Full Name:

Smart Grids Technologies
Module 2, Lab 1 - 21/03/2025
Admittance Matrix Calculus

1 Organization

1.1 Objectives

This lab session covers the basics for the calculation of the network admittance matrix. It is assumed that you are already familiar with the basic concepts of the nodal admittance model. The goal of the lab is to provide you with a tutorial as well as with a documented toolbox to be used later on in the course. First, you will be asked to compute by hand the admittance matrix of a small-scale electrical network. Then, you will be requested to verify your calculations using MATLAB, in order to familiarize yourself with the provided toolbox.

1.2 Report

This report will not be graded; however, its submission is mandatory to unlock access to Quiz 2. The purpose of the questions within this document is to enhance your comprehension of the subject matter. Your acquired knowledge from all three laboratories of **Module 2** will be evaluated in Quiz 2 scheduled for Monday, April 28. The deadline for submission of this report is Sunday, March 30 at 23:55. Do not forget to write your full name in the corresponding box on top of this page.

2 Theory

This section briefly recapitulates the basics of the per-unit system and nodal admittance matrix calculus. If you are already familiar with these topics, feel free to skip it, and proceed directly to Sec. 3.

2.1 Per-Unit System

Introduction The analysis of complex power systems can be simplified using the concept of *per-unit systems*. A per-unit system system is obtained by selecting a common set of *base values*, and expressing all parameters with respect to these. That is, the analysis is performed in *relative units*, or *per unit* (p.u.), rather than *absolute units*. The definition of any quantity (e.g., power, voltage, current, impedance) in the per-unit system is

$$\text{Quantity (p.u.)} = \frac{\text{Quantity (absolute units)}}{\text{Associated base value (absolute units)}}. \tag{1}$$

Observe that the arguments of complex powers $\bar{S}$, voltages $\bar{V}$, currents $\bar{I}$, impedances $\bar{Z}$, etc. are not affected by this conversion. One advantage of per-unit systems is that electrical networks with different voltage levels (i.e., which contain transformers) can be represented using impedances only.

Once the *base power* A_b (i.e., P or Q) and the *base voltage* V_b are selected, the *base current* I_b and the *base impedance* Z_b (or *base admittance* Y_b) can be calculated.

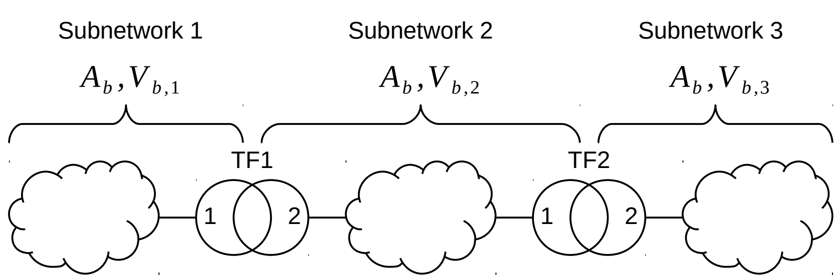


Figure 1: Base values in an electrical network with transformers.

Selection of Base Values In the presence of transformers, a network is devided into subnetworks at different voltage levels (see Fig. 1).

- The base voltages on each side of a transformer (each subnetwork) are selected.
- The base power is the same for the entire network under study.
- All the other base values (current, impedance, admittance), for each subnetwork, are derived from chosen base voltages and the base power.

First, consider a *single-phase* system. Let $A_{b,\text{1P}}$ be the single-phase base power, and $V_{b,\text{LN}}$ the *line-to-neutral* base voltage. Then, the base current I_b and the base impedance Z_b can be calculated as follows

$$I_b = \frac{A_{b,\text{1P}}}{V_{b,\text{LN}}}, \tag{2}$$

$$Z_b = \frac{V_{b,\text{LN}}^2}{A_{b,\text{1P}}}. \tag{3}$$

Now, consider a *three-phase* system. Let $A_{n,\text{3P}}$ be the three-phase base power, and $V_{b,\text{LL}}$ the *line-to-line* base voltage. Then, the base current I_b and the base impedance Z_b can be calculated as follows

$$I_b = \frac{A_{b,\text{3P}}}{\sqrt{3}V_{b,\text{LL}}}, \tag{4}$$

$$Z_b = \frac{V_{b,\text{LL}}^2}{A_{b,\text{3P}}}. \tag{5}$$

Change of Base Let A_b', V_b' and A_b'', V_b'' define two different bases. The following relations can be used to perform a change of base

$$\frac{\bar{S}'\ (\text{p.u.})}{\bar{S}''\ (\text{p.u.})} = \frac{A_b''}{A_b'}, \tag{6}$$

$$\frac{\bar{V}'\ (\text{p.u.})}{\bar{V}''\ (\text{p.u.})} = \frac{V_b''}{V_b'}, \tag{7}$$

$$\frac{\bar{I}'\ (\text{p.u.})}{\bar{I}''\ (\text{p.u.})} = \frac{I_b''}{I_b'} = \frac{A_b''}{A_b'}\frac{V_b'}{V_b''}, \tag{8}$$

$$\frac{\bar{Z}'\ (\text{p.u.})}{\bar{Z}''\ (\text{p.u.})} = \frac{Z_b''}{Z_b'} = \left(\frac{V_b''}{V_b'}\right)^2 \frac{A_b'}{A_b''}, \tag{9}$$

$$\frac{\bar{Y}'\ (\text{p.u.})}{\bar{Y}''\ (\text{p.u.})} = \frac{Y_b''}{Y_b'} = \left(\frac{V_b'}{V_b''}\right)^2 \frac{A_b''}{A_b'}. \tag{10}$$

Further Information If you still have doubts regarding the use of relative units, please check the exercise on per-unit calculus on moodle.

2.2 Nodal Admittance Matrix

Nodal Analysis Consider an electrical network whose *nodes* are labeled as $n \in \mathcal{N} \coloneqq \{1, \cdots, N\}$, and the *ground* as $g \in \mathcal{G} \coloneqq \{0\}$. Furthermore, let $\ell_k \in \mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ be the *branches* and $t_n \in \mathcal{T} \coloneqq \mathcal{N} \times \mathcal{G}$ the *shunts*. A branch $\ell_k = (i, j)$ $(i, j \in \mathcal{N})$ is associated with a *branch admittance* $\bar{y}_{\ell_k} = \bar{y}_{ij}$, and a shunt $t_i = (i, 0)$ with a *shunt admittance* $\bar{y}_{t_i} = \bar{y}_{i0}$. Let $\bar{V}_i$ the nodal voltage phasor at i, and $\bar{I}_i$ the nodal current phasor at i (i.e., the net current injected by the generators / absorbed by the loads connected to i). The electrical network is described by the following system of linear equations

$$\begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_n \\ \vdots \\ \bar{I}_N \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \cdots & \bar{Y}_{1n} & \cdots & \bar{Y}_{1N} \\ \vdots & & \vdots & & \vdots \\ \bar{Y}_{n1} & \cdots & \bar{Y}_{nn} & \cdots & \bar{Y}_{nN} \\ \vdots & & \vdots & & \vdots \\ \bar{Y}_{N1} & \cdots & \bar{Y}_{Nn} & \cdots & \bar{Y}_{NN} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \vdots \\ \bar{V}_n \\ \vdots \\ \bar{V}_N \end{bmatrix} \tag{11}$$

or in matrix form

$$\bar{\mathbf{I}} = \bar{\mathbf{Y}}\bar{\mathbf{V}} \tag{12}$$

$\bar{\mathbf{Y}}$ is called the *nodal admittance matrix*. Its entries are calculated as follows. The diagonal elements $\bar{Y}_{ii}$ are the sum of the admittances of all branches and shunts connected to the node i (i.e., starting or ending in i). That is

$$\bar{Y}_{ii} = \bar{y}_{i0} + \sum_{(i,x)\in\mathcal{L}} \bar{y}_{ix} + \sum_{(x,i)\in\mathcal{L}} \bar{y}_{xi}. \tag{13}$$

The off-diagonal elements $\bar{Y}_{ij}$ $(i \neq j)$ are the negative of the corresponding branch admittance (if there exists a branch between i and j). That is

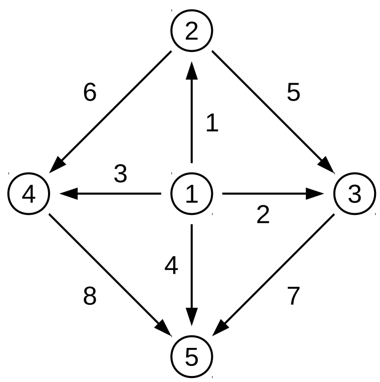
$$\bar{Y}_{ij} = \begin{cases} -\bar{y}_{ij} & \text{if } (i, j) \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

For small electrical networks, $\bar{\mathbf{Y}}$ can be constructed using (13)–(14).

Automated Construction For large electrical networks, $\bar{\mathbf{Y}}$ can be constructed in an automated manner. Namely

$$\bar{\mathbf{Y}} = \mathbf{A}_{\mathfrak{B}}^T \bar{\mathbf{Y}}_{\mathcal{L}} \mathbf{A}_{\mathfrak{B}} + \bar{\mathbf{Y}}_{\mathcal{T}} \tag{15}$$

where



(a) Graph diagram.

branch/node	1	2	3	4	5
$\ell_1 = (1, 2)$	+1	−1	0	0	0
$\ell_2 = (1, 3)$	+1	0	−1	0	0
$\ell_3 = (1, 4)$	+1	0	0	−1	0
$\ell_4 = (1, 5)$	+1	0	0	0	−1
$\ell_5 = (2, 3)$	0	+1	−1	0	0
$\ell_6 = (2, 4)$	0	+1	0	−1	0
$\ell_7 = (3, 5)$	0	0	+1	0	−1
$\ell_8 = (4, 5)$	0	0	0	+1	−1

(b) Incidence matrix in table form.

Figure 2: Example for the construction of $\mathbf{A}_{\mathfrak{B}}$.

- $\bar{\mathbf{Y}}_{\mathcal{L}}$ is the *primitive branch admittance matrix*, which is given by

$$\bar{\mathbf{Y}}_{\mathcal{L}} \coloneqq \mathrm{diag}_{\ell_k \in \mathcal{L}}(\bar{y}_{\ell_k}) = \mathrm{diag}_{(m,n)\in\mathcal{L}}(\bar{y}_{mn}). \tag{16}$$

- $\bar{\mathbf{Y}}_{\mathcal{T}}$ is the *primitive shunt admittance matrix*, which is given by

$$\bar{\mathbf{Y}}_{\mathcal{T}} \coloneqq \mathrm{diag}_{t_n \in \mathcal{T}}(\bar{y}_{t_n}) = \mathrm{diag}_{(n,0)\in\mathcal{T}}(\bar{y}_{n0}). \tag{17}$$

- $\mathbf{A}_{\mathfrak{B}}$ is the *incidence matrix* of the *branch graph* $\mathfrak{B} = (\mathcal{N}, \mathcal{L})$, which is given by

$$\mathbf{A}_{\mathfrak{B}} : \; A_{kn} = \begin{cases} +1 & \text{if } \ell_k = (n, \cdot) \in \mathcal{L} \\ -1 & \text{if } \ell_k = (\cdot, n) \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

An example of $\mathbf{A}_{\mathfrak{B}}$ can be found in Fig. 2. Note that we can choose directions of the graph branches arbitrarily.

Model of a Transformer *Transformers* allow stepping the voltage up or down. In this lab, we will see how such devices can be incorporated into the nodal admittance matrix. A transformer can be modelled by an ideal transformer with a ratio $1 : n$ in series with a short-circuit impedance $\bar{Z}_{sc}$ (here in the secondary side), as shown in Fig. 3. Here, we neglected the transformer's shunt admittance, $\bar{Y}_0$. **Recall that, in this case,** $n = \dfrac{V_{n,2}}{V_{n,1}}$**!**

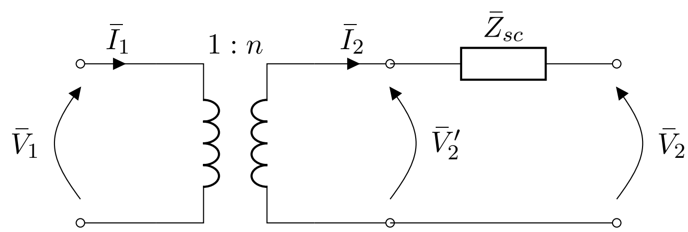


Figure 3: Model of a transformer. The short-circuit impedance $\bar{Z}_{sc}$ is referred to the secondary side.

As discussed in the lecture on the per-unit method, the complexity of the transformer model depends on the choice of the base voltages $V_{b,1}$ and $V_{b,2}$ on the primary and secondary side, respectively. If the base voltages are chosen so that their ratio is equal to the nominal transformer ratio

$$\frac{V_{b,2}}{V_{b,1}} = n, \tag{19}$$

the per-unit equivalent circuit shown in Fig. 4 is obtained (i.e., a branch element $\bar{y}_{sc} = \bar{z}_{sc}^{-1}$ only). Note that the p.u. short-circuit admittance $\bar{y}_{sc}$ is obtained using chosen base values of the secondary side of the transformer.

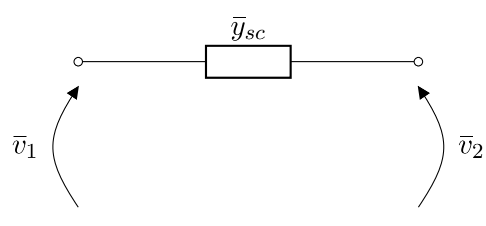


Figure 4: Using two base voltages such that $V_{b,2} = nV_{b,1}$.

Conversely, in the case when

$$\frac{V_{b,2}}{V_{b,1}} \neq n, \tag{20}$$

the per-unit equivalent circuit shown in Fig. 5 is obtained (i.e., a complete π-section equivalent) where

$$m = n\left(\frac{V_{b,2}}{V_{b,1}}\right)^{-1}. \tag{21}$$

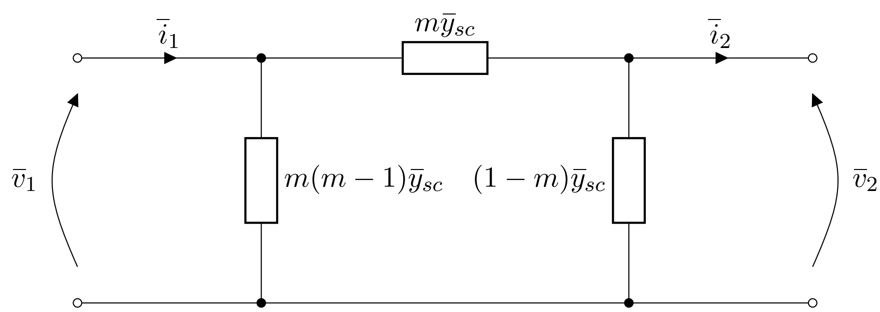


Figure 5: Using two base voltages such that $V_{b,2} \neq nV_{b,1}$.

Property. In per-unit, short-circuit impedances $\bar{z}'_{sc}$ and $\bar{z}''_{sc}$ referred to the primary and secondary side of a transformer with ratio $\frac{V_{n,2}}{V_{n,1}} = n$, respectively, are related as

$$\bar{z}'_{sc} = \frac{1}{m^2}\bar{z}''_{sc}, \tag{22}$$

where m is the per-unit transformer ratio, $m = n\left(\frac{V_{b,2}}{V_{b,1}}\right)^{-1}$.

Proof. Short-circuit impedance seen from primary and secondary sides, $\bar{Z}'_{sc}$ and $\bar{Z}''_{sc}$, respectively, in absolute units are related as:

$$\bar{Z}'_{sc} = \frac{1}{n^2}\bar{Z}''_{sc}. \tag{23}$$

Therefore, in per-unit, with base voltages $V_{b,1}$ and $V_{b,2}$ and base power A_b, one reads:

$$\bar{z}'_{sc} = \frac{\bar{Z}'_{sc}}{Z_{b,1}} = \frac{\bar{Z}'_{sc}}{\frac{V_{b,1}^2}{A_b}} = \frac{\frac{1}{n^2}\bar{Z}''_{sc}}{\frac{V_{b,1}^2}{A_b}\cdot\frac{V_{b,2}^2}{V_{b,2}^2}} = \frac{1}{n^2\left(\frac{V_{b,2}}{V_{b,1}}\right)^{-2}}\cdot\bar{z}''_{sc} = \frac{1}{m^2}\bar{z}''_{sc}. \quad \blacksquare \tag{24}$$

Corollary. If the base voltages ratio is equal to the transformer's ratio, $\frac{V_{b,2}}{V_{b,1}} = n$, the per-unit short-circuit impedances referred to the primary and secondary side are equal:

$$\overline{z}'_{sc} = \overline{z}''_{sc}. \tag{25}$$

Proof. It follows directly from (24), since for $\frac{V_{b,2}}{V_{b,1}} = n$ the per-unit transformer ratio is $m = 1$. ■

Note that the property holds for arbitrary base power, $A_b > 0$. If we choose $V_{b,1} = V_{n,1}$ and $V_{b,2} = V_{n,2}$, then obviously, both per-unit short-circuit impedances, referred to primary and secondary sides, are equal. That is the reason why on the nameplate of a transformer, it is usually not specified which side the short-circuit per-unit impedance is referred to.

2.3 References

1. John J. Grainger and William D. Stevenson, "Power System Analysis", McGraw-Hill, 1994.

2. Leon O. Chua and Pen-Min Lin., "Computer-Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques", Prentice-Hall, 1975.

3 Exercises

3.1 Calculation by Hand

In this part of the lab, you will compute the nodal admittance matrix of the small-scale power grid shown in Fig. 6. It consists of six buses B1-B6, which are connected by five lines L1-L5. The nominal voltage is 4.16 kV (line-to-line, RMS). The lines are represented by π-section equivalents, which are characterized by the *transmission line parameters* R', X', and B' along with the line length l. These parameters are listed in Tab. 1.

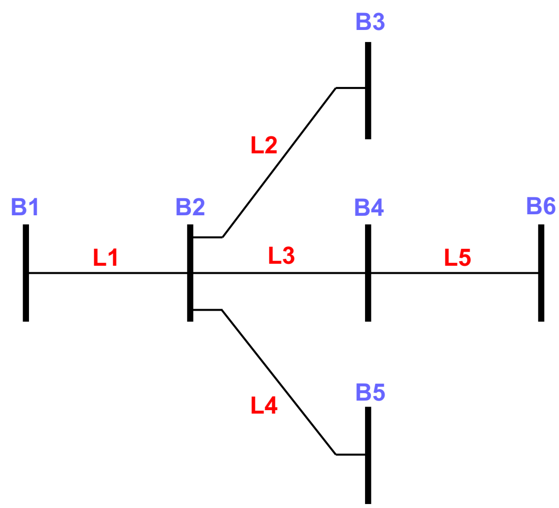


Figure 6: Topology of the small-scale power grid.

Table 1: Electrical parameters of the small-scale power grid.

Line	From	To	R' $\left(\frac{\Omega}{\text{km}}\right)$	X' $\left(\frac{\Omega}{\text{km}}\right)$	B' $\left(\frac{\text{S}}{\text{km}}\right)$	l (km)
L1	B1	B2	0.151	0.298	$1.196 \cdot 10^{-6}$	1.2
L2	B2	B3	0.122	0.331	$1.231 \cdot 10^{-6}$	1.4
L3	B2	B4	0.143	0.324	$1.215 \cdot 10^{-6}$	1.1
L4	B2	B5	0.169	0.254	$1.137 \cdot 10^{-6}$	0.7
L5	B4	B6	0.173	0.325	$1.225 \cdot 10^{-6}$	2

Q1/ Construct the incidence matrix $\mathbf{A}_{\mathfrak{B}}$ of the power grid. In doing so, respect the numbering of the buses and lines according to Fig. 6. How many different matrices $\mathbf{A}_{\mathfrak{B}}$ can we construct for such numbering?

[A1]

Q2/ Consider the per-unit base given by $A_b = 6$ MVA and $V_b = 4.16$ kV. Calculate the following quantities for the power grid (all in p.u.):

i. The primitive branch admittance matrix $\bar{\mathbf{Y}}_{\mathcal{L}}$.

ii. The primitive shunt admittance matrix $\bar{\mathbf{Y}}_{\mathcal{T}}$.

iii. The nodal admittance matrix $\bar{\mathbf{Y}}$. *Here, you are encouraged to use* (13)–(14).

For the sake of simplicity, list only the non-zero elements of these matrices (i.e., the diagonal elements of $\bar{\mathbf{Y}}_{\mathcal{L}}$ and $\bar{\mathbf{Y}}_{\mathcal{T}}$, and the non-zero elements of $\bar{\mathbf{Y}}$).

[A2]

Suppose that the operating voltage of only line L5 needs to be increased in order to augment its power transfer capacity. More precisely, the nominal voltage shall be increased from 4.16 kV to 4.50 kV. To this end, two transformers are installed at the ends of the line as shown in Fig. 7. The electrical parameters of the transformers are listed in Tab. 2.

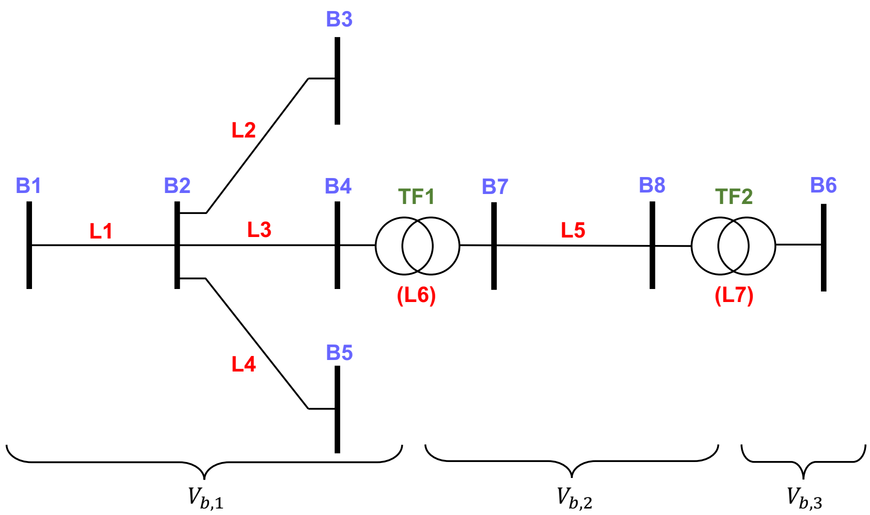


Figure 7: Modified topology with added transformers.

Component	Parameters
TF1	$A_n = 10$ MVA, $V_{n,1} = 4.16$ kV, $V_{n,2} = 4.50$ kV, $z_{sc} = 6\%$, $\cos\phi_{sc} = 0.22$
TF2	$A_n = 10$ MVA, $V_{n,1} = 4.20$ kV, $V_{n,2} = 4.50$ kV, $V_{sc} = 5\%$, $P_{sc} = 140$ kW

Table 2: Electrical parameters of the two transformers.

Q3/ The new topology is given in Fig. 7.

(a) Construct the new incidence matrix $\mathbf{A}_{\mathfrak{B}}$ (respect the numbering of the buses and lines according to Fig. 7).

(b) The transformers now introduce different voltage levels in the grid. Which conditions must be satisfied such that the transformers in the p.u. system can be represented by the *simple model* shown in Fig. 4? How many different solutions for $(V_{b,1}, V_{b,2}, V_{b,3})$ can you propose?

(c) Taking into account the conditions from (b), if we set $V_{b,1} = 4.16$ kV, what are the values of $V_{b,2}$ and $V_{b,3}$?

[A3]

Consider the base values $A_b = 6$ MVA and $V_{b,1} = 4.16$ kV, and $V_{b,2}$ and $V_{b,3}$ determined in the previous question.

Q4/ Calculate the equivalent circuit parameters of TF1 and TF2.

[A4]

Q5/ Calculate the following quantities for the modified power grid (all in p.u.):

i. The primitive branch admittance matrix $\bar{\mathbf{Y}}_{\mathcal{L}}$.
ii. The primitive shunt admittance matrix $\bar{\mathbf{Y}}_{\mathcal{T}}$.
iii. The nodal admittance matrix $\bar{\mathbf{Y}}$.

As before, list only the non-zero elements of these matrices.

[A5]

3.2 Calculation using MATLAB

In this part of the lab, you will compute the nodal admittance matrix using a simple MATLAB toolbox[1]. Download the MATLAB source code from

```
https://moodle.epfl.ch/mod/folder/view.php?id=1288651a94
```

The folder contains the following items:

- The script `main.m`, which calculates the nodal admittance matrix $\bar{\mathbf{Y}}$.
- Two configuration files `data_lines.txt` and `data_transformers.txt`, where you will have to specify the electrical parameters of the grid.
- Two template functions `build_lines.m` and `build_transformers.m`, where you will have to write code to construct the equivalent circuits of the lines and transformers.
- Two auxiliary functions `build_parameters.m` and `print_matrix.m`, which you do not need to edit.

The calculations are done in 4 steps, which are briefly explained now.

Step 1: Electrical Parameters

In the first step, the electrical parameters of lines and transformers are read from `data_lines.txt` and `data_transformers.txt`, respectively.

Each row in `data_lines.txt` corresponds to a line of the power grid, and contains the following entries (separated by tabulators):

1. Index of the bus where the line starts.
2. Index of the bus where the line ends.
3. Per-unit-length resistance R' in Ω/km.
4. Per-unit-length reactance X' in Ω/km.
5. Per-unit-length susceptance B' in S/km.
6. Length l in km.

[1]For those who are not familiar with Matlab, a general tutorial is available online under `https://ch.mathworks.com/help/index.html`.

7. Base voltage V_b for the line in V.

Each row in `data_transformers.txt` corresponds to a transformer in the power grid, and contains the following entries (separated by tabulators):

1. Index of the bus where the primary side is connected.
2. Index of the bus where the secondary side is connected.
3. Nominal power A_n.
4. Nominal voltage $V_{n,1}$ on the primary side.
5. Nominal voltage $V_{n,2}$ on the secondary side.
6. Short-circuit resistance r_{sc} in p.u.
7. Short-circuit reactance x_{sc} in p.u.
8. Base voltage $V_{b,1}$ on the primary side in V.
9. Base voltage $V_{b,2}$ on the secondary side in V.

One line is already configured in the files provided with the template code.

Step 2: Per-Unit Models

In the second step, the per-unit models of lines and transformers are built. The base power A_b needs to be specified:

```
% !!! put the correct base value here !!!
A_b = 6e6;      % base value for the power in VA
```

The π-section equivalent circuits of lines and transformers are constructed by the functions `build_lines` and `build_transformers`, respectively. Each function builds an array of structs with the following fields (see Fig. 8)

- `i`: the start node of the line.
- `j`: the end node of the line.
- `Y_ij`: the branch admittance between start node i and end node j.
- `Y_i_ij`: the shunt admittance on the side of the start node i.
- `Y_j_ij`: the shunt admittance on the side of the end node j.

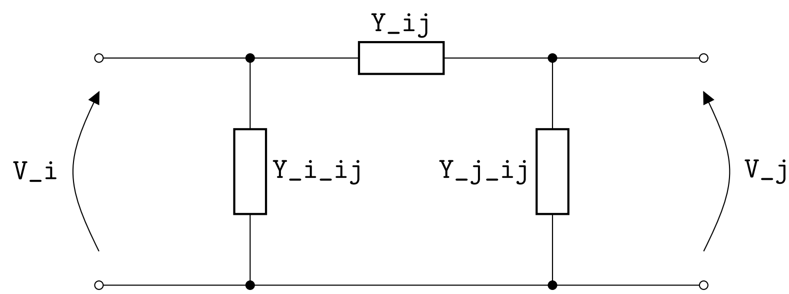


Figure 8: π-section equivalent in the MATLAB script.

In the files provided with the template code, only the function interfaces, as well as some basic functionality, are given. You will be asked to implement the rest later on. Please add your own code only where indicated by:

```
% **********
% !!! write your own code here !!!
% **********
```

Step 3: Incidence Matrix & Primitive Admittance Matrices

In the third step, the incidence matrix $\mathbf{A}_{\mathfrak{B}}$ and the primitive admittance matrices $\bar{\mathbf{Y}}_{\mathcal{T}}$ and $\bar{\mathbf{Y}}_{\mathcal{L}}$ are built from the π-section equivalents. This is done by the auxiliary function `calculate_parameters`. You do not need to change anything here, but you can of course take a look at the code if you like.

Step 4: Nodal Admittance Matrix

In the fourth step, the nodal admittance matrix is computed using (15):

```
Y = A.' * Y_L * A + Y_T;
```

Furthermore, the diagonal elements of $\bar{\mathbf{Y}}_{\mathcal{L}}$, $\bar{\mathbf{Y}}_{\mathcal{T}}$, and nonzero upper diagonal elements of $\bar{\mathbf{Y}}$ are printed to the console using the auxiliary function `print_matrix`.

Now, it is your turn to write some code and to perform some analyses. First, finish the code that builds the π-section equivalents:

Q6/ Complete the functions `build_lines` and `build_transformers` where indicated, and paste your code here.

[A6]

Then, configure the original power grid as specified in Fig. 6 and Tab. 1:

Q7/ Complete the file `data_lines.txt` with the missing lines L2-L5, and paste the content of the completed file here. Run the script `main.m` and observe the console output in step 4. Are the results in accordance with what you have obtained in **Q2**?

[A7]

Do the same for the modified power grid treated in **Q3** and **Q5**:

Q8/ In order to equip line L5 with transformers (see Sec. 3.1), modify the file `data_lines.txt` and complete the file `data_transforers.txt`, and paste the contents of the updated files here. Run the script `main.m` again, and observe the console output in step 4. Are the results in accordance with what you have obtained in **Q5**?

[A8]

Finally, let's analyse the results when only one base voltage is used for the modified grid.

Q9/ Modify the files such that unique base voltage $V_b = V_{b,1} = 4.16$ kV is used for the entire grid. Run the script `main.m` again. Observe the differences in $\bar{\mathbf{Y}}_{\mathcal{L}}$, $\bar{\mathbf{Y}}_{\mathcal{T}}$, and $\bar{\mathbf{Y}}$. How many elements of each of these matrices differ from the corresponding elements of matrices from the previous question? List them (including numerical values).

[A9]

After the coding part, three theoretical questions conclude the lab.

Q10/ Consider a network with K voltage levels. Is it always possible to set base voltages $(V_{b,1}, \ldots, V_{b,K})$, such that all the transformers are represented by *simple models* in per unit system (refer to transformer models in Fig. 3 and Fig. 4) if the network is

(a) radial

(b) meshed?

[A10]

Q11/ To establish a coherent per-unit system, in power system analysis, one usually chooses arbitrarily base power A_b for the entire system and a base voltage V_b, for each voltage level, and then derives all the other base quantities (I_b, Z_b, Y_b). What other possibilities could exist, and how many alternatives do we have? *For instance, can we arbitrarily choose A_b and Z_b and derive all the other base quantities?*

[A11]

Q12/ Is it possible to fully reconstruct (i.e., obtain all the elements) admittance matrix $\bar{\mathbf{Y}} \in \mathbb{C}^{N \times N}$, $N \geqslant 2$, by knowing *only* its diagonal elements $\bar{Y}_{11}, \ldots, \bar{Y}_{NN}$ and the primitive shunt admittance matrix $\bar{\mathbf{Y}}_{\mathcal{T}}$ (the incidence matrix $\mathbf{A}_{\mathfrak{B}}$ is unknown)? If not, what is the minimum number of additional non-diagonal elements necessary to reconstruct it? Detail your reasoning.

[A12]