Smart Grids Technologies
Module 1, Lab 2 - 03/03/2025
DFT-based Synchrophasor Estimation: Interpolated DFT (IpDFT)

1 Organization of the lab

During the previous Laboratory session you have learned about the DFT, about aliasing and spectral leakage and about reducing their effects. During this lab session you will analyse the synchrophasor estimation technique called Interpolated DFT (IpDFT), encountered during Lecture #4. In particular, you will learn to:

- Analytically formulate an IpDFT-based algorithm for synchrophasor estimation;
- Experimentally assess its performance.

Moreover, you will make use of the concepts you have learned so far to build your first PMU prototype and characterize its performance.

This report will not be graded; however, its submission is mandatory. The purpose of the questions within this document is to enhance your comprehension of the subject matter. Your acquired knowledge from **all three laboratories of Module 1** will be evaluated in a quiz scheduled for **Monday, March 17th** from **9:15 to 10:00**. The deadline for submission of the reports is **Sunday, March 16th** at **23:55**.

2 Theoretical Background

The IpDFT is a technique that enables us to extract the parameters of a sinusoidal waveform by processing the highest DFT bins of the related DFT spectrum. In particular, the IpDFT enables the mitigation of the effects generated by *incoherent sampling* (i.e., when the frequency of the tone under analysis is not an integer multiple of the frequency resolution) by:

- Applying suitably shaped window functions to reduce the *long range spectral leakage* effects;
- Interpolating the highest DFT bins of the spectrum to minimize the *scalloping* effects.

Indeed, in case of incoherent sampling, the peak value of the continuous spectrum of the tone under analysis is located between two consecutive DFT bins and can be expressed as (Lecture #4, Slide 6):

$$k_{peak} = k_m + \delta \tag{1}$$

being $-0.5 \le \delta < 0.5$ a fractional correction term and k_m the index of the highest bin.

The IpDFT problem lies in finding the correction term δ (and, consequently, the fundamental tone's parameters $\{f_0, A_0, \varphi_0\}$) that better approximates the exact location of the main spectrum tone.

Lecture #4 has defined the 2-points IpDFT algorithm for the rectangular and the Hanning windows. For the **rectangular** window, the fractional term δ can be computed by interpolating the 2 highest DFT bins as (Lecture #4, Slide 10):

$$\hat{\delta} = \varepsilon \, \frac{|X(k_m + \varepsilon)|}{|X(k_m)| + |X(k_m + \varepsilon)|} \tag{2}$$

where:

$$\varepsilon = \pm 1 = \operatorname{sgn}\left(|X\left(k_m + 1\right)| - |X\left(k_m - 1\right)|\right), \tag{3}$$

Consequently, the waveform parameters can then be computed as follows:

$$\hat{f}_0 = (k_m + \hat{\delta})\Delta f \tag{4}$$

$$\hat{A}_0 = |X(k_m)| \left| \frac{\pi\hat{\delta}}{\sin(\pi\hat{\delta})} \right| \tag{5}$$

$$\hat{\varphi}_0 = \angle X(k_m) - \pi\hat{\delta} \tag{6}$$

The rectangular function is characterized by the narrowest main lobe but at the same time it exhibits the highest side lobes, i.e. those with the lowest decay rate.

In order to reduce the effects of long range spectral leakage generated by the side-lobe levels, IpDFT algorithms typically adopt bell-shaped windows. The most common is the **Hanning (Hann)** window, as it offers a good trade-off between the main-lobe width and side-lobe levels. In this case, the correction term δ is defined as (Lecture #4, Slide 12):

$$\hat{\delta} = \varepsilon \, \frac{2\,|X(k_m+\varepsilon)| - |X(k_m)|}{|X(k_m)| + |X(k_m+\varepsilon)|} \tag{7}$$

And consequently:

$$\hat{f}_0 = (k_m + \hat{\delta})\Delta f \tag{8}$$

$$\hat{A}_0 = |X(k_m)| \left| \frac{\pi\hat{\delta}}{\sin(\pi\hat{\delta})} \right| \left| \hat{\delta}^2 - 1 \right| \tag{9}$$

$$\hat{\varphi}_0 = \angle X(k_m) - \pi\hat{\delta} \tag{10}$$

The main assumptions behind the formulation of the IpDFT technique are the following:

1. The input signal is characterized by time-invariant parameters within the analysis window;
2. The Nyquist-Shannon sampling theorem is satisfied, i.e. no aliasing is present;
3. The DFT bins used to perform the interpolation are only generated by the positive image of the tone under analysis.

When any of these assumptions is not met, the IpDFT may produce erroneous results.

Q1/ How can you choose the analysis parameters to minimize the error if Assumption #1 is violated. What are the undesireable consequences of making these design choices?

[A1]

Q2/ How can you choose the analysis parameters to minimize the error if Assumption #2 is violated. What are the undesireable consequences of making these design choices?

[A2]

Q3/ How can you choose the analysis parameters to minimize the error if Assumption #3 is violated. What are the undesireable consequences of making these design choices?

[A3]

2.1 Synchrophasor Estimation Performance

According to the IEC/IEEE 60255-118-1:2018 Std., the performance of a synchrophasor estimation algorithm can be assessed using the following performance indicators:

- The Total Vector Error (TVE), i.e. the normalised Euclidean distance between the estimated synchrophasor ($\hat{X}_{Re} + j\hat{X}_{Im}$) and the reference one ($X_{Re} + jX_{Im}$):

$$\text{TVE} = \sqrt{\frac{(\hat{X}_{Re} - X_{Re})^2 + (\hat{X}_{Im} - X_{Im})^2}{X_{Re}^2 + X_{Im}^2}} \cdot 100 \tag{11}$$

- The Amplitude Error (AE) is defined as the estimated amplitude minus the reference amplitude.

- The phase Error (pE) is defined as the estimated phase minus the reference phase.
- The Frequency Error (FE), is computed as the absolute value of the difference between the true frequency (f) and the estimated one ($\hat{f}$):

$$\mathrm{FE} = \left| f - \hat{f} \right| \tag{12}$$

2.2 References

1. Chapter 3"*DFT-based synchrophasor estimation processes for Phasor Measurement Units applications: algorithms definition and performance analysis*", in the book *"Advanced Techniques for Power System Modelling, Control and Stability Analysis"* edited by F. Milano, IET 2015.

2. "IEEE/IEC International Standard - Measuring relays and protection equipment - Part 118-1: Synchrophasor for power systems - Measurements," in IEC/IEEE 60255-118-1:2018 , vol., no., pp.1-78, 19 Dec. 2018

3 LabVIEW Implementation

From Moodle, download the folder *"SGT_Lab2_IpDFT"* and open the LabVIEW project called *"SGT_Lab2_IpDFT.lvproj"*. The folder *"subVIs"* includes the VIs you have developed during the previous laboratory session.

TASK 1 - PMU Implmentation – 2-point IpDFT: Open the subVI called *"IpDFT.vi"*. Based on the provided skeleton, **modify its implementation in order to include the 2-point IpDFT** for the Hanning and for the rectangular window.

TASK 2 - Error estimation: Based on the provided skeleton, **modify the subVI** called *"PMU_TVE&FE.vi"* to compute the error quantities referenced in Section 2.1 above: TVE, FE, pE and AE.

Q4/ Analyse the block scheme of *"PMU.vi"* and describe it. How is the reporting of the waveforms managed?

[A4]

4 PMU experimental validation

Congrats! You should now have a working PMU. Let's test it out!
Let's first observe the windowed signal (top graph). In order to answer questions **Q6** through **Q8**:

- Set the window length to 5 periods at 50 Hz;
- Set the rated power system frequency to 50 Hz;
- Set the reporting rate to 10 fps;

Run the *"PMU.vi"* and describe the **graph of the acquired waveform** in the scenarios proposed in Q6 through Q8. Explain your observations.

Q5/ Set the frequency of the generated signal to 50 Hz. Looking at the graph of the acquired waveform, compare the results for a window length of 5 and 5.5 periods. How do the waveform and subPPS change?

[A5]

Q6/ Set the window length to 5 periods and the frequency of the generated signal to 50 Hz. Change the reporting rate to 10, 50 and 51 fps. How do the waveform and subPPS change?

[A6]

Q7/ Set the window length to 5 periods and the reporting rate to 50 fps. Change the fundamental frequency of the generated signal to 49.9, 50 and 50.1 Hz. How do the waveform and subPPS change?

[A7]

Assess the accuracy of the IpDFT-based estimations of frequency, amplitude and phase by comparing them with the reference values.

- Fix the window length to 60 ms;
- Set the rated power system frequency to 50 Hz;
- Set the reporting rate to 50 fps;

For each case below, compare the performance of the rectangular and the Hanning window functions. Which window performs better and why?

Q8/ A single-tone signal at 50 Hz, phase 0 rad.

[A8]

Q9/ A single-tone signal at 50 Hz, phase 1 rad.

[A9]

Q10/ A single-tone signal at 49 Hz, phase 0.75 rad.

[A10]

Q11/ A multi-tone signal characterized by a fundamental tone at 50 Hz and a 10% harmonic tone at 150 Hz (phase 0.2 rad for both tones).

[A11]

Q12/ A multi-tone signal characterized by a fundamental tone at 49.5 Hz and a 10% harmonic tone at 148.5 Hz (phase 0.8 rad for both tones).

[A12]