MA1 Acoustic Labs 3
Modeling of sound sources in COMSOL Multiphysics

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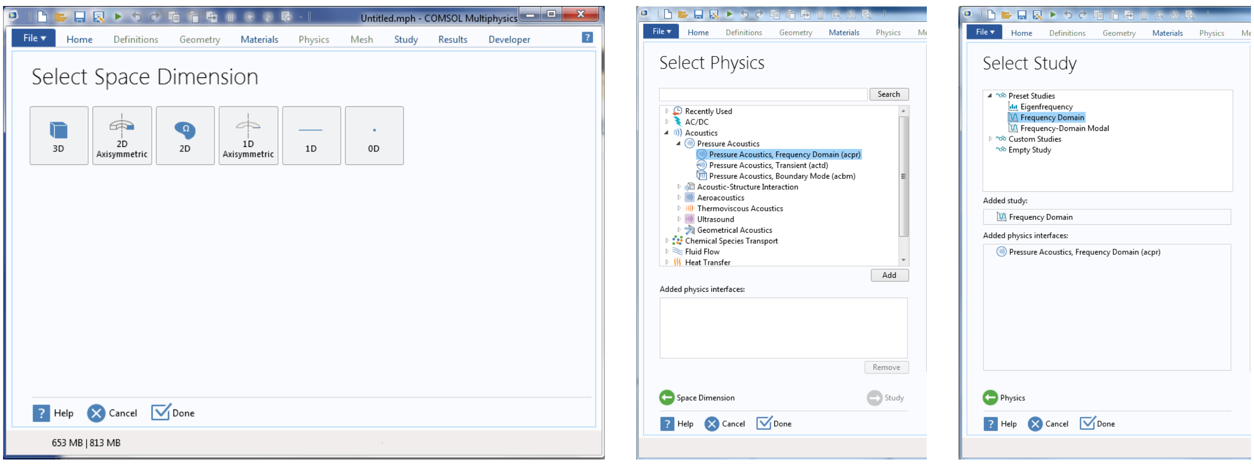
1 Exercise 1: Radiation of the pulsating sphere

We will show in this simple exercise how to model using a Finite Element Method (FEM) the pulsating sphere in COMSOL Multiphysics. This exercise should be considered as a template for the other following exercises.

1.1 Model settings

In COMSOL Multiphysics 6.0, create a new **2D-axisymmetric** model with stationary pressure acoustic physics.



(a) (b) (c)

Figure 1: Setting the model

First click on "New/Model Wizard" and select **2D-axisymmetric** on the "select space dimension" tab (Fig1a). Then select the physics by clicking on **Acoustics/Pressure Acoustics/Pressure Acoustics, Frequency Domain (acpr)** and click on "Add". The select the study by clicking on **Preset Studies/Frequency Domain** (Fig 1c). Click on "Done" to validate.

1.2 Building the model

1.2.1 Entering the model parameters and variables

You will now enter the parameters of the model. We will consider a sphere of radius $a = 1$ cm, with volume velocity $q_0 = 0.001$ m^3/s. We will also enter the sound celerity $c_{air} = 343$ m s^{-1} and mass density $\rho_{air} = 1.2$ kg m^{-3}. In the Model Builder tab, right-click on **Global definitions→Parameters**. In the "Parameters" table, enter the parameters as given in Fig. 2.

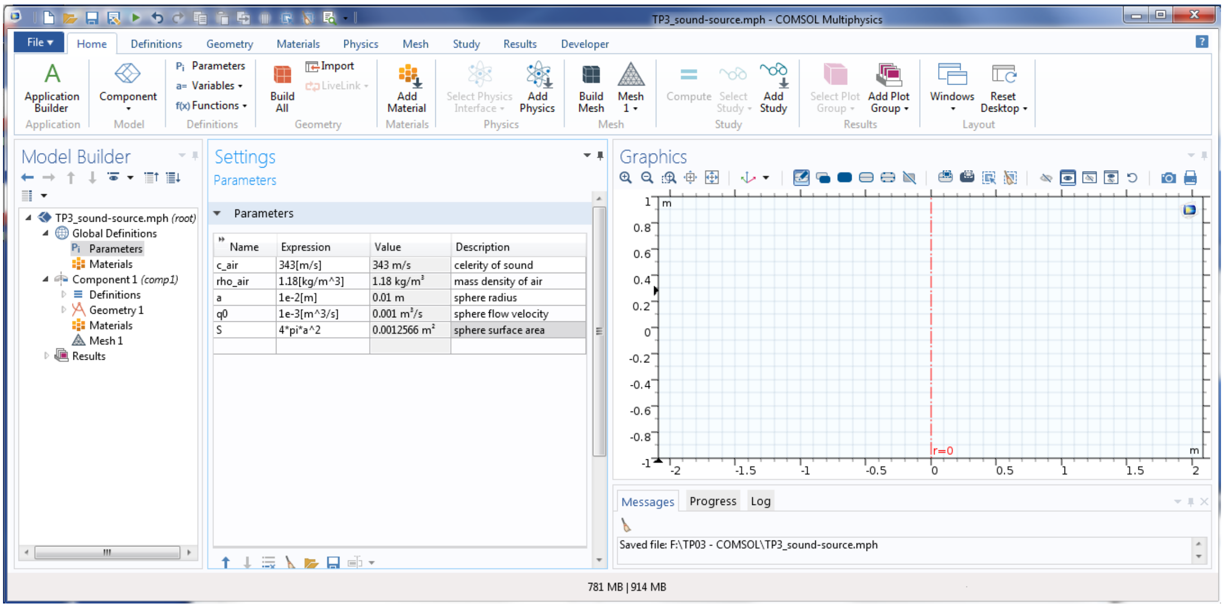


Figure 2: Parameters of the model

In the Model Builder tab, right-click on **Global definitions→Variables**. In the "Variables" table, enter the parameters as indicated in Fig. 3.

1.2.2 Drawing the geometry

Drawing the propagating medium, including non-reflecting boundaries
Now you will draw the geometry of the propagation medium and pulsating sphere. First, right-click on **Component 1/Geometry 1 → Circle**, and enter the dimensions as indicated in Fig. 4 for the propagation medium (sphere of diameter 1 m). It will create a first object called "Circle 1 (c1)".

Adding a Perfectly-matched layer (PML)
The limit of the medium should be set as absorbing such that no sound wave is reflected from the boundaries. For that purpose, the "Perfectly Matched Layer" (PML) property will be used, and one needs to define a "Layer" at the border of the domain, that will become a non-reflecting boundary. In the parameters of the **Circle 1**, go to "Layers" and select "Layer 1". Write down 0.1 in the "Thickness (m)" column. Then click on "Build all objects", It should resemble Fig. 4.

To indicate this new geometry is absorbent ("PML"), right-click on
Component 1/Definitions→Perfectly Matched Layer,

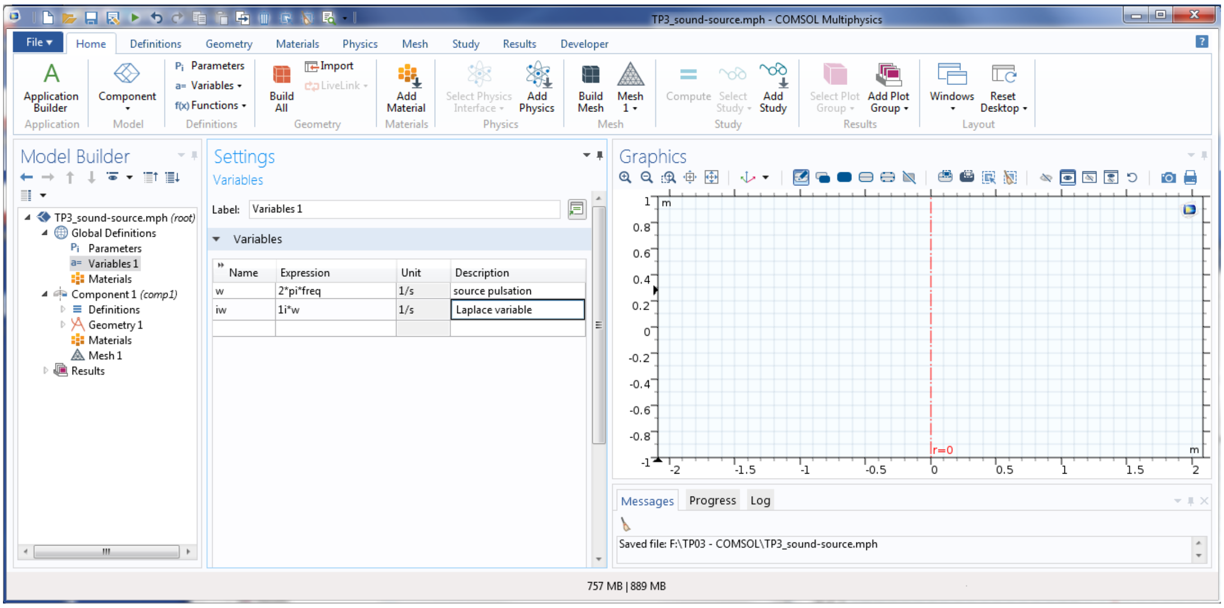


Figure 3: Variables of the model

and select the two subdomains (1 and 3) in the "Domain Selection". Do not change any other settings.

1.2.3 Subtracting a pulsating sphere

Now create the geometry of the pulsating sphere as for the propagating medium, except the radius that should be entered as "**a**". This will create a "Circle 2 (c2)" in the geometry list. In order to subtract this sphere from the propagating medium, right-click on **Component 1/Geometry→Booleans and Partitions→Difference**. In the settings tab, select the geometry "c1" in the "Objects to add", and on Geometry "c2" in the "Objects to subtract". Then, click on the "Build All" icon to see the propagation geometry including the sphere drawn on the "Graphics" tab.

1.3 Materials definition

Set the propagating medium as air. Right-click on **Component 1/Materials→Blank Material**, and enter the value ρ_{air} and c_{air} in the "Material settings" tab. Domains 1, 2 and 3 should be present in the "Geometry Entity Selection" panel at the top of the tab.

1.4 Setting the physics of the model

Now let's set the properties of the pulsating sphere. It should have a uniform radial velocity on its surface. To set this, right-click on **Component 1/Pressure Acoustics, Frequency Domain(acpr)→Normal velocity**. This will set the "inward" normal velocity at the surface of the pulsating sphere. Select the boundaries of the hollow sphere of radius a (boundaries 8 and 9) in the "Boundary selection" box, and type the velocity in the bottom of the "Normal Velocity" tab.

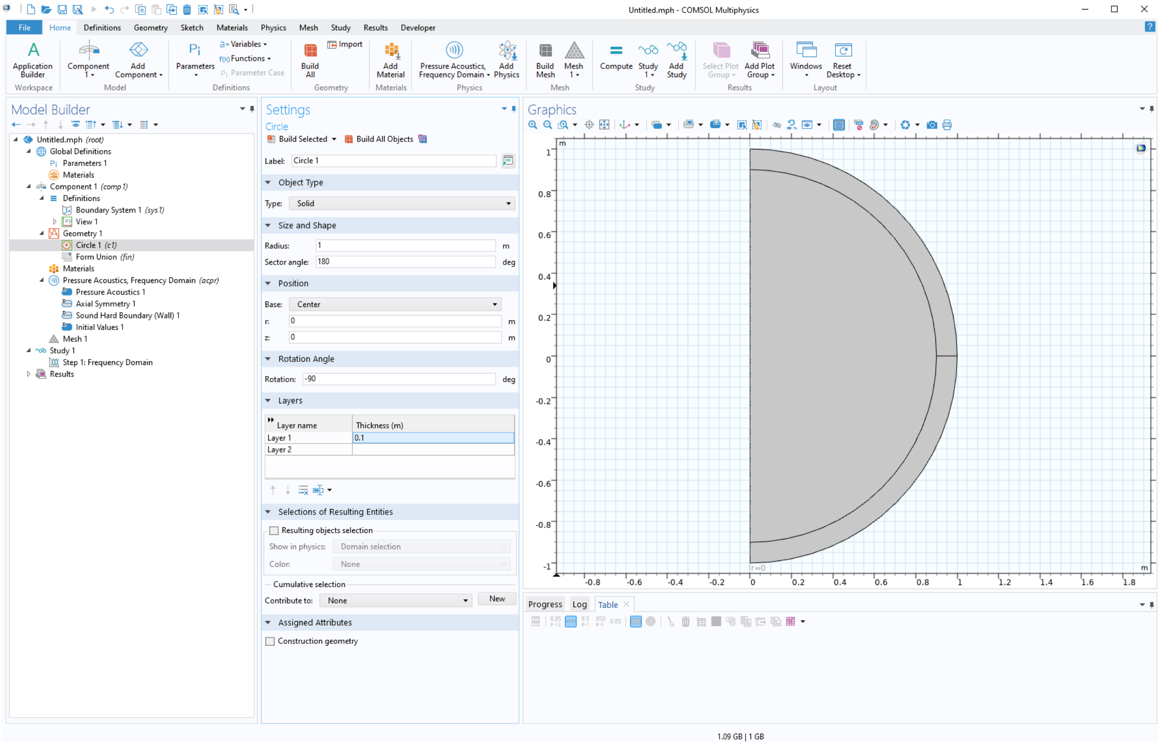


Figure 4: Geometry of the model

Question: Since we want a constant volume flow velocity q_0, what expression for acceleration should be written in the "Inward Velocity" settings ?

1.5 Meshing the model

Right-click on **Component 1/Mesh→Free Triangular**. In the **Component 1/Mesh1/Size** settings tab, click on "Custom", and chose a value for the Maximum Element Size.

Question: We will proceed the simulation up to $f = 4000$ Hz. The meshing should always be capable of processing 6 nodes per wavelength. What should be the maximum element size?

1.6 Setting the study

On the **Component 1/Study1/Step1:Frequency Domain** settings, enter the frequency values: $125, 250, 500, 1000, 2000, 4000$.

Run the computation by right-clicking on **Component 1/Study1→Compute**.

1.7 Analyzing the results

1.7.1 Directivity

You have many ways to view the results. On usual characteristics of sound sources is the polar diagram of the directivity. Right-click on **Component 1/Results→Polar Plot Group**.

Right-click on **Component 1/Results/Polar Plot Group→Line Graph**. In the Line Graph settings, select the propagating medium boundaries (boundaries 7 and 10) in the "Selection" box. In the "Expression" box, switch to "Sound Pressure Level" (acpr.Lp).

Then click on "Plot in" icon. **At this stage, discuss with the assistant to improve the rendering.** It is advised to define the angle θ thanks to COMSOL variable r (horizontal) and z (vertical), with the COMSOL function $atan2(x, y)$. Make sure to set the 0 rad pointing towards the top of the figure.

1.7.2 Geometrical damping

Right-click on **Component 1/Results→1D Plot Group**. Right-click on **Component 1/Results/1D Plot Group6→Line Graph**. In the Line Graph settings, select the axis of symmetry line (boundary 3) in the "selection" box. In the expression box, leave "acpr.p_t". Plot the curves. What do you think of these functions? Compare these results to the analytical expression given in the book "Audio" of Mario Rossi (See Eq. (1.139), page 37):

$$p_{sp} = Z_c k\, q \frac{exp\left[-jk(r-a)\right]}{4\pi r(ka-j)} \tag{1}$$

Note: to compare a simulation result with an analytical formulation, you should first enter the analytic expression of p_{sp} in the variable section (see **Component 1/Definition/Variables**). Then you can plot in the **Results** section any of the variables defined in this section.

It is also recommended to define a distance called *dist* instead of r (**beware:** the variable r already exists as a coordinate in COMSOL !) from the source center as a variable in the variable section.

1.7.3 Radiation impedance

We will now process the radiation impedance of the pulsating sphere, as defined in the book "Audio" in page 38. For this part of the exercise, you will first need to change the frequency vector: in the **Component 1/Study1/Step1:Frequency Domain** settings, and define an frequency vector ranging from 125 to 4'000 Hz in a logarithmic series of 100 values.

- Entry method: *Logarithmic*
- Start: *125*
- Stop *4000*
- Steps per decade: *100*

You are now asked to compute the total force applied to the surface of the sphere. For that, you will need to define a new node in the model by right-clicking on **Component 1/Definition→Nonlocal Couplings→Integration**. In the settings, chose "boundary" for **Geometry Entity Level**, and select boundaries 8 and 9 corresponding to the sphere surface. Don't forget to tick the option "Compute integral in revolved geometry". You can now integrate any quantity on the surface of the sphere, for instance the pressure.
Then right-click on **Component 1/Definition→Variables** to define the variables to be process with the model. You can then define the total pressure force applied on the surface as

$F = intop1(p)$ (p is the dependent variable, i.e. the pressure).
Define then the normalized radiation impedance $z_r = \frac{1}{\rho_{air}.c_{air}} \frac{F}{q_0}$. Compare the result with the theory.

1.7.4 Monopole

A monopole can be defined as a pulsating sphere the radius of which tends towards 0. Based on the preceding model, process the acoustic field of a point source (monopole) of flow velocity q_0 (you should specify the source as **Component 1/Pressure Acoustics, Frequency Domain (acpr)→Points→Monopole point source**). Compare the results with the theory provided in the book "Audio" by Mario Rossi.

Note: Disable "Normal velocity" in the physics "Pressure Acoustics, Frequency Domain", and disable "Circle2" and "Difference1" in "Geometry1".

2 Exercise 2: Radiation of the oscillating sphere

An oscillating sphere is a vibrating sphere, the velocity of which is axial along the symmetry axis (here z) (rather than radial as in the preceding exercise). You can refer to the "Audio" book, p 311

Question: how would you define the angle θ from the symmetry axis Oz? Define a new variable *theta* in the **Global Definition** section.

Repeat the preceding exercise, but with an axial velocity rather than a radial velocity. **Note:** Make sure you enter the correct normal acceleration in this model.

Comment the results. Compare to the theory provided in the book "Audio" of Mario Rossi.

3 Exercise 3: Radiation of the doublet, the dipole, and the cardioid source

Modify the model of the monopole to simulate the radiation of a dipole (two monopoles with opposite volume velocity) and a cardioid source (one dipole + one monopole at the center of the dipole, see the "Audio" book, p 309), with distance between monopoles being d=1cm.

4 Exercise 4: Beamformers

Consider an array of N=10 ponctual sources, distant of d=5 cm between each other. The array acoustic center is O and the reference flow velocity is given as q_0=10^{-3} m^3/s.

1. process the field of an uniform array (especially the directivity).

2. process the field of a phased array of lineic phase shift of $\beta z_i = -kz_i$, where z_i is the position of the ith source.

3. process the field of a uniform antenna of length $l = (N-1)d$

4. process the field of a phased antenna of lineic phase shift of $\beta z = -kz$, where z is the position of an elementary antenna at position z