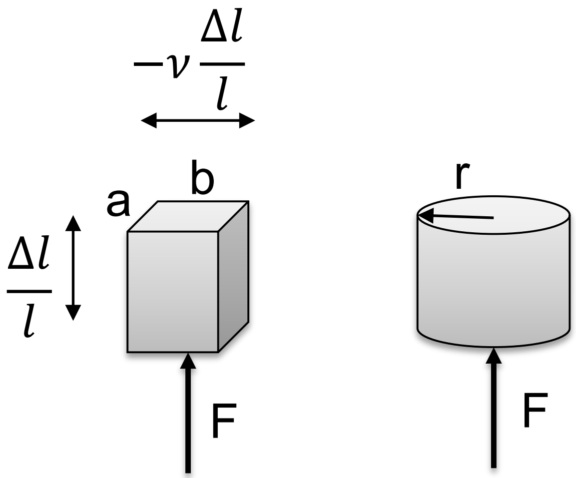
- $R = \rho \frac{l}{A}$

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A}$$

- The surface is mainly rectangular or circular

$$A = a \cdot b \; or \; A = \pi r^2$$

- $\frac{\Delta A}{A} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \; or \; \frac{\Delta A}{A} = 2\frac{\Delta r}{r} \Rightarrow \frac{\Delta A}{A} = -2\nu\frac{\Delta l}{l}$



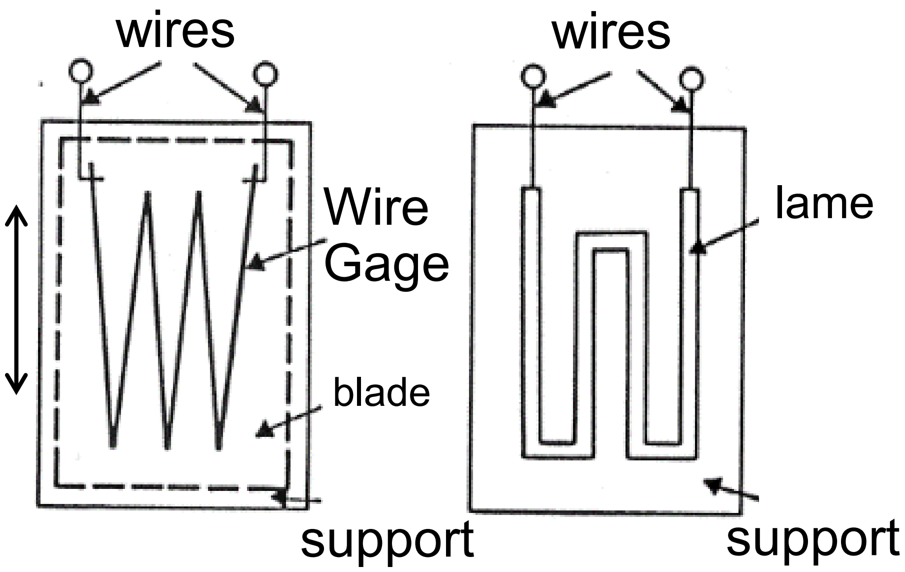
$$\boldsymbol{\frac{\Delta R}{R} = (1 + 2\nu)\frac{\Delta l}{l} + \frac{\Delta\rho}{\rho}}$$

Metallic conductor

$$\frac{\Delta\rho}{\rho} = C\frac{\Delta V}{V} = C(1-2\nu)\frac{\Delta l}{l}$$

V : volume ($V = A \cdot l$)
C : Bridgman correction factor

$$\frac{\Delta R}{R} = (1+2\nu)\frac{\Delta l}{l} + \frac{\Delta\rho}{\rho}$$



- The relative variation of the gage resistance is:

$$\frac{\Delta R}{R} = \{(1+2\nu)+(1-2\nu)C\}\frac{\Delta l}{l} = K\frac{\Delta l}{l}$$

$$K = (1+2\nu)+(1-2\nu)C$$

$$\boxed{\frac{\Delta R}{R} = K\frac{\Delta l}{l}}$$

  - K : gage factor
    (for metals $C \cong 1$ and $\nu \cong 0.3$ and $K \cong 2$)

Semiconductor

- Resistivity changes with doping: p-type (deficit of electron) or n-type (excess of electron)

$$\frac{\Delta R}{R} = (1+2\nu)\frac{\Delta l}{l} + \frac{\Delta \rho}{\rho}$$

- $$\frac{\Delta\rho}{\rho} = \pi \cdot \sigma = \pi \cdot Y \frac{\Delta l}{l}$$

  π : piezoresistivity coefficient

- The relative variation of the gage resistance is :

$$\frac{\Delta R}{R} = \{(1+2\nu) + \pi \cdot Y\} \cdot \frac{\Delta l}{l}$$

$$K = (1+2\nu) + \pi \cdot Y \approx \pi \cdot Y$$

$$\frac{\Delta R}{R} = K \frac{\Delta l}{l}$$

- $K \approx 100 - 200$ for a semiconductor gage

In summary

$$\frac{\Delta R}{R} = K\frac{\Delta l}{l}$$

- Metallic strain gage
  - More sensitive to mechanical strain
  - K=2 to 4

- Semiconductor strain gage
  - More sensitive to resistivity (piezoresistif effect)
  - K=100 to 200

Temperature effect

$R(T_1)=R(T_o)(1+\alpha_R(T-T_o))$

- Thermal expansion of the gage: $l_j=l_{jo}(1+\lambda_j(T-T_o))$
- Thermal expansion of the support: $l_s=l_{so}(1+\lambda_s(T-T_o))$
- At T_o, $l_{jo}=l_{so}=l_o$; At T, $l_s-l_j=l_o(\lambda_s-\lambda_j)\Delta T$ with $\Delta T=T-T_o$

$$\left.\frac{\Delta R}{R}\right|_{expansion} = K\frac{l_s-l_j}{l_o} = K(\lambda_s-\lambda_j)\cdot\Delta T$$

- $$\left.\frac{\Delta R}{R}\right|_T = \{\alpha_R + K(\lambda_s-\lambda_j)\}\cdot\Delta T = \beta\cdot\Delta T$$
  - α_R : temperature coefficient , ° C^{-1}
  - λ_S : expansion coefficient of support, ° C^{-1}
  - λ_j : expansion coefficient of gage, ° C^{-1}

Temperature effect – Comparison

$$\left.\frac{\Delta R}{R}\right|_T = \left\{\alpha_R + K(\lambda_s - \lambda_j)\right\} \cdot \Delta T = \beta \cdot \Delta T$$

Metallic strain gages	Semiconductor strain gages (piezoresistive)
$\alpha_R = 0.01 \text{ to } 0.04\,\% / ^\circ C$ relatively low If $\lambda_S - \lambda_j$ low, β stays low Compensation for temperature for $\beta > 1.5 \cdot 10^{-6} / ^\circ C$	– α_R high - K varies with temperature Compensation for temperature mainly necessary $\lambda_j = 3.2 \cdot 10^{-6} / ^\circ C << \lambda_S$ - Sensors integrated in a silicon support $\lambda_S - \lambda_j = 0$

Comparison

Strain Gage	Metal	Semi-conductor
Linearity	High	Medium
Sensitivity	Low	High
Thermal sensitivity	Low	High

RESISTIVE SENSORS

Part IV- Direct measurement of arterial pressure

(Strain gage application)

Measuring arterial blood pressure

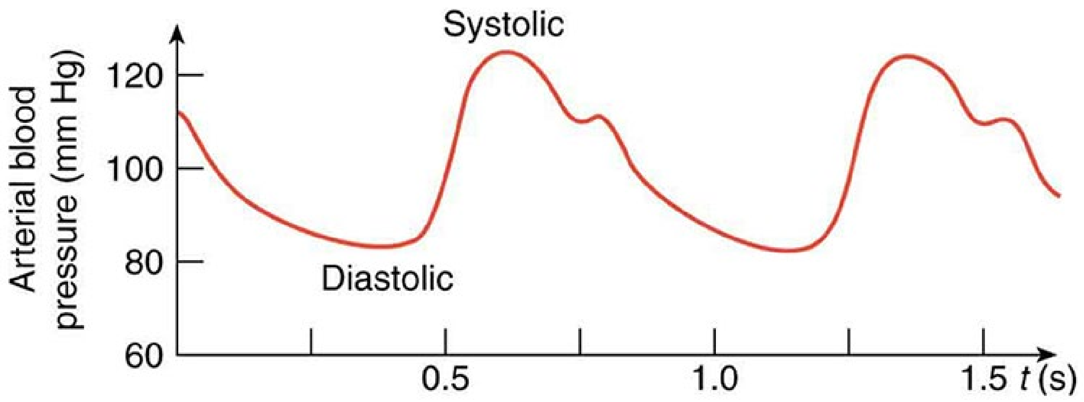
Direct measurement

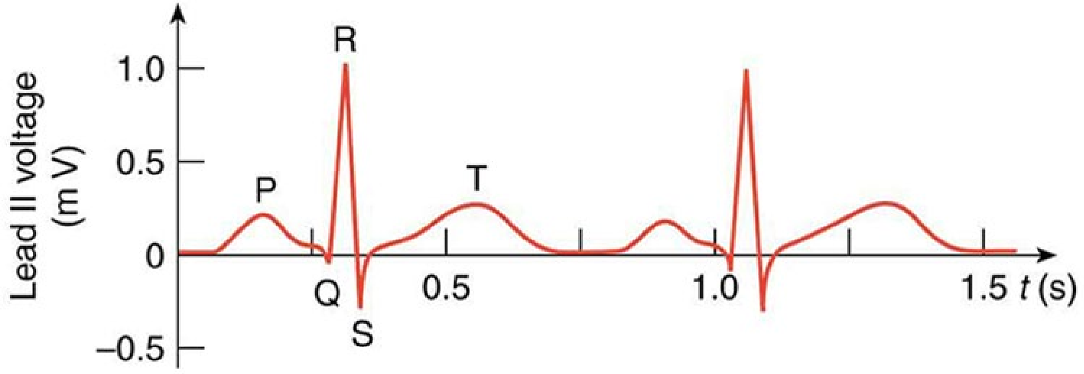
- **Extravascular** sensor: the vascular pressure is coupled via a catheter (filled with liquid) to a pressure sensor located outside the body
  Long response time

pressure | liquid | sensor

- **Intravascular** sensor : "Catheter-tip sensors",
  the sensor is located at the extremity of the catheter (e.g. in direct contact with the blood)
  Short response time

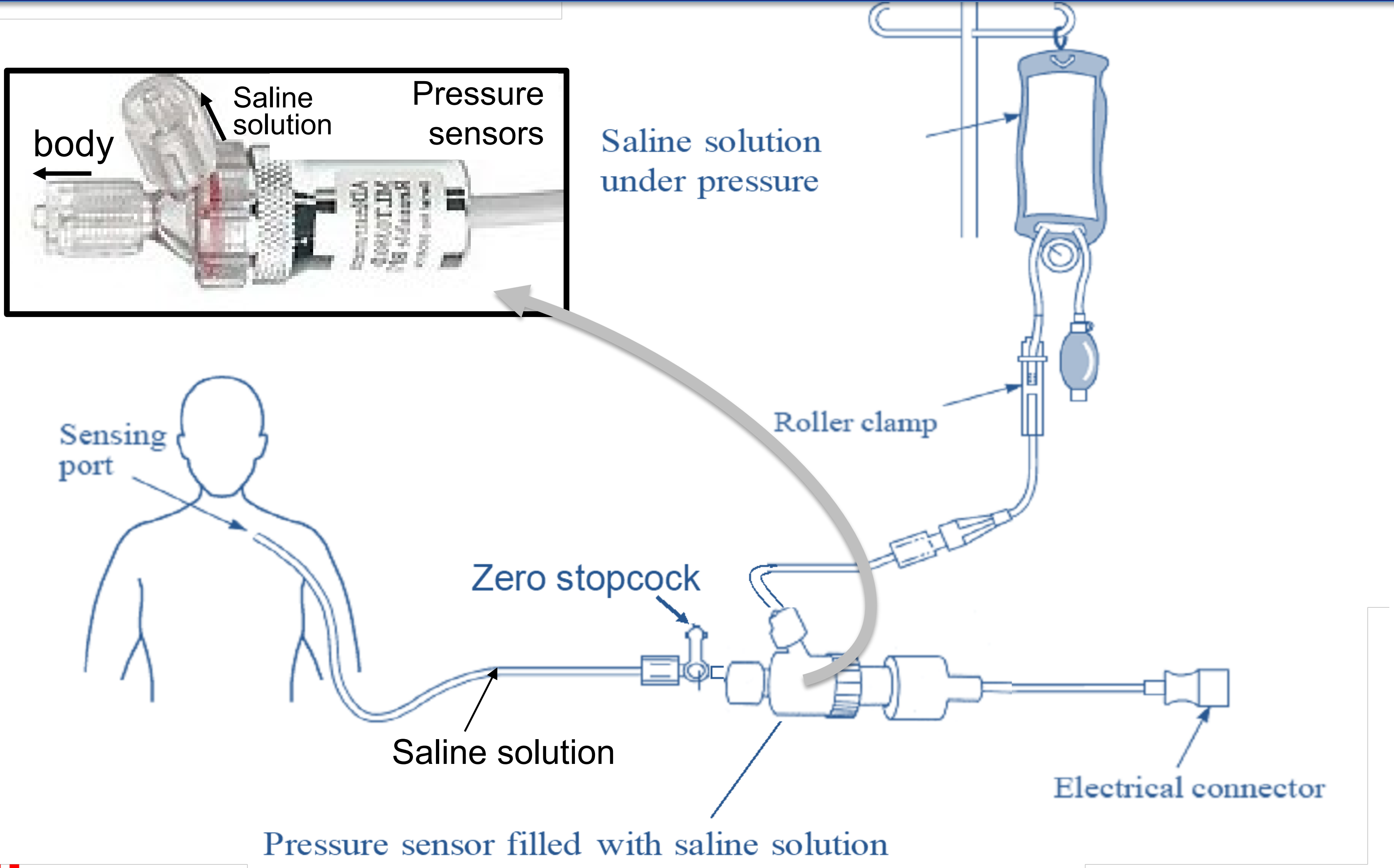
pressure | sensor



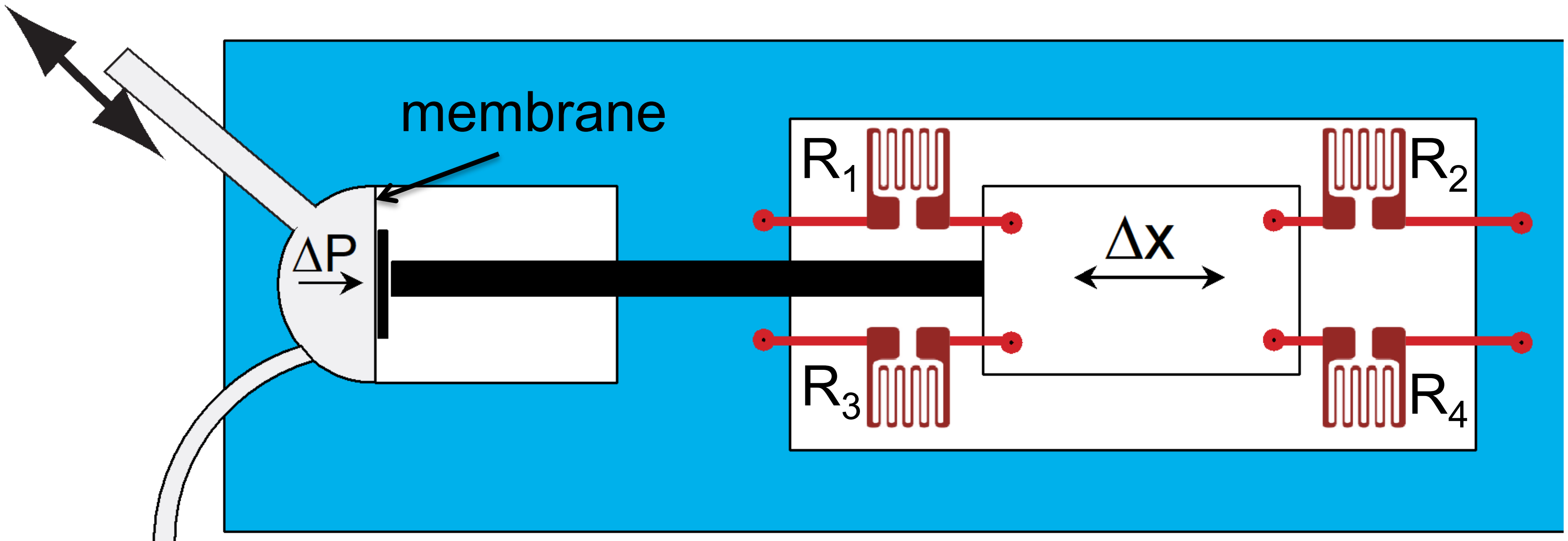


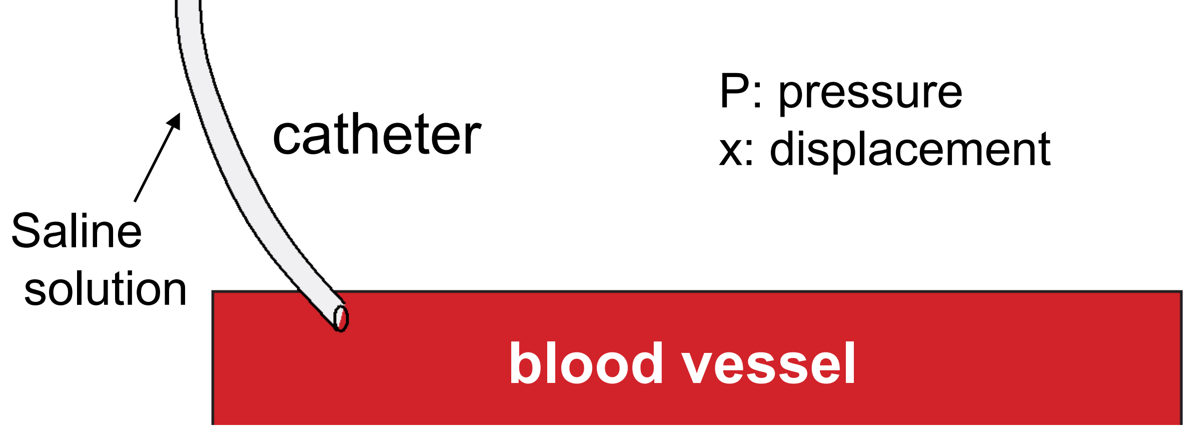
systolic pressure: maximum
diastolic pressure: minimum

Extravascular sensor



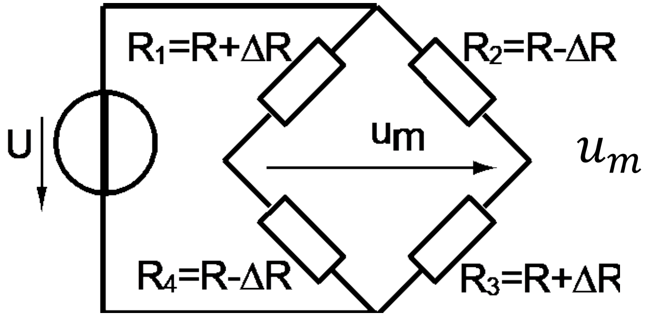
Extravascular sensor





P: pressure
x: displacement

Conditioning circuit: Wheatstone bridge

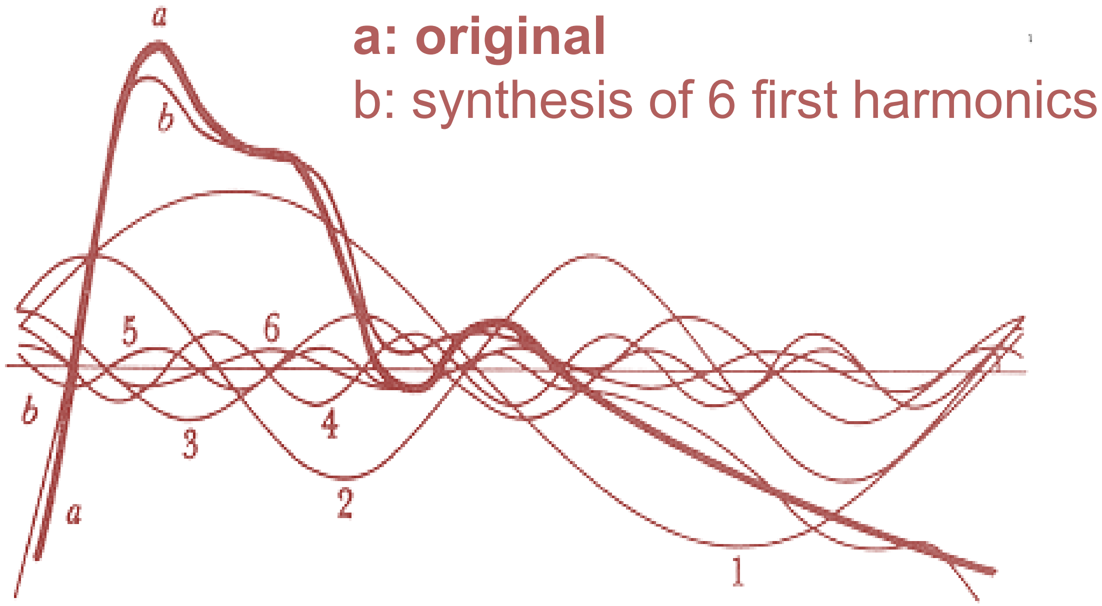


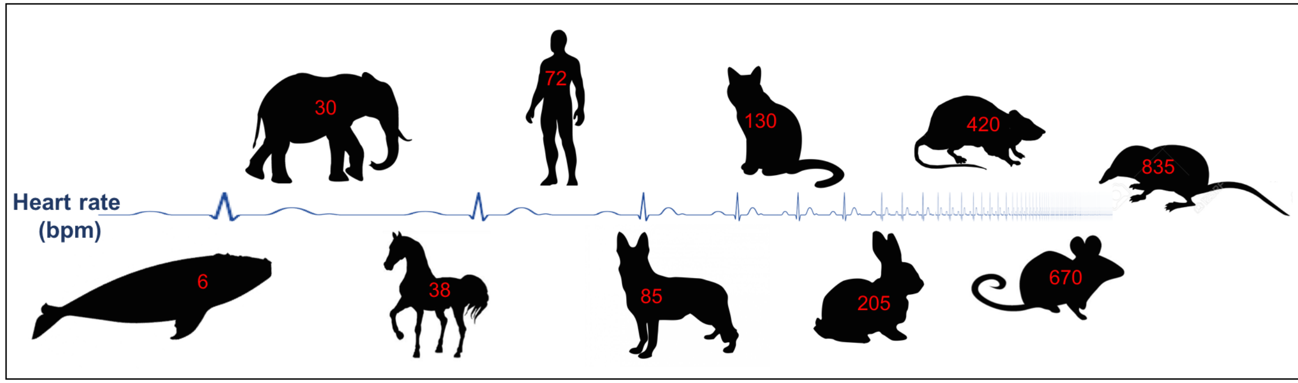
$$u_m = U\frac{\Delta R}{R}$$

$$\frac{\Delta R}{R} = K\frac{\Delta x}{x} = K'\frac{\Delta P}{P}$$

Dynamic properties of pressure measurement systems

- Arterial pressure bandwidth in humans: 0 to 50 Hz





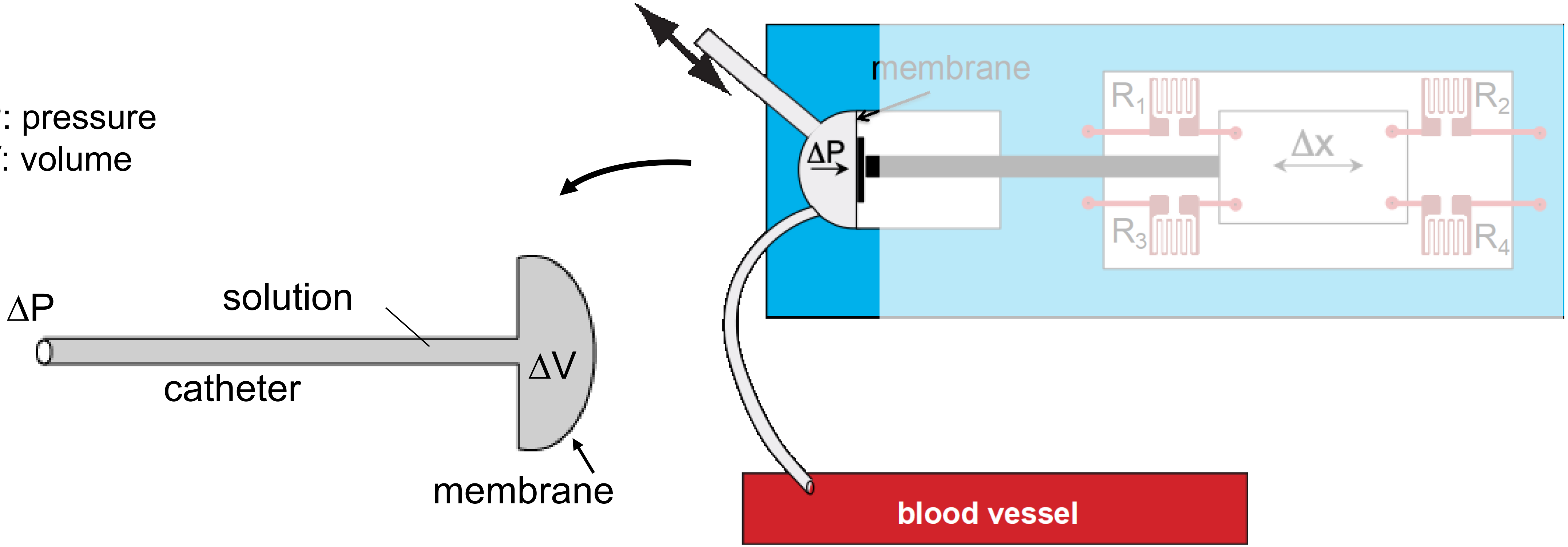
- Need for transfer function estimation of the system

Modelling the catheter – liquid – membrane system

analogy

- Hydraulic system ⟶ electrical system

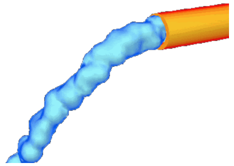
P: pressure
V: volume



Analogous Variables: Electrical-Hydraulic

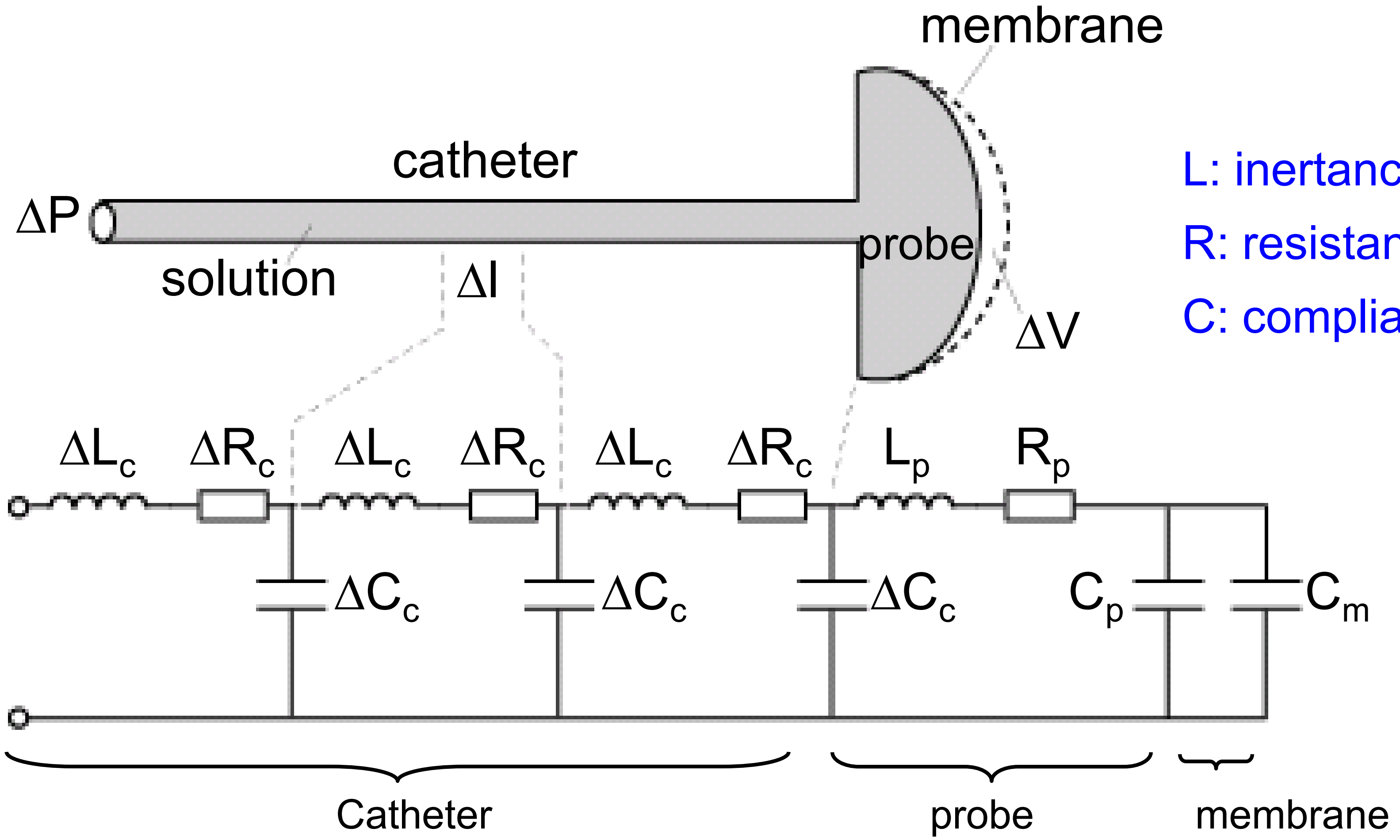
$power \leftarrow \times$

Quantity	Electrical System	Pressure System
Flowing quantity	**Current** (Amps = Coul/s)	**Flow** (mL/s)
Volume quantity	**Charge** (Coul)	**Volume** (mL)
Energy Density	**Volt** (Joule/Coul)	**Pressure** (kPa) (Pa=F/Area = J/Vol)
Energy Density to maintain Flow	**Resistance** (Ohms = Volt/Amps)	**Resistance** kPa / (mL/s)
Volume maintained by Energy Density	**Capacitance** (Farads = Coul/Volt)	**Compliance** mL / kPa
Energy Density to maintain d/dt (Flow)	**Inductance** (Henry = Volt/(Amps/s))	**Inertance** (kPa / (ml/s^2)



Modelling the catheter – liquid – membrane system

- Hydraulic system $\xrightarrow{\text{analogy}}$ electrical system



L: inertance

R: resistance

C: compliance

Modelling the catheter – liquid – membrane system

- Measure of energy density: Pressure (ΔP)→voltage
- Measure of flow quantity: Flow (Q)→current
- The resistance of the catheter liquid is:

$$R_c = \frac{\Delta P}{Q} = \frac{\Delta P}{v \cdot A} \quad Pa \cdot s / m^3 \qquad (Q = v \cdot A)$$

  - v : average speed of liquid
  - A : cross-sectional area of catheter

- Laminar flow, the Poiseuille equation*:
  - η : viscosity of the liquid
  - l_c : length of the catheter
  - r_c : radius of the catheter

$$R_c = \frac{8\eta l_c}{\pi \cdot r_c^4}$$

*See Moodle: Annex_HydraulicResistanceCalculation

Modelling the catheter – liquid – membrane system

- Inertance of the liquid

Electrical circ:
L=u/(di/dt)

$$L_c = \frac{\Delta P}{\frac{dQ}{dt}} \quad \text{with} \quad \frac{dQ}{dt} = \frac{dv}{dt} \cdot A = a \cdot A \qquad \text{and } \Delta P = \frac{Force}{surface} = \frac{m \cdot a}{A}$$

$$m = \rho \cdot V = \rho \cdot A.l$$

$$L_c = \frac{m \cdot a}{A.a \cdot A} = \boxed{L_c = \frac{m}{A^2}}$$

a: acceleration
m: mass of the liquid
ρ: density
A: Section
l: length

- Compliance of catheter liquid:
  - E_c : Volumetric elastic modulus

$$\boxed{C_c = \frac{\Delta V}{\Delta P} = \frac{1}{E_c}}$$

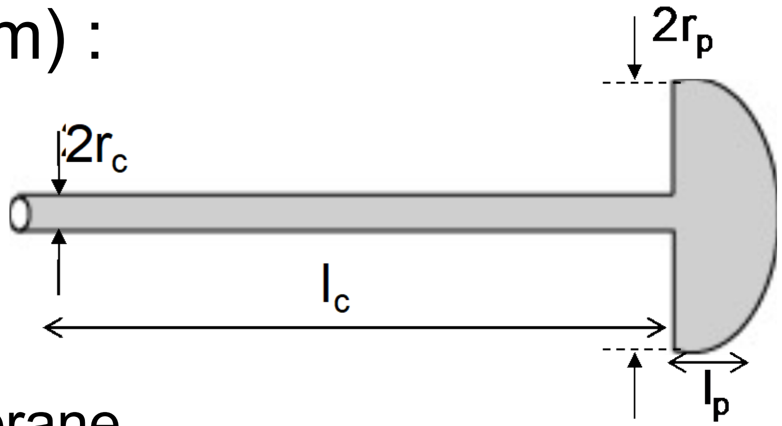
Electrical circ:
C=Q/V

Modelling the catheter – liquid – membrane system

- For the probe(p), catheter (c) and membrane (m) :

$$C_{p,c,m} = \frac{1}{E_{p,c,m}} \qquad R_{p,c} = \frac{8\eta l_{p,c}}{\pi \cdot r_{p,c}^2} \qquad L_{p,c} = \frac{m_{p,c}}{A_{p,c}^2}$$

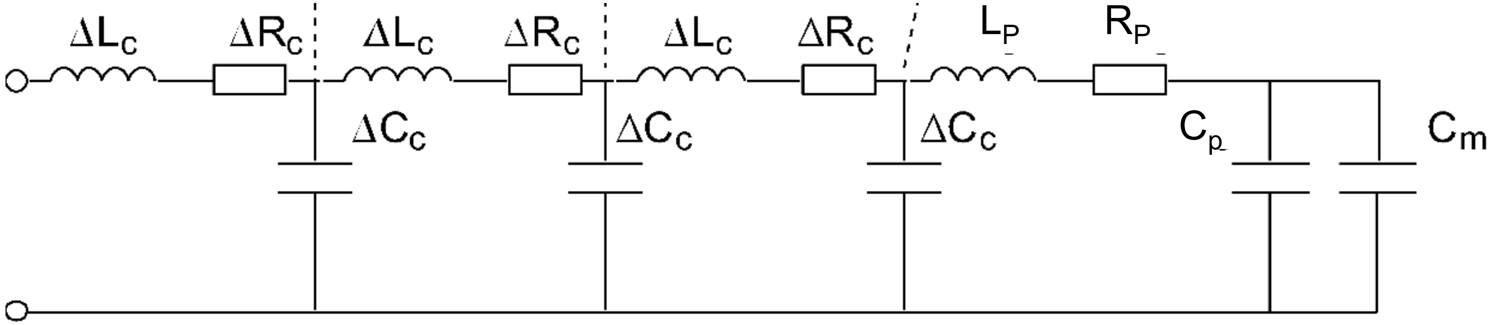
  - $L_{p,c}$: length of probe or catheter
  - $R_{p,c}$: radius of probe or catheter
  - $E_{p,c,m}$: elastic modulus of the probe, catheter or membrane



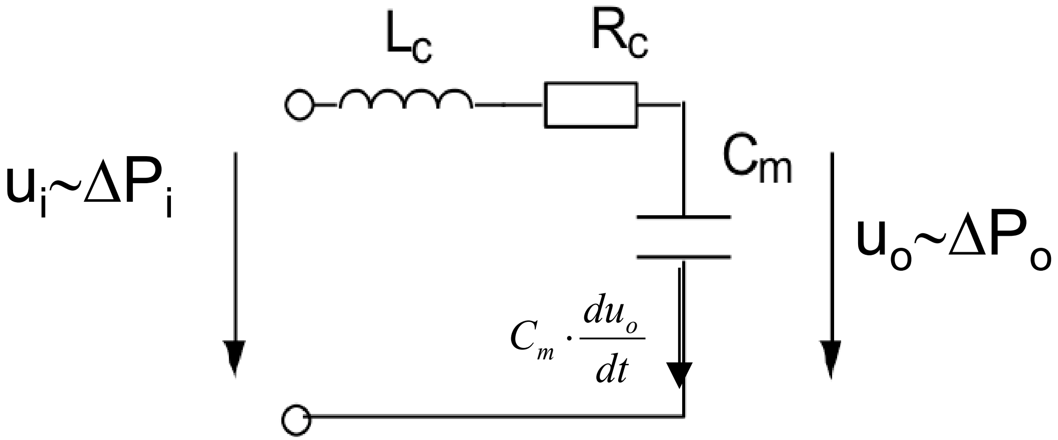
- For $l_P << l_c$ and $r_P >> r_c$: $R_c >> R_P$ and $L_c >> L_P$
- For $E_m << E_c$ and $E_m << E_P$: $C_m >> C_P$ and $C_m >> C_c$

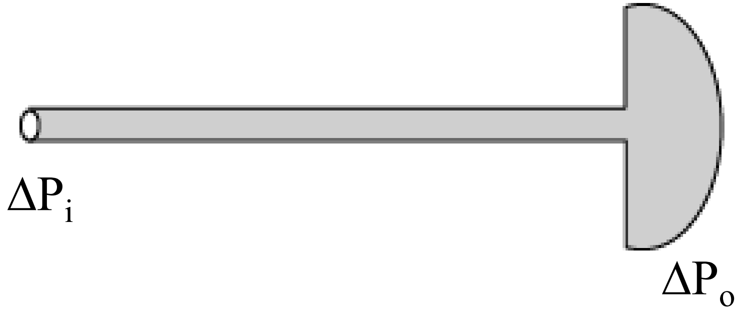
$$\boxed{C_m = \frac{1}{E_m}} \qquad \boxed{R_c = \frac{8\eta l_c}{\pi \cdot r_c^4}} \qquad \boxed{L_c = \frac{m_c}{A_c^2}}$$

Modelling the catheter – liquid – membrane system



$R_c >> R_P$ and $L_c >> L_P$ $C_m >> C_P$ and $C_m >> C_c$

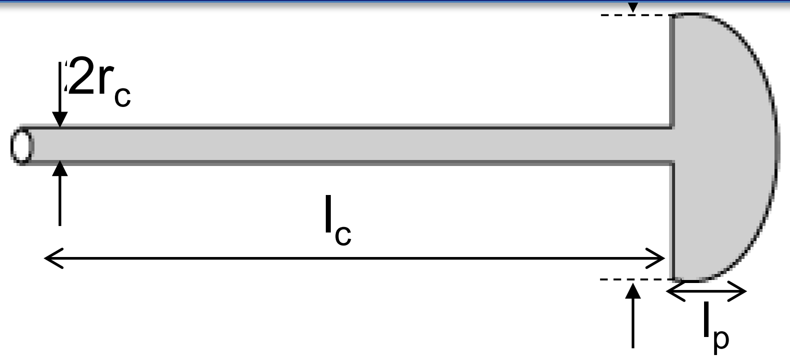




$$u_i(t) = L_c C_m \frac{d^2 u_o(t)}{dt^2} + R_c C_m \frac{du_o(t)}{dt} + u_o(t)$$

Modelling the catheter – liquid – membrane system

- Second order system



$$u_i(t) = L_c C_m \frac{d^2 u_o(t)}{dt^2} + R_c C_m \frac{du_o(t)}{dt} + u_o(t)$$

Characteristic equation

$$u_i(t) = \frac{1}{\omega_o^2} \frac{d^2 u_o(t)}{dt^2} + \frac{2\zeta}{\omega_o} \frac{du_o(t)}{dt} + u_o(t)$$

$$C_m = \frac{1}{E_m} \qquad R_c = \frac{8\eta l_c}{\pi \cdot r_c^4}$$

- $\omega_o = \frac{1}{\sqrt{L_c C_m}}$

$$\omega_o = \pi \cdot r_c \sqrt{\frac{E_m}{\pi \rho l_c}}$$

$$L_c = \frac{m_c}{A_c^2} = \frac{\rho A_c l_c}{A_c^2} = \frac{\rho l_c}{\pi r_c^2}$$

- $\zeta = \frac{R_c}{2} \sqrt{\frac{C_m}{L_c}}$

$$\zeta = \frac{4\eta}{r_c^3} \sqrt{\frac{l_c}{\pi \rho E_m}}$$

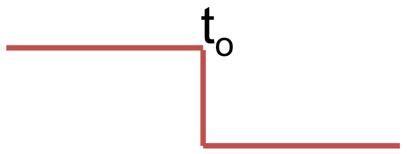
ρ: density

Modelling the catheter – liquid – membrane system

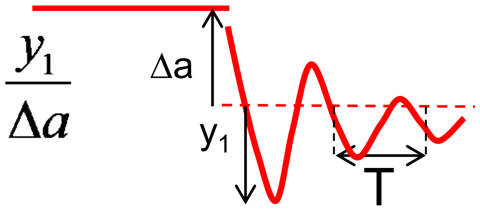
- Step response

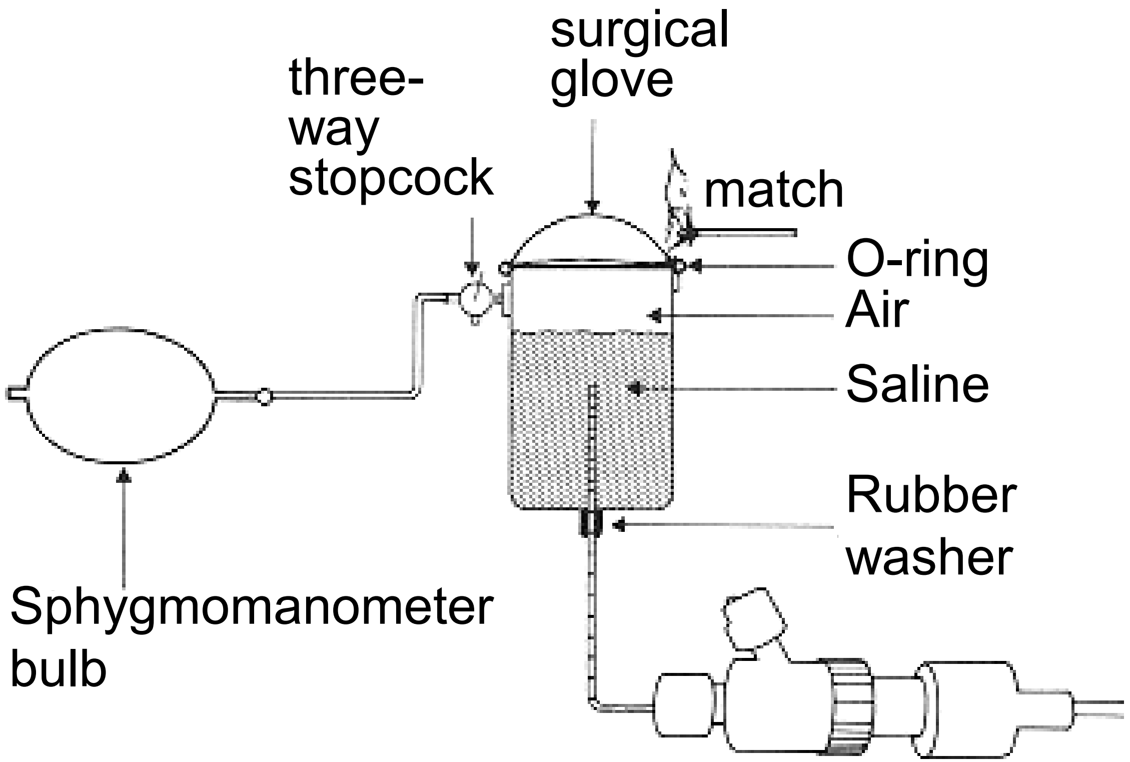
$$\zeta = \frac{1}{\sqrt{\left(\frac{\pi}{\ln D}\right)^2 + 1}}$$

$$\omega = \frac{2\pi}{T\sqrt{1-\zeta^2}}$$



$$D = \frac{y_1}{\Delta a}$$

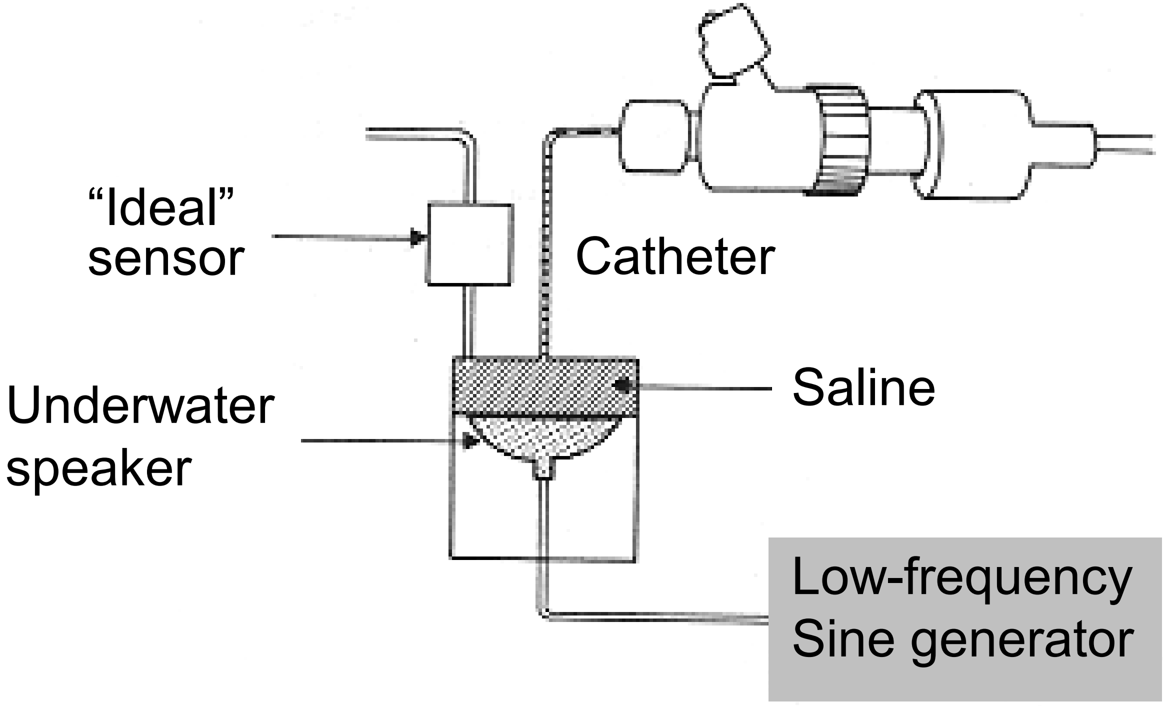




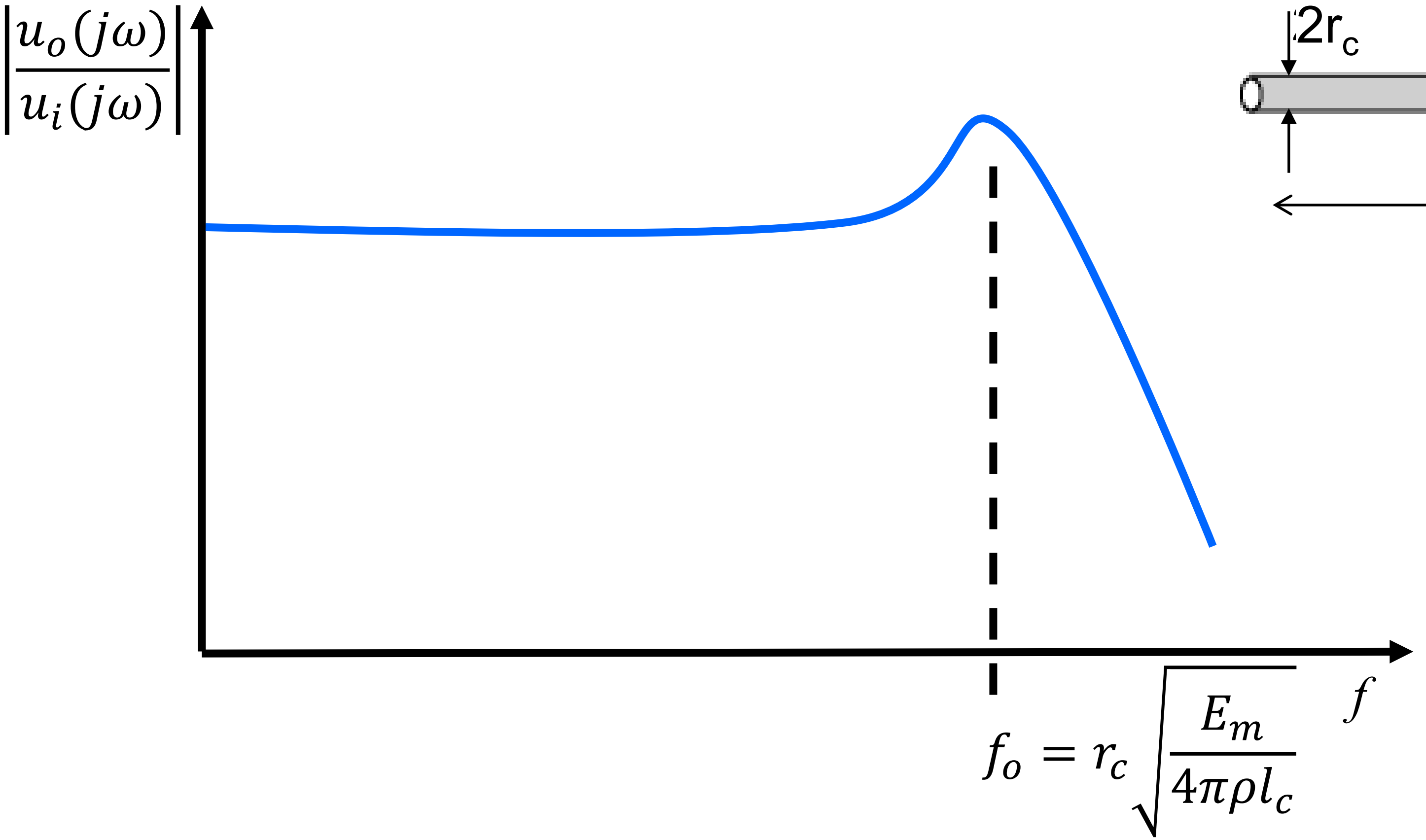
At time t$_o$ the glove is burst

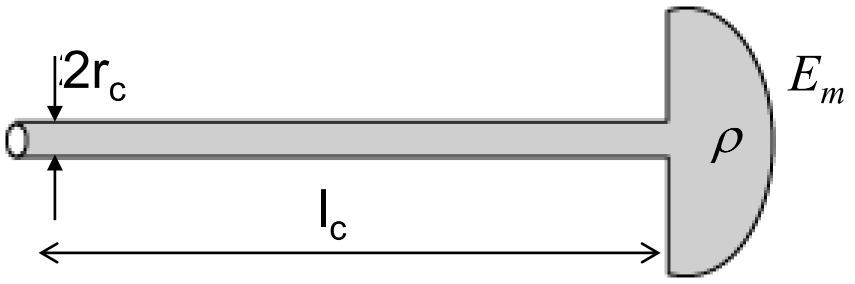
Modelling the catheter – liquid – membrane system

- Response to a sinusoidal signal



Frequency range of extravascular sensor





The frequency range is limited by the size of catheter (l,r), the membrane elasticity (E_m) and density (ρ) of the liquid

→ low pass behavior

Intravascular sensor

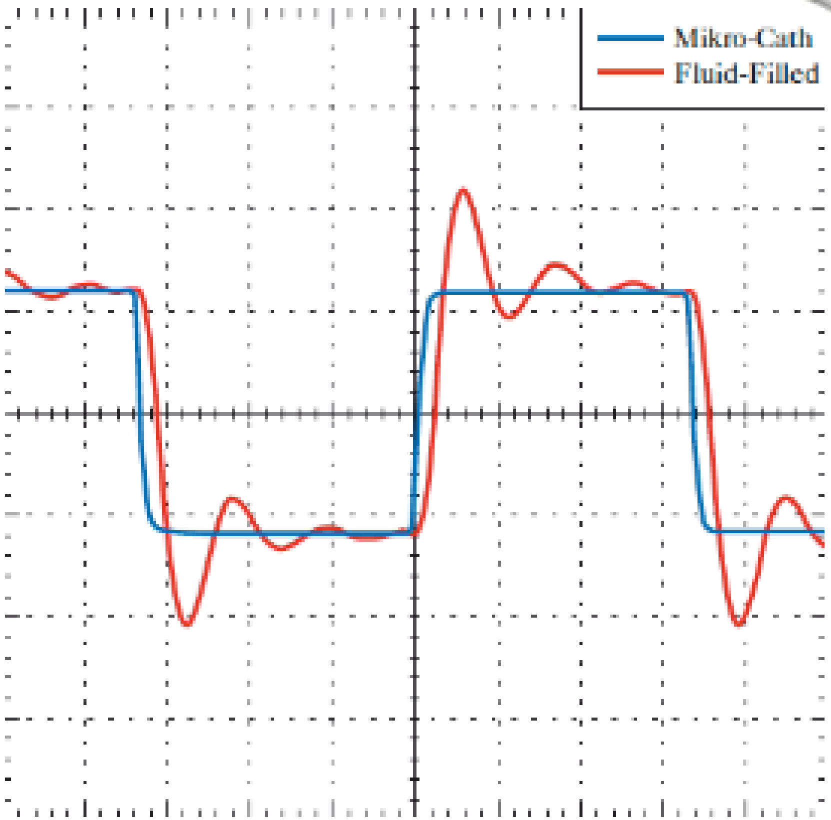
- Move the sensor at the tip of catheter and avoid the RLC circuit which reduces the frequency band
- Example: "Gaeltec"
  - Structure: metallic diaphragm with deposed gauges
  - Excitation: 5V AC r.m.s. maximum or 1V DC maximum
  - Bridge resistance: 1.5kΩ nominal
  - Sensitivity: 5μV/V/mmHg
  - Range: 0 - 150mmHg for urology
  - Range with temperature compensation:15-40° C
  - Temperature offset: < 0.05%FS/° C
  - Temperature sensitivity: < 0.2%/° C
  - Linearity error and hysteresis: < ±1%FS BSL
  - Surpressure: 600mmHg

- Example catheter diameter: F5
  - F: French scale (0.33 *mm*)
  - F5=1.65mm

Millar intravascular sensors

Specification

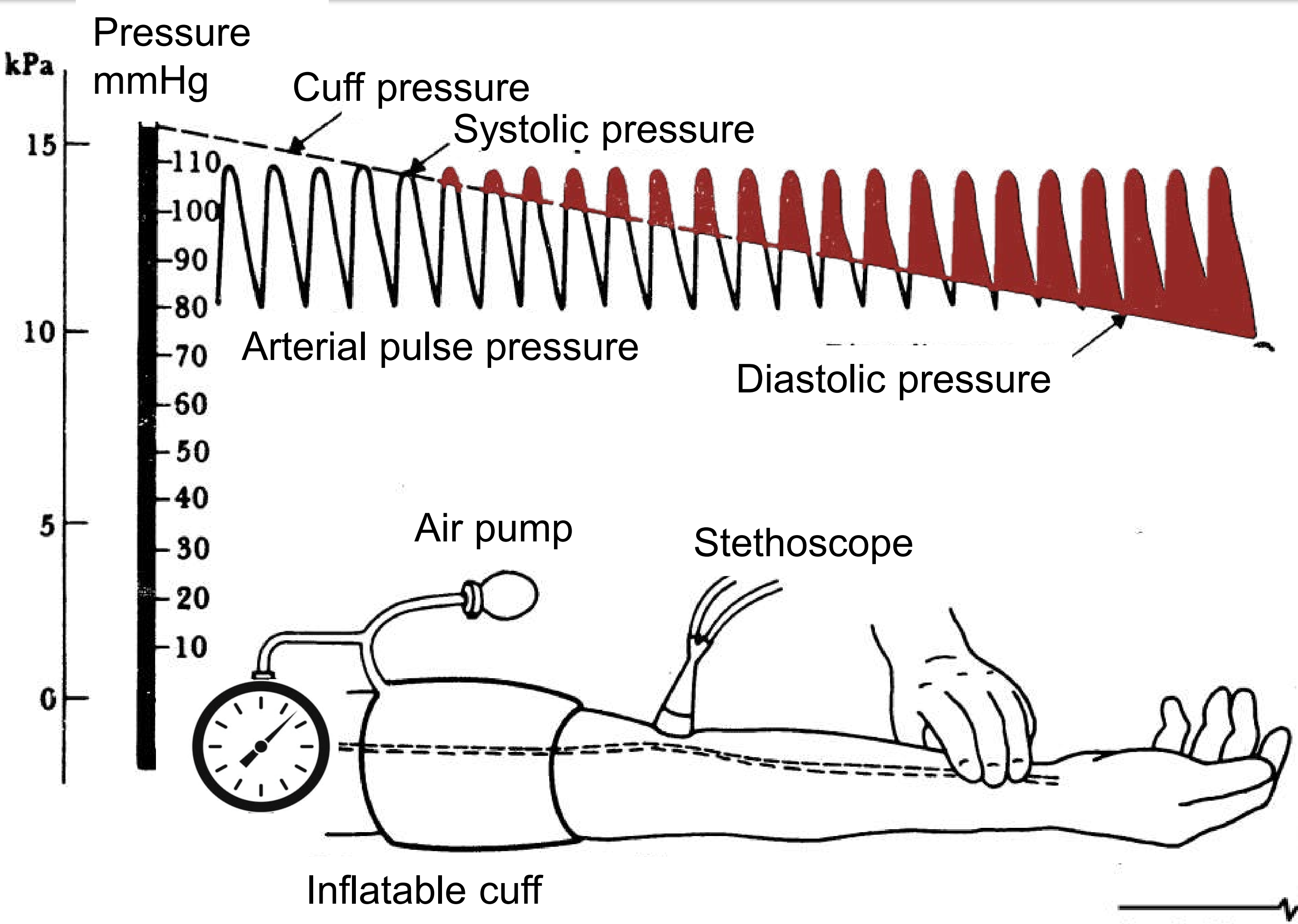
Part Number	825-0101
Catheter Material	Nylon
Effective Length	120 cm
French Size (Sensor)	3.5F (1.2 mm o.d.)
French Size (Catheter Body)	2.3F (0.9 mm o.d.)
Tip Characteristics	Straight
Connector Type	Low Profile
Reusable	No
Ship Sterile	Yes

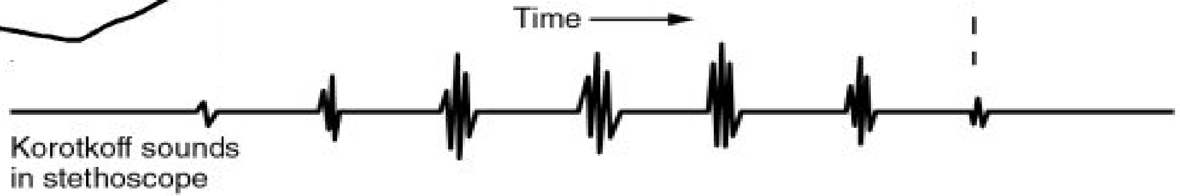


RESISTIVE SENSORS

Part V- Indirect measurement of arterial pressure

Measuring arterial pressure: Indirect measurement





Korotkoff method

- The cuff pressure is raised above the systolic pressure e.g. 180mmHg: The artery is blocked preventing all circulation of blood.
- The cuff pressure is decreased by 2-3mmHg/s.
- When the systolic pressure = occlusive pressure, the blood begins to flow again
- The stethoscope detects an audible sound (Korotkoff). The pressure value corresponds to the systolic pressure (around 120 mmHg).
- When the cuff pressure reaches the diastolic pressure, the Korotkoff sound disappears (~ 80 mmHg)

Korotkoff method

- The Korotkoff sounds goes through 5 phases:
  - phase 1 : the initial sound (cuff pressure = systolic pressure)
  - phase 2 : the intensity of the sound increases
  - phase 3 : the sound reaches its maximum intensity
  - phase 4 : the sound is muffled and muted
    (cuff pressure = diastolic pressure)
  - phase 5 : the sound disappears
- **Origin**
  - Turbulences, vibration of the arterial walls
- **Accuracy**
  - underestimates the systolic pressure by 1 to 13 mmHg
  - overestimates the diastolic pressure by 8 to 18 mmHg

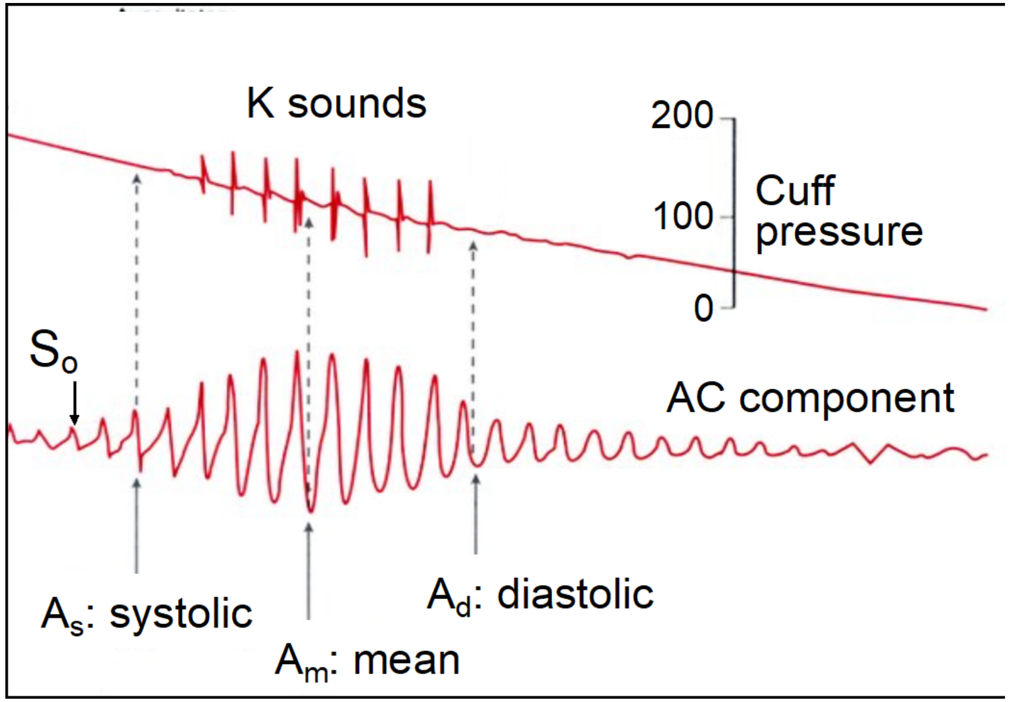
Oscillometry method

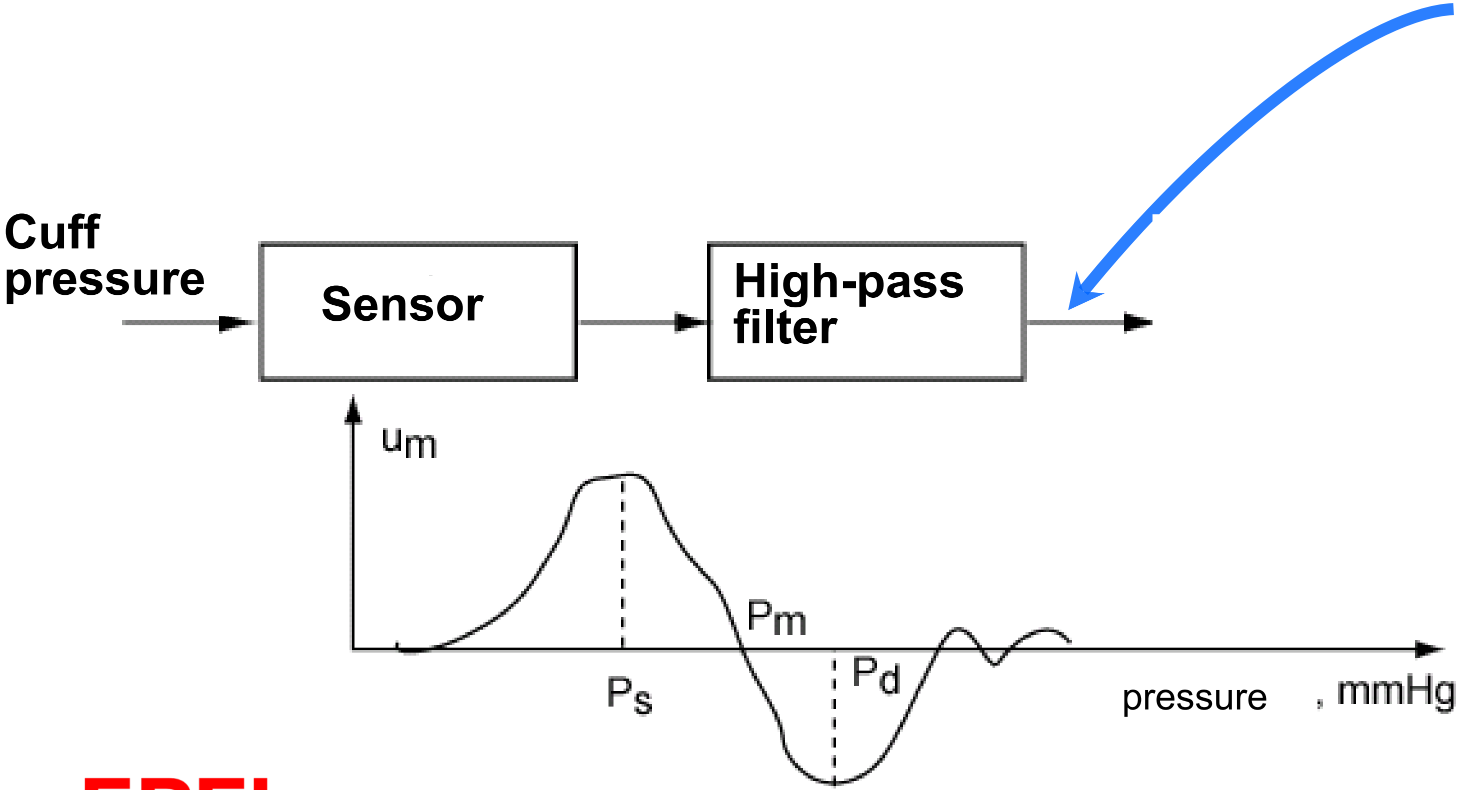
- Identical to the Korotkoff method; however the measurement of systolic and diastolic pressures is done directly by reading the cuff pressure and not by the stethoscope
- Change of volume under the cuff at each pulse
- Change of air volume inside the cuff
- Change in cuff pressure

Oscillometry method

at S_o the cuff pressure starts to decrease (AC component)

- A_s = systolic pressure estimated by the Korotkoff method (Ps)
- A_d = diastolic pressure estimated by the Korotkoff method (Pd)
- A_m = mean arterial pressure (Pm)

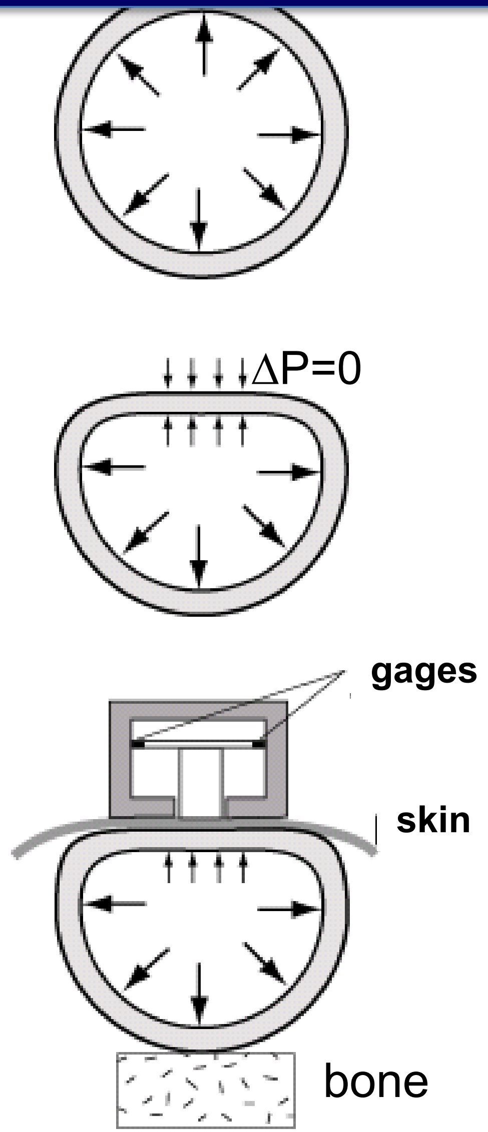




Tonometry

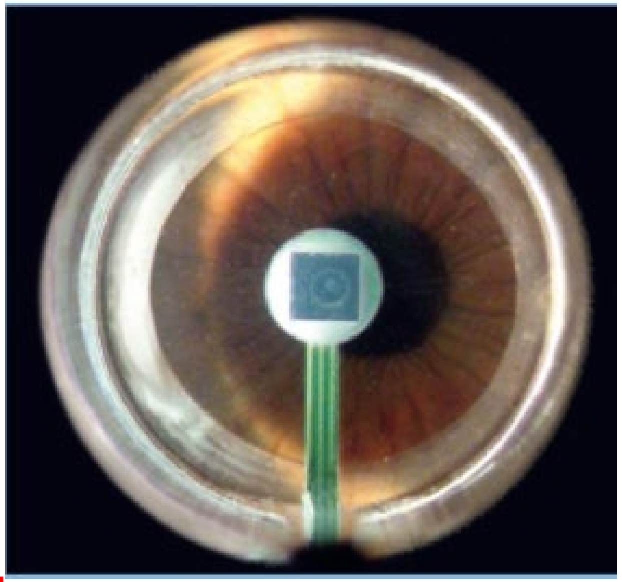
- Vessel wall partially flattened
- Pressure gradient vanishes through flattened part
- Pressure measured outside vessel = arterial pressure

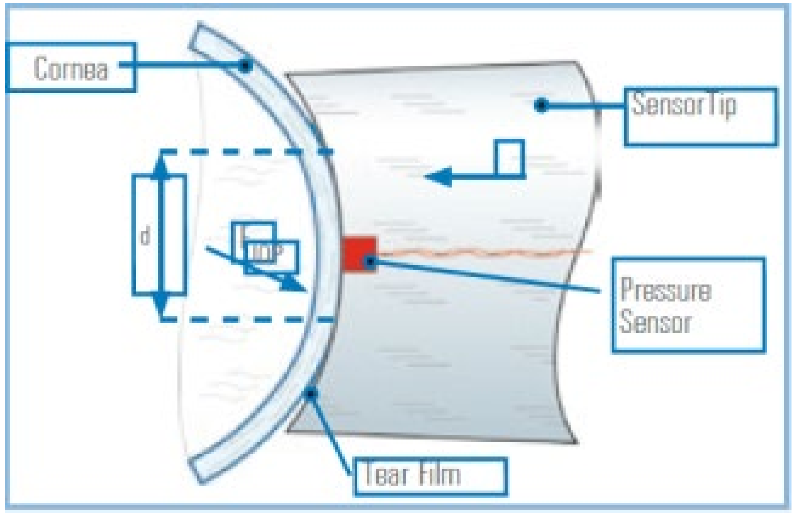
- **Applications**
  - Measuring heart rate
  - Measuring ocular pressure



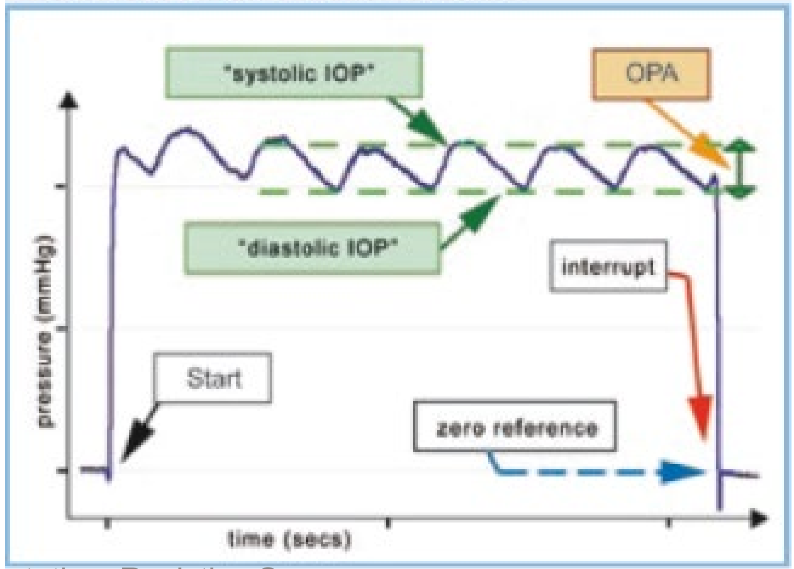
Example: Ziemer tonometer

SensorTip with contour-matched contact surface





This is what the PASCAL tonometer measures:



RESISTIVE SENSORS

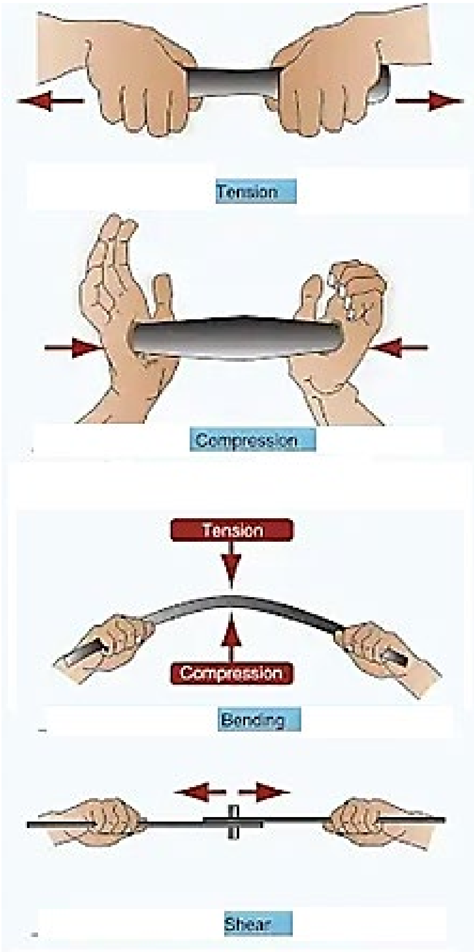
Part VI- Force measurement: instrumented implant

Kinetic measurement

Force sensors

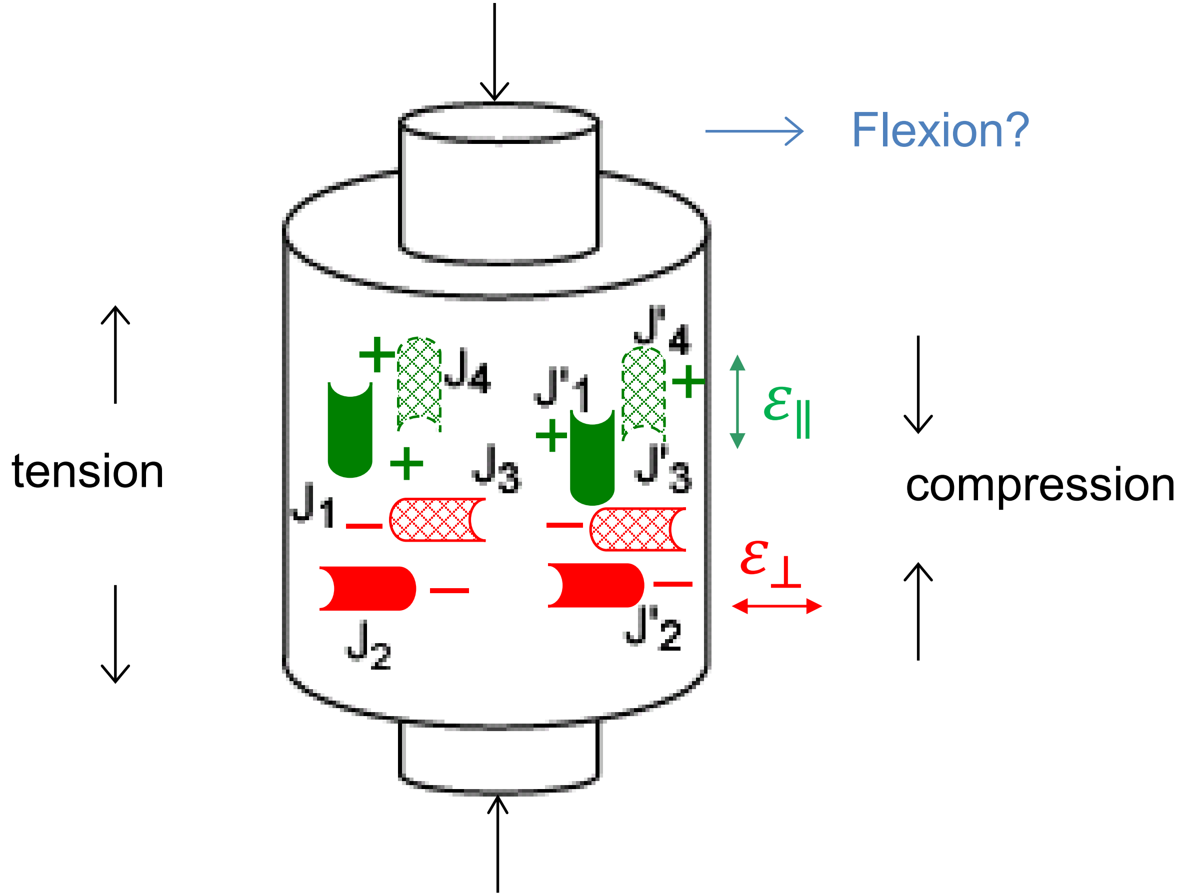
- The force is transformed to deformation via a test specimen
- Testing in tension, compression, flexion or shear

- The deformation is measured by a metallic or semiconductor strain gage mounted on a Wheatstone bridge.

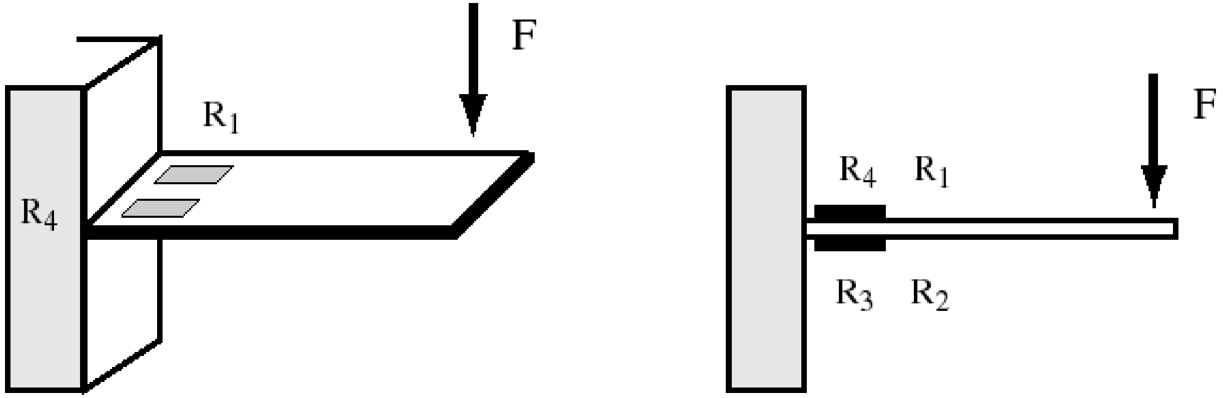


Kinetic measurement

- Example of a test specimen in tension – compression mode



Example
flexion mode



$$u_o = u_a - u_b = \frac{R_2}{R_1 + R_2}u - \frac{R_4}{R_3 + R_4}u$$

$$= \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_2).(R_3 + R_4)}u$$

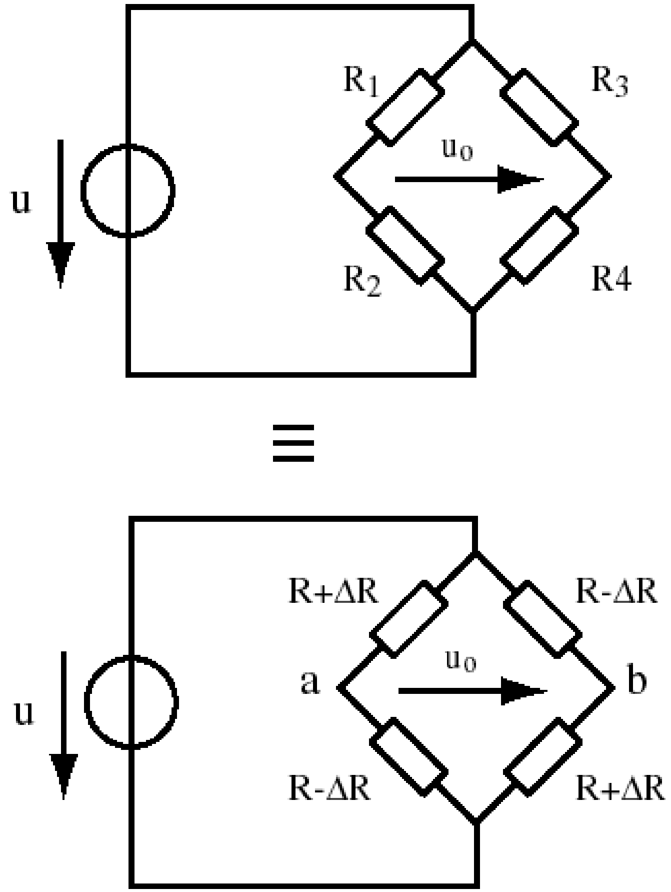
For $R_1 = R_2 = R_3 = R_4 = R$

$$u_o = \frac{(R - \Delta R)^2 - (R + \Delta R)^2}{(2R)(2R)}u = -\frac{\Delta R}{R}u$$

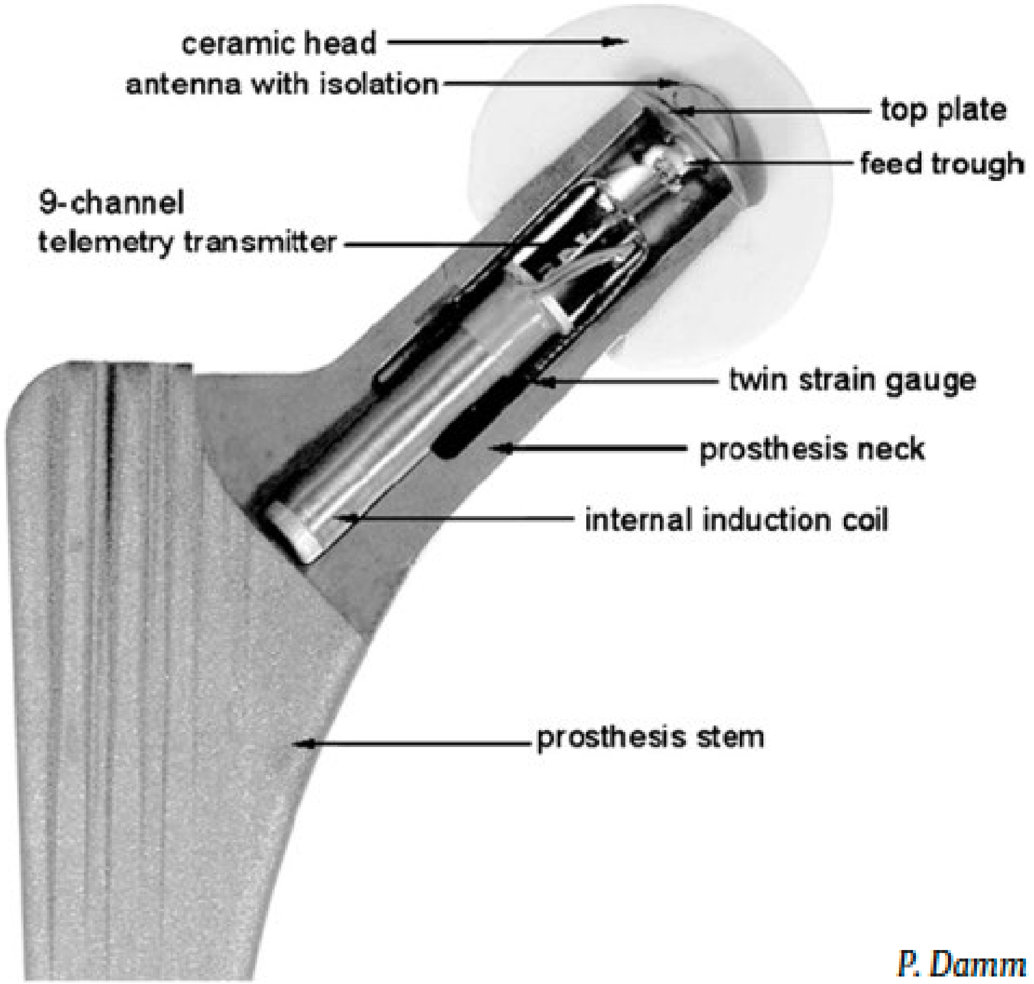
$$\frac{\Delta R}{R} = K\frac{\Delta l}{l} = K\varepsilon$$

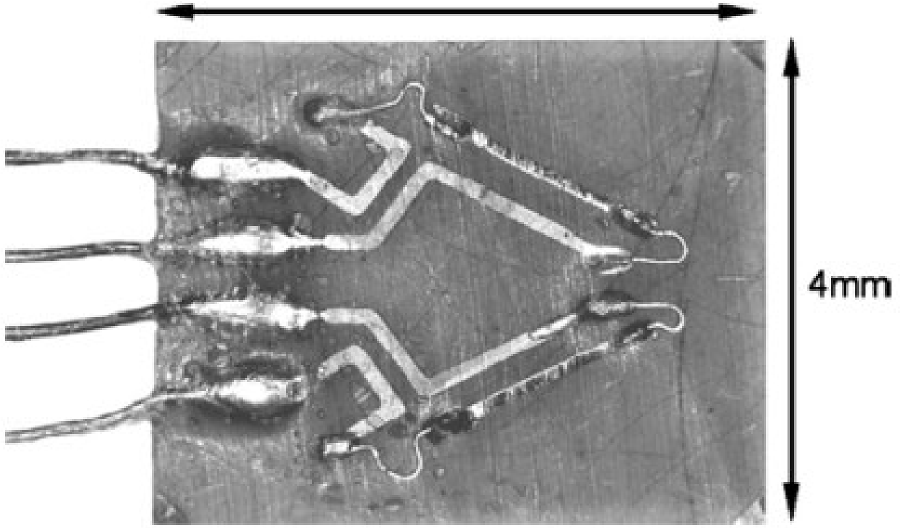
Relative variation of resistance for metallic strain gage

$$\Rightarrow \quad u_o = -K\frac{\Delta l}{l}u = -K' uF$$



Instrumented prosthesis



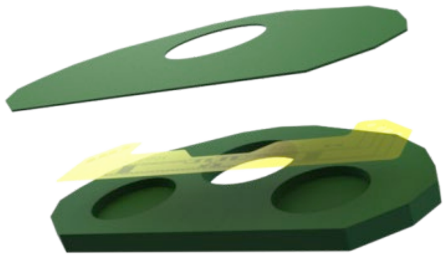


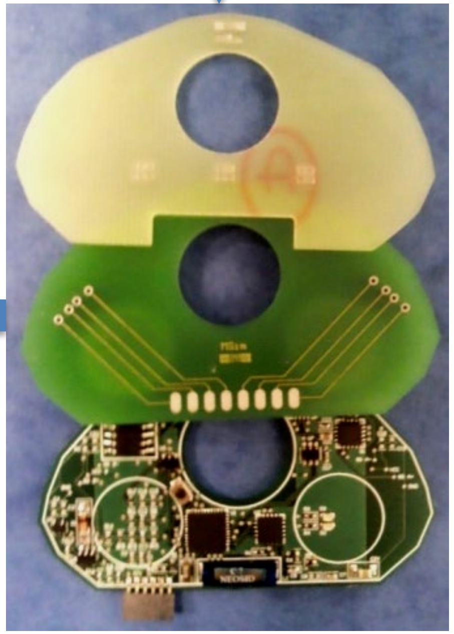
P. Damm et al. / Medical Engineering & Physics 32 (2010) 95–100

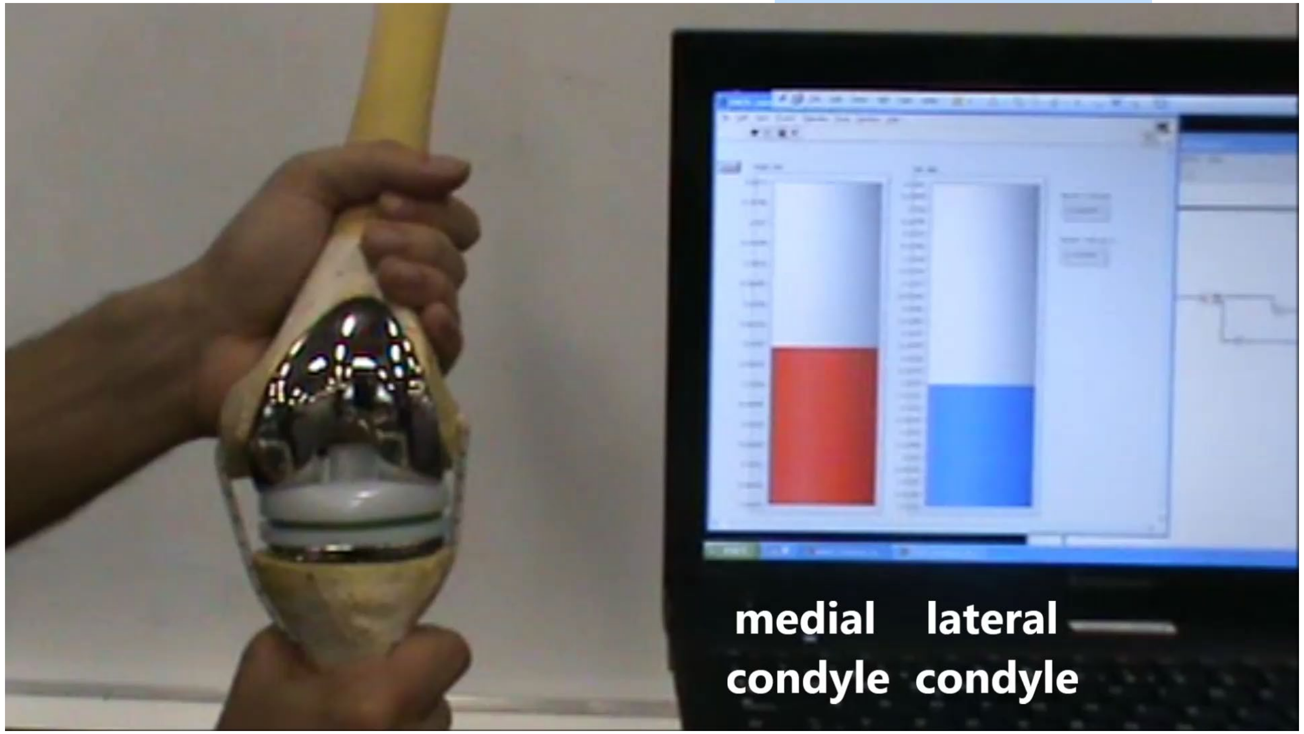
Measurement of force and symmetry

- SImOS

Smart **Im**plants for **O**rthopaedics **S**urgery







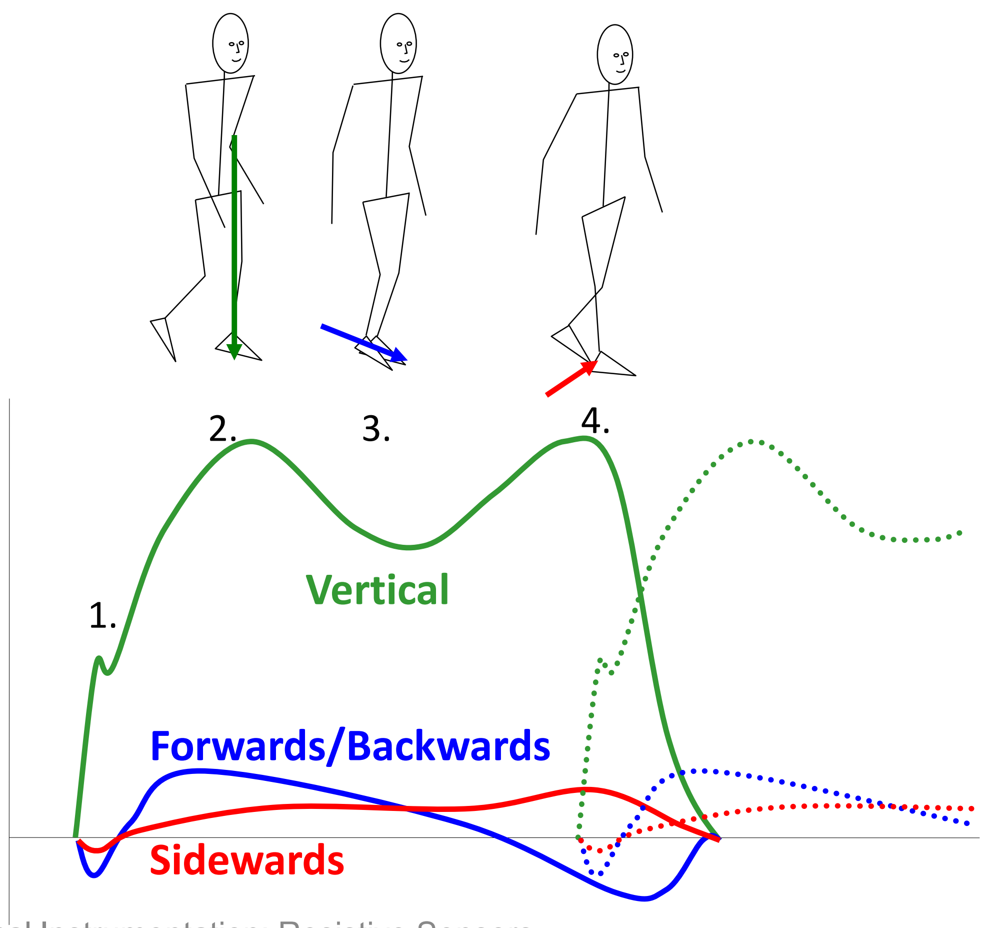
RESISTIVE SENSORS

Part VII- Force plate: application in biomechanics

Force platform

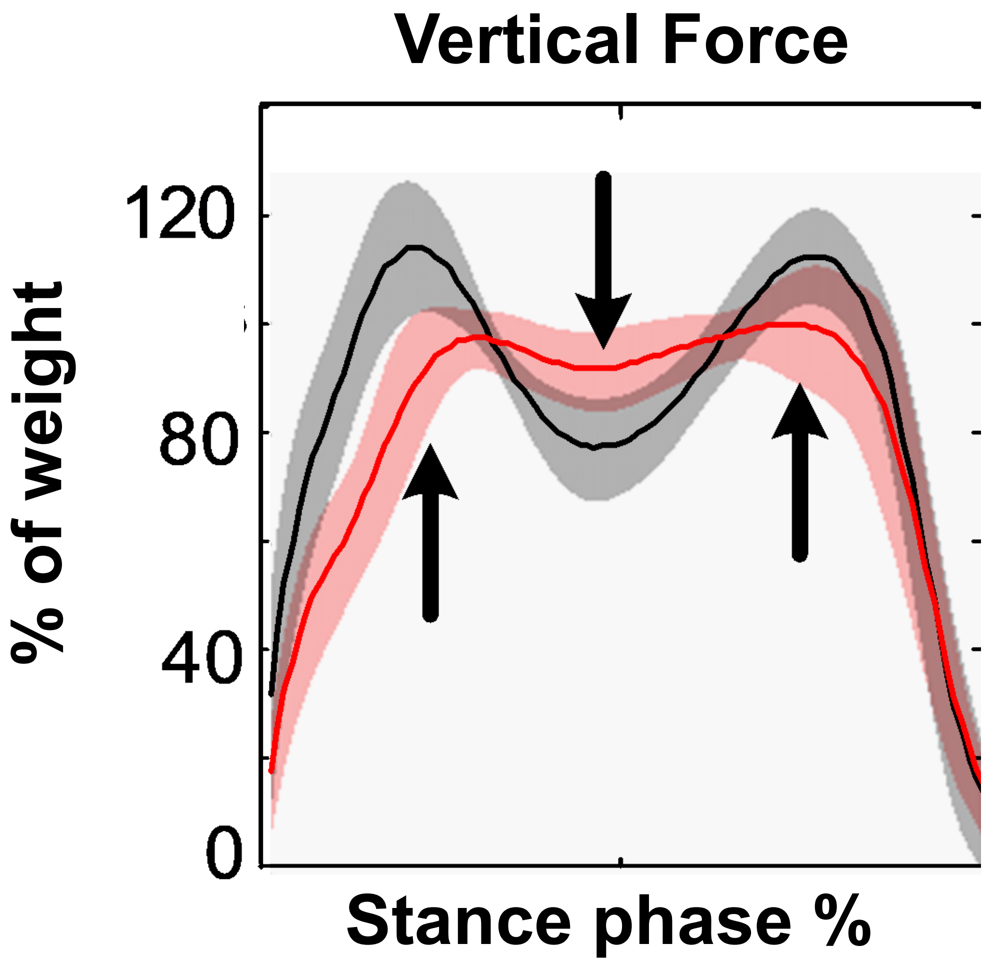
- The force platform allows:
  - the measurement of external forces
  - the estimation of the center of pressure (CoP)
  - the measurement of joint forces and moments

Force platform: ground reaction force



1. Heel Strike
2. Weight Acceptance
3. Mid Stance Phase
4. Push Off

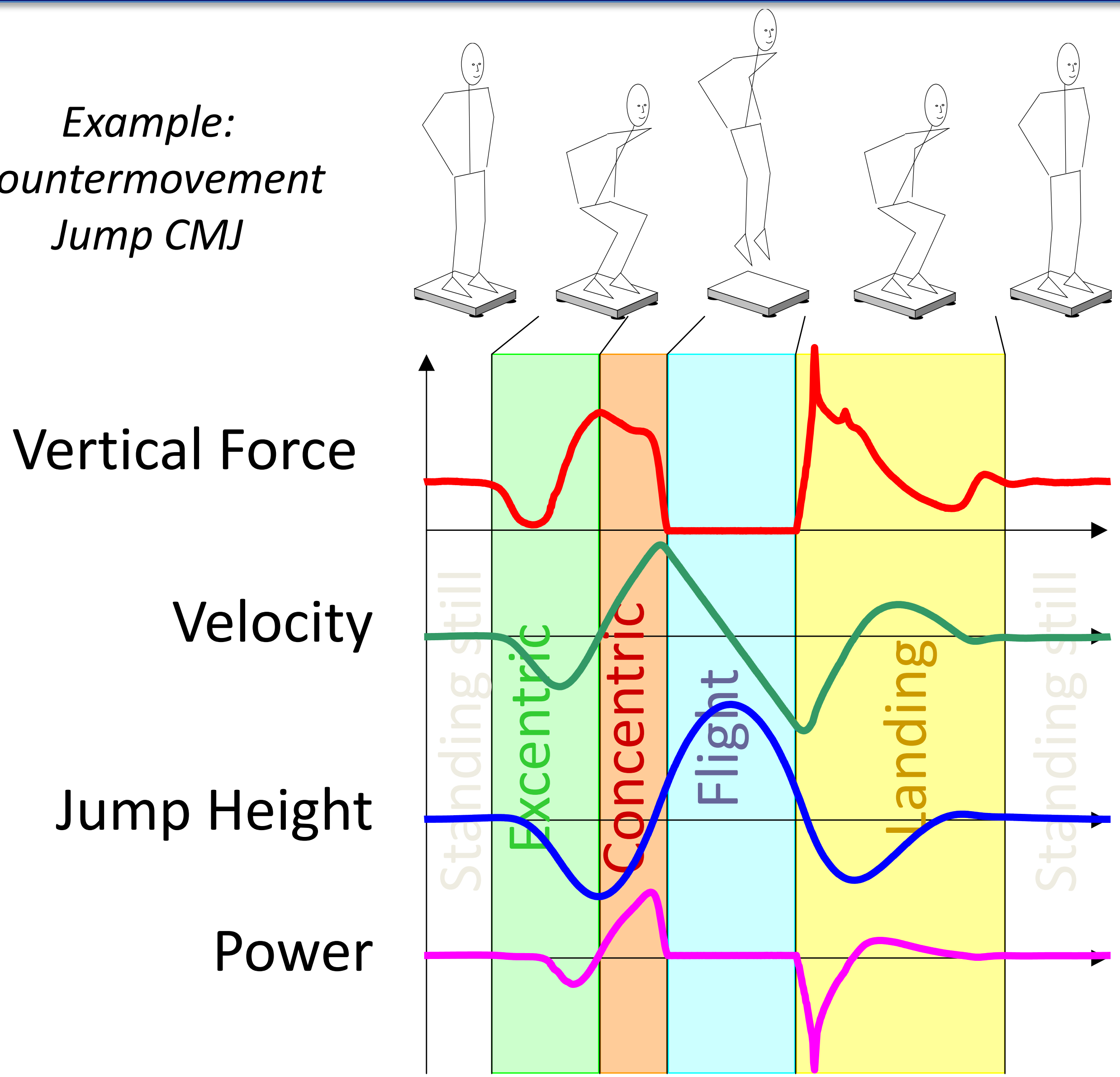
Example: vertical ground reaction force during walking



Healthy subjects
Patients with ankle arthritis

Force platform: jump

Example: Countermovement Jump CMJ

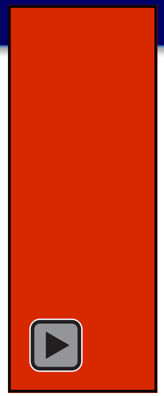


$$F = measured$$

$$v = \int a = \int \frac{F}{m}$$

$$s = \int v$$

$$P = F \times v$$

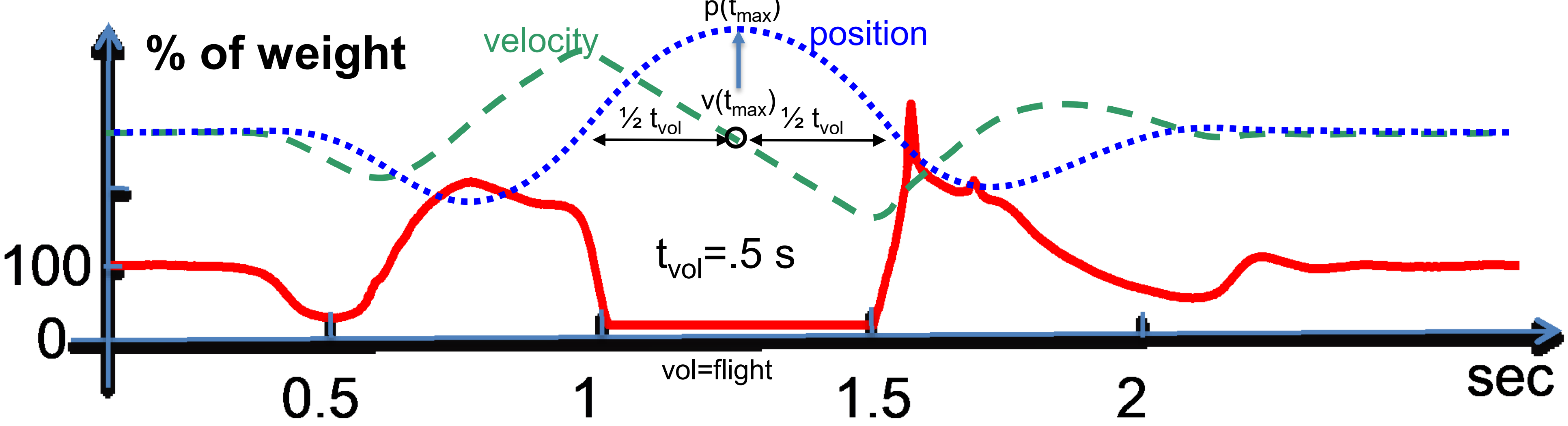


Example: calculating the jump height

Show that: $h_{vol} = \frac{g}{2} \cdot \left(\frac{t_{vol}}{2}\right)^2 = \frac{9.8}{2} \cdot 0.25^2 \approx 30cm$

$$mgh_{\max} = \frac{1}{2} m v^2 (t_{\max}) = \frac{1}{2} m g^2 \left(\frac{t_{vol}}{2}\right)^2$$

$$v(t) = gt \Rightarrow v(t_{\max}) = gt_{\max} \approx g\frac{t_{vol}}{2}$$



Posturography: CoP Sway During Standing

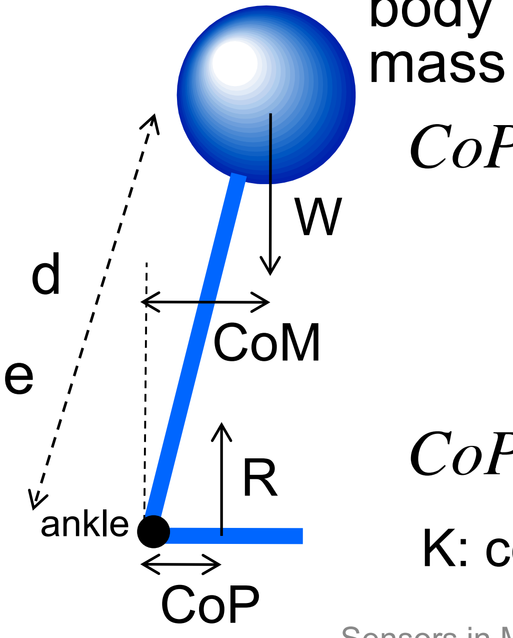
- Body sway is controlled by:
  - Vestibular system
  - Visual system
  - Proprioceptive system

$$R \cdot CoP - W \cdot CoM = I \cdot \dot{\omega}$$

$R = W$

$\dot{\omega}$: angular acceleration

I: moment of inertia of total body about ankle



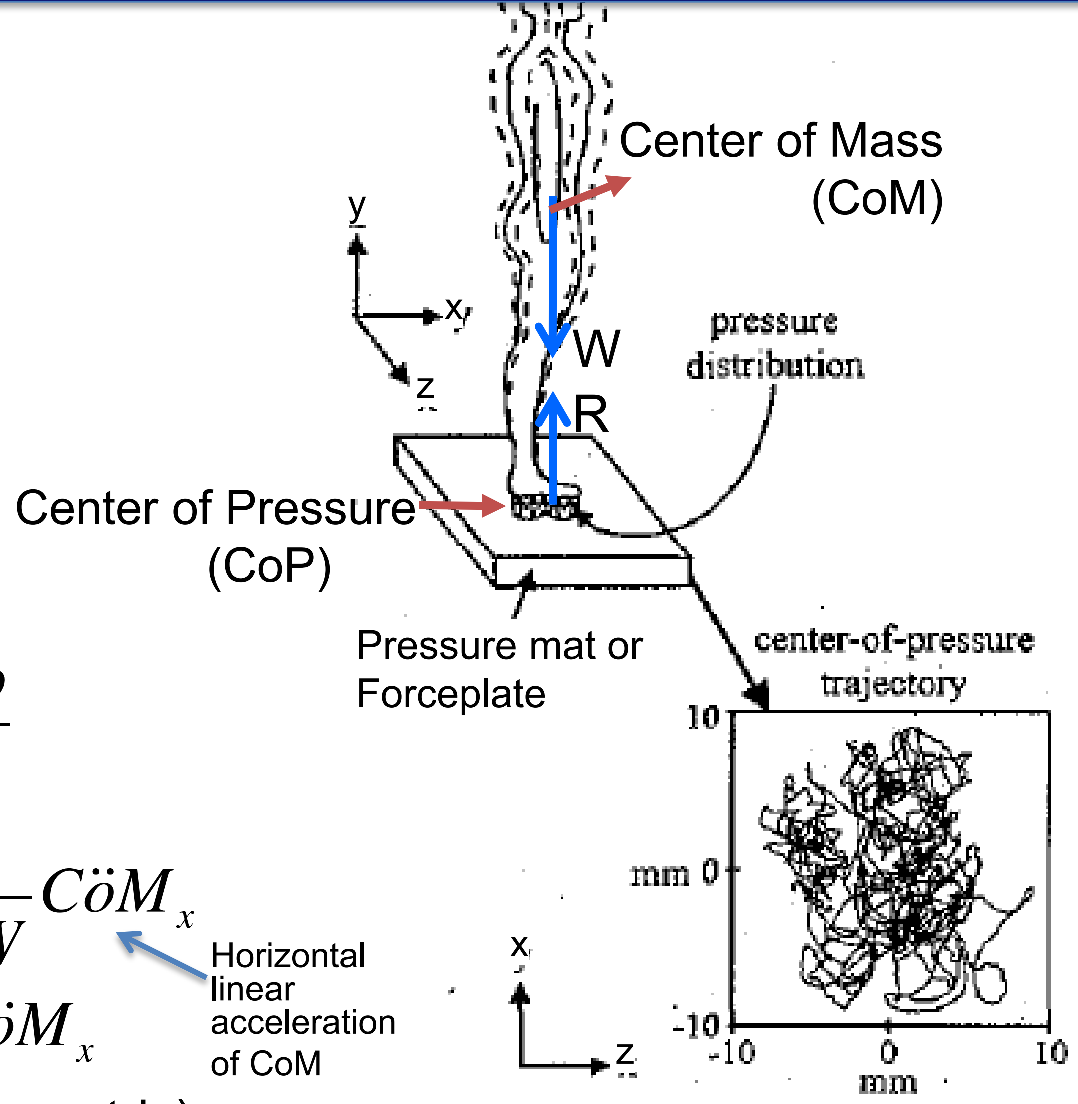
$$CoP - CoM = \frac{I \cdot \dot{\omega}}{W}$$

$$= \frac{I}{d \cdot W} C\ddot{o}M_x$$

Horizontal linear acceleration of CoM

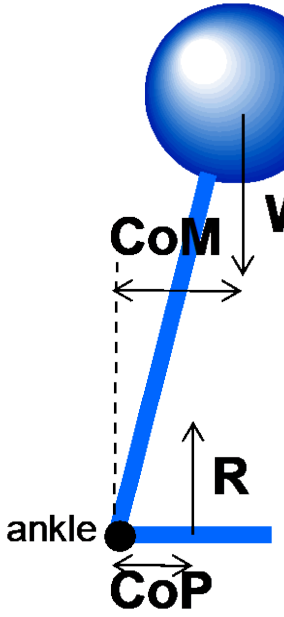
$$CoP - CoM = KC\ddot{o}M_x$$

K: constant (anthropometric)

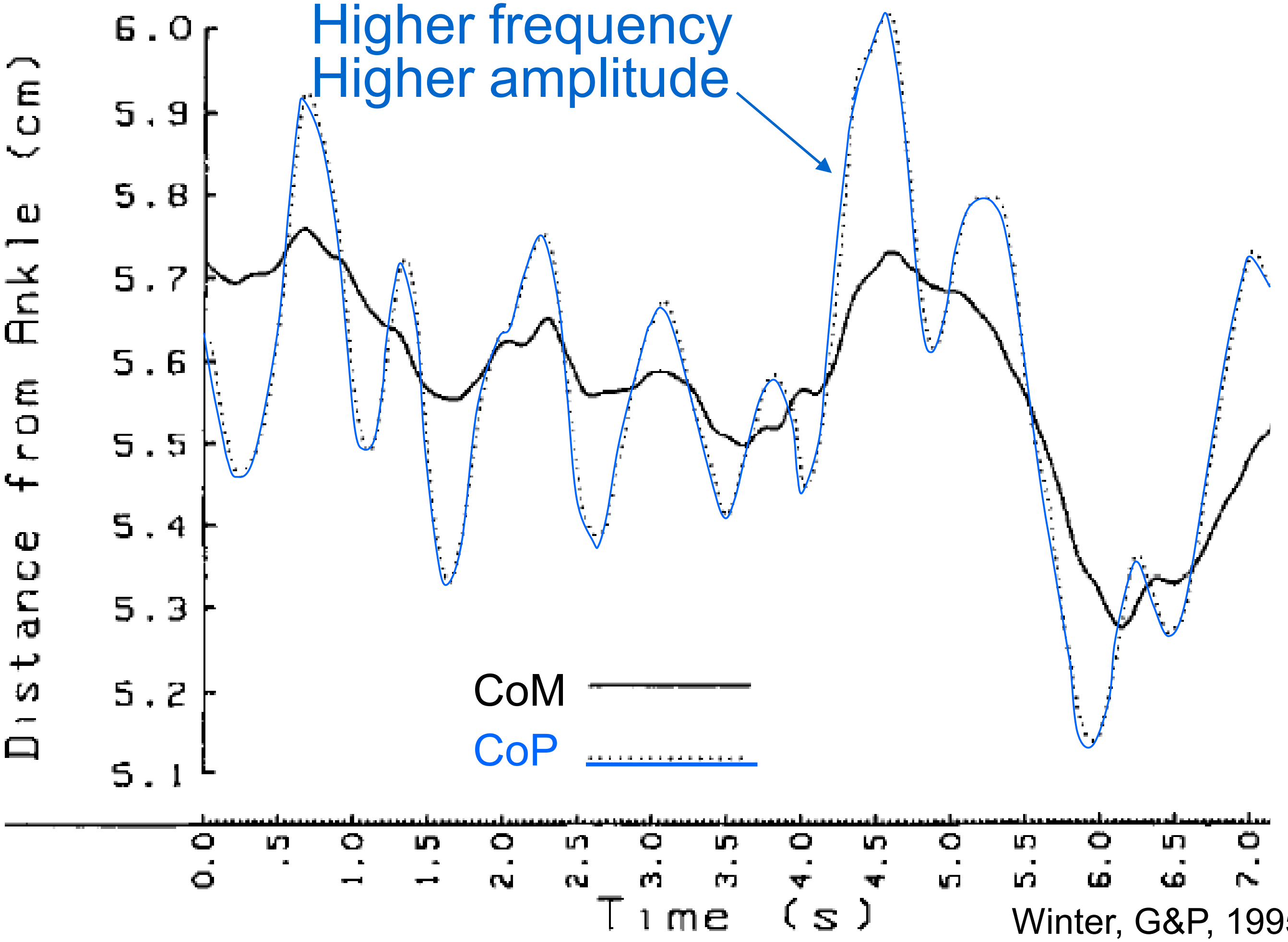


Quite Standing Posture: CoM versus CoP

To keep balance:
CoP should vary more and faster in order to keep CoM range low and stable



$$CoP - CoM = \frac{I \cdot \dot{\omega}}{W}$$



Winter, G&P, 1995

Postural sway in clinical assessment

Objective:

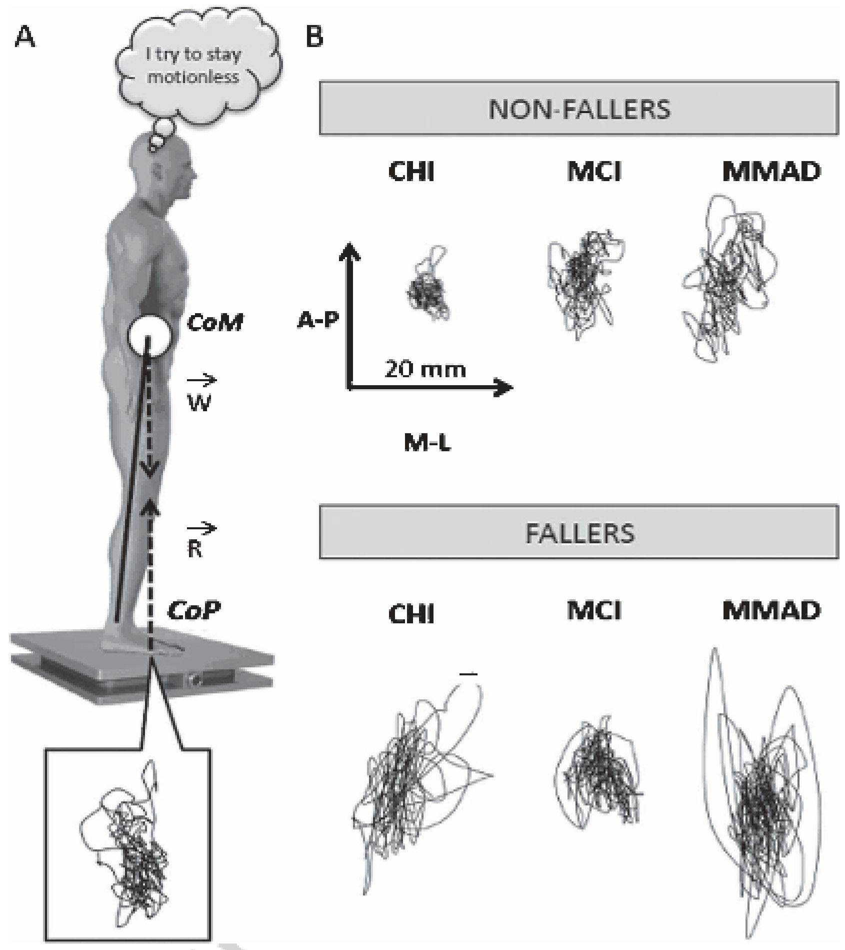
To examine the center-of-pressure (COP) velocity association with cognitive status and history of falls, in cognitively healthy individuals (CHI), patients with mild cognitive impairment (MCI), and with mild-to-moderate Alzheimer's disease (MMAD).

Conclusion:

identifying people with and without cognitive impairment who are at risk of falls risk via the evaluation of the postural control strategies might be a valuable window of opportunity for fall-prevention interventions.

Mignardot, Jean-Baptiste, et al. "Postural sway, falls, and cognitive status: a cross-sectional study among older adults." *Journal of Alzheimer's Disease* 41.2 (2014): 431-439.

J.-B. Mignardot et al. / Postural Sway, Cognitive Status, and Falls



. 1. Representative examples of center-of-pressure (COP) trajectories recorded using a force platform (A), as a function of the cog
us (CHI, MCI, and MMAD) and fall risk (non-fallers versus fallers) (B). CHI, cognitive healthy individual; MCI, mild cognitive impair
IAD, mild-to-moderate dementia; AP, anteroposterior axis; ML, medio-lateral axis.

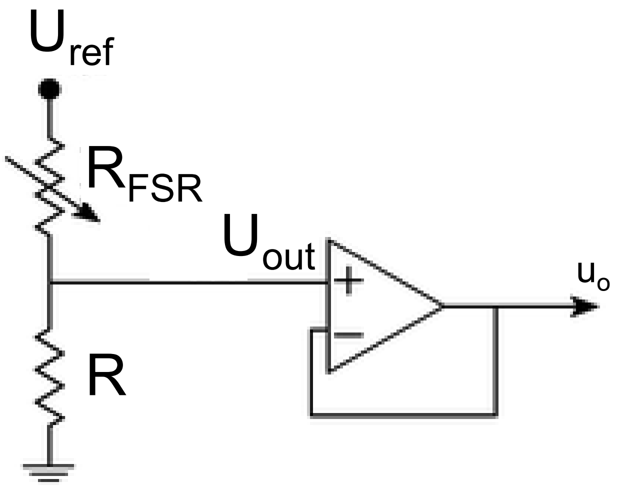
RESISTIVE SENSORS

Part VIII- Wearable force measurement: gait analysis

Plantar sensors

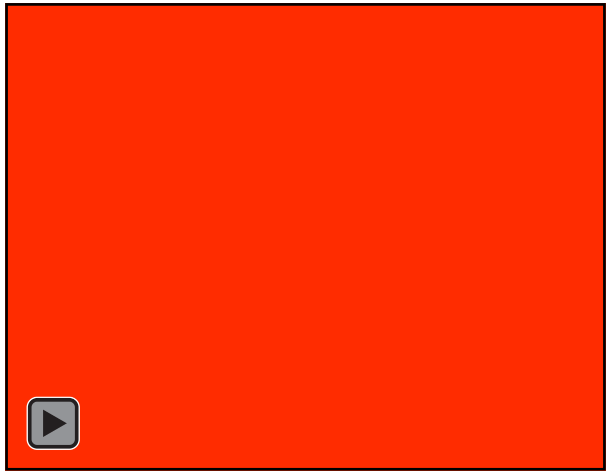
- **The force is inversely proportional to the resistance**
- 3 regions of operation
  - Between 0 and 20gr the resistance changes very rapidly ("footswitch")
  - >20gr 1/R
  - Saturation
- Low precision
- Particular characteristics :
  - flexibility
  - Limited lifetime
  - low thickness, sensitivity
  - simple to use
- FSR + inverting amplifier
  linear tension-force


FSR: Force Sensing Resistor

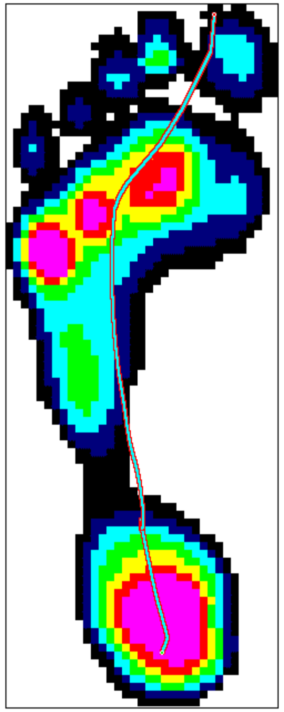


$$u_o = \frac{R}{R+R_{FSR}} U_{ref} = \frac{1}{1+\frac{R_{FSR}}{R}} U_{ref} = \frac{R}{R_{FSR}} U_{ref}$$

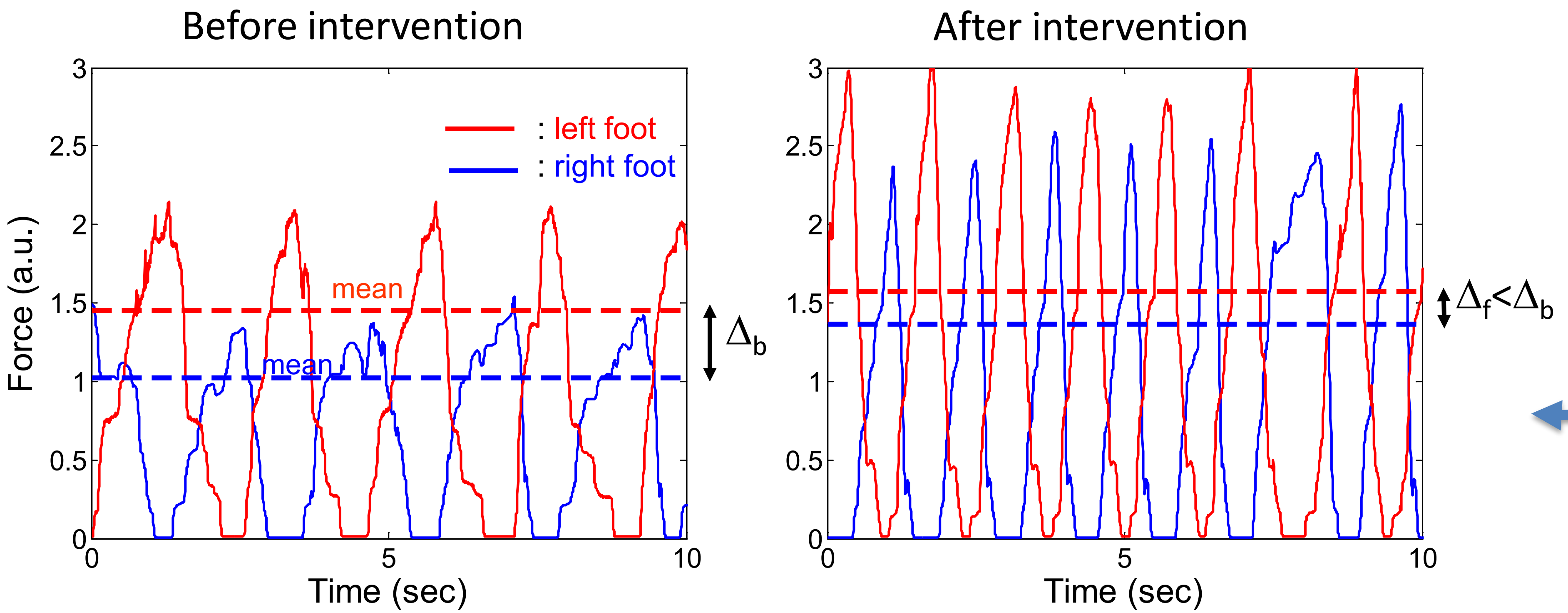
Plantar sensor

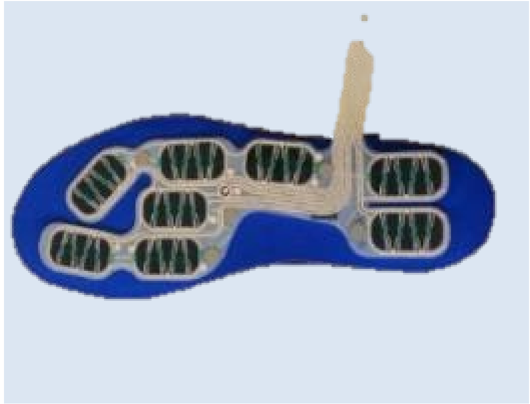


- Applications
  - measure the pressure distribution under the foot
  - Platform with high number of sensors (cells)
  - Integrate the sensors inside an insole
  - Diabetes
  - Functional Electrical Stimulation (FES)
  - Gait analysis



Example: Foot loading during walking after hip fracture surgery

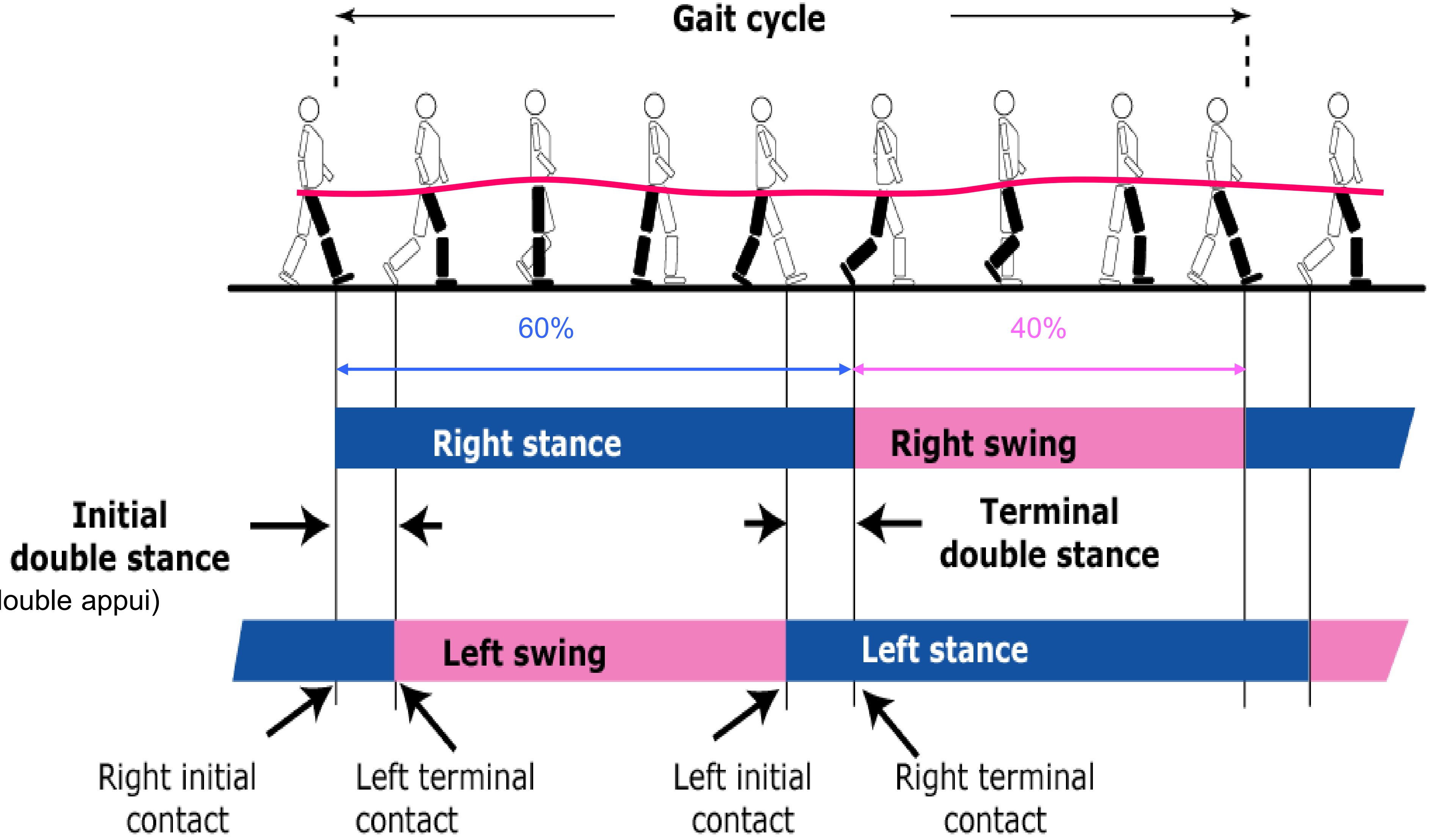




Sum of all sensors.

Moufawad El Achkar et al., EPFL thesis, 2016

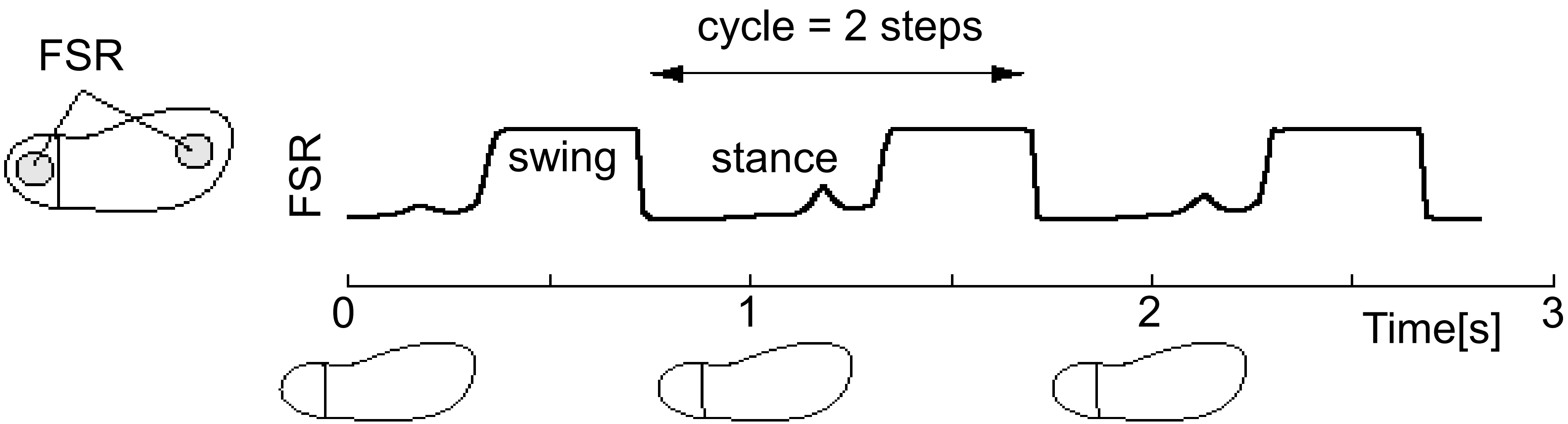
Temporal gait analysis



- (double appui)

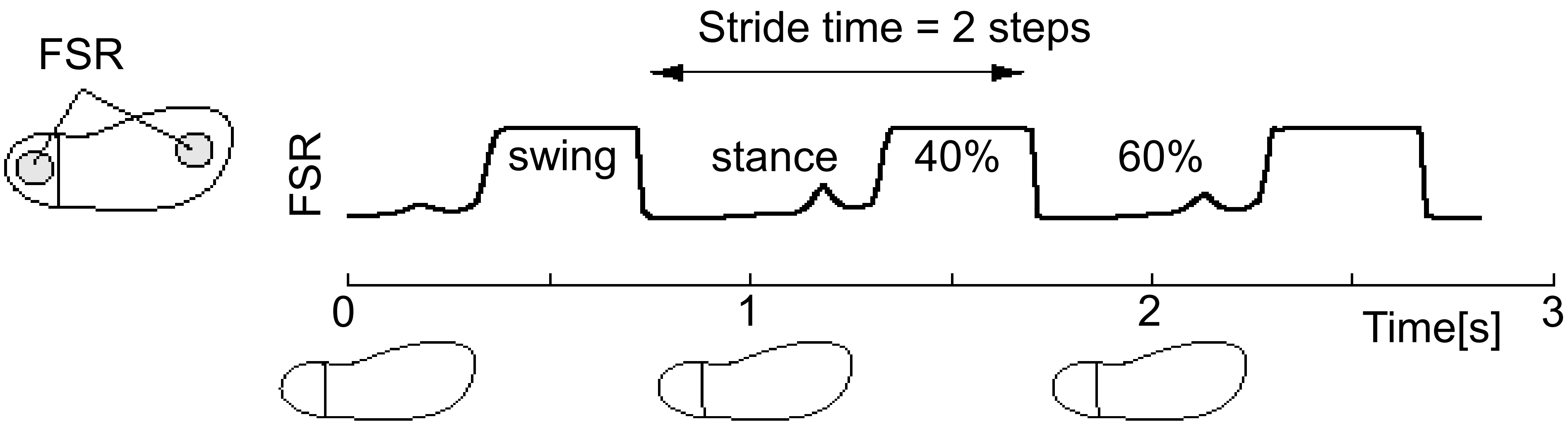
Measuring the temporal gait parameters

- Stance and swing phases, initial and terminal double stance, ground impact (heel strike), heel and toe off, rolling of the foot.
- Sensors are fixed directly on the sole of the foot or incorporated in an insole.

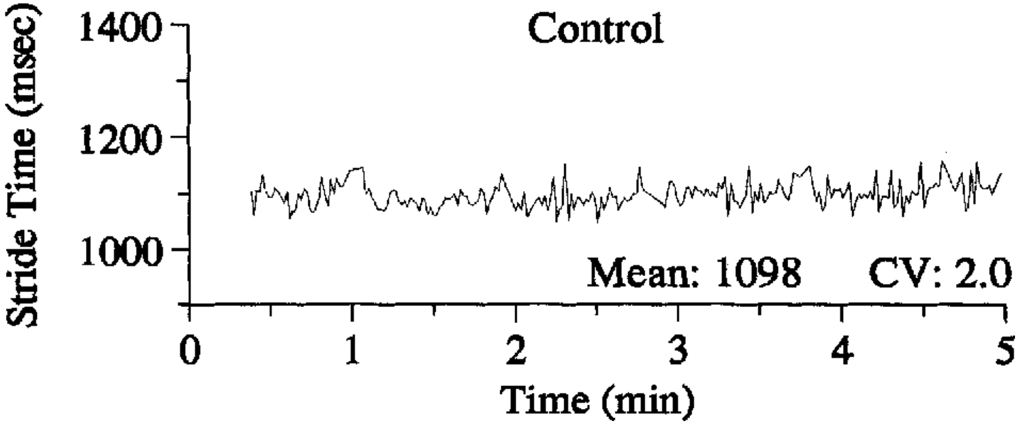


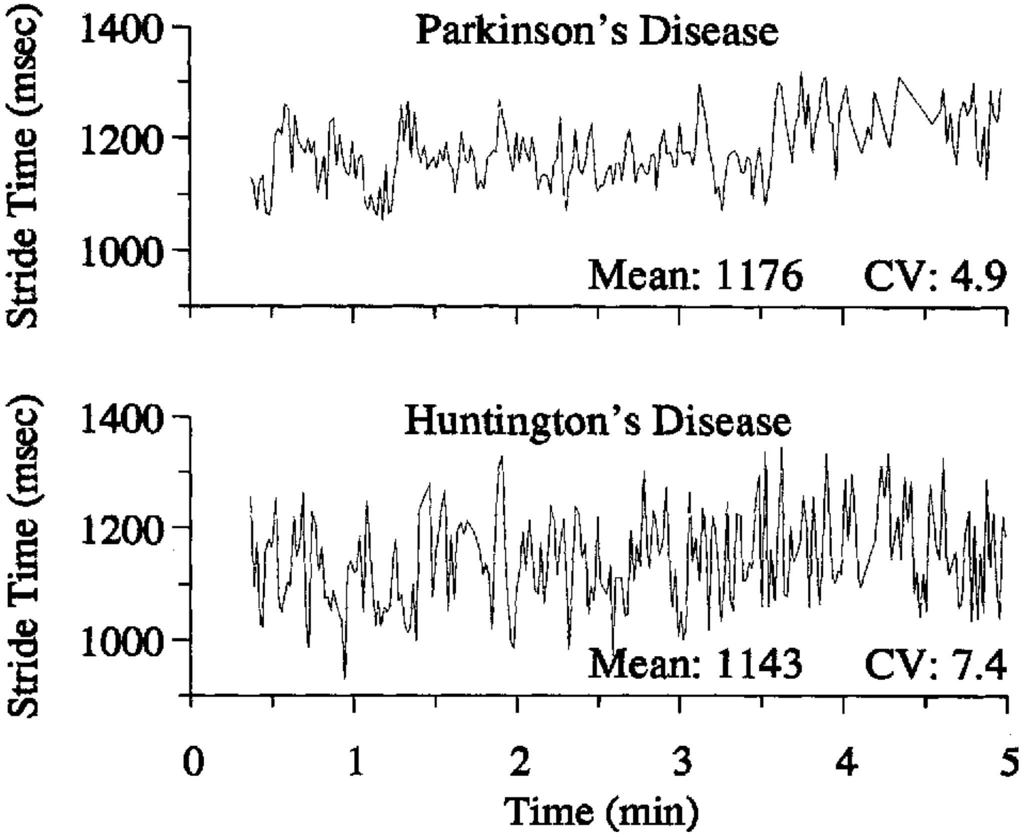
Measuring the temporal gait parameters

- Application
  - Distinguish normal from pathological gait
  - Better understand the muscle activity of the lower limbs



Example: increase Stride time variability with disease





CV=SD/mean, %

Hausdorff, Jeffrey M. et al. *Movement disorders* 1998