EE-465 - W12
LCL FILTER

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Grid connected converters employ some kind of filter at their output, with the role:

- to provide inductive behavior and allow for control of active and reactive power exchange with the grid
- to provide attenuation of PWM caused harmonics

We have considered simple L filter, which is not always sufficient. High order filters, such as LCL are often used, as they offer:

- better attenuation of switching harmonics
- smaller filter size (physically)

There also some issues related to resonances, that must be considered during implementation

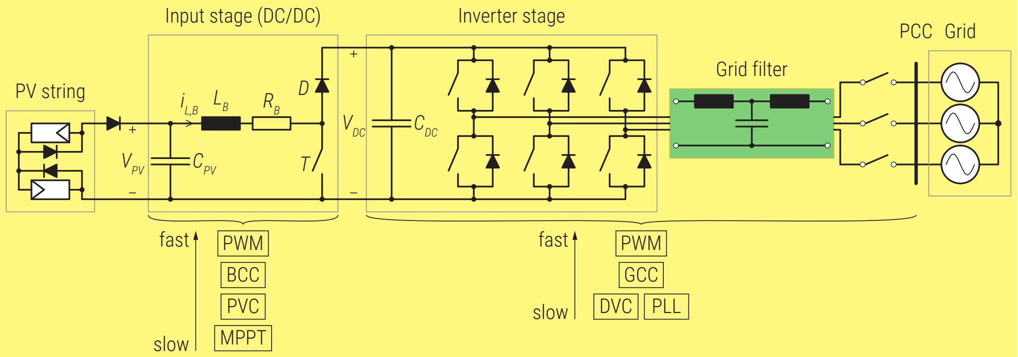

Figure 1 PV double-stage grid connected converter.

Power exchange between point A and B (grid connected converter):

$$S = \underline{V}_A \underline{I}^* = V_A \left[\frac{V_A - V_B \angle - \delta}{Z \angle \theta} \right]^* = \frac{V_A^2}{Z} \angle \theta - \frac{V_A V_B}{Z} \angle (\theta + \delta)$$

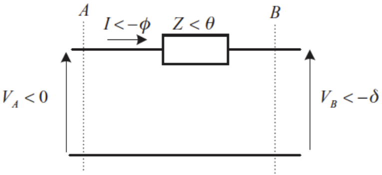

Figure 2 Converter - Impedance - Grid.

Active and Reactive powers are (δ is the power angle and θ is the power factor angle):

$$P_A = \frac{V_A^2}{Z} \cos \theta - \frac{V_A V_B}{Z} \cos(\theta + \delta), \qquad Q_A = \frac{V_A^2}{Z} \sin \theta - \frac{V_A V_B}{Z} \sin(\theta + \delta)$$

With $R = Z \cos(\theta)$ and $X = Z \sin(\theta)$ we have:

$$P_A = \frac{V_A}{R^2 + X^2} \left[R(V_A - V_B \cos \delta) + X V_B \sin \delta \right]$$

$$Q_A = \frac{V_A}{R^2 + X^2} \left[- R V_B \sin \delta + X(V_A - V_B \cos \delta) \right]$$

Neglecting R due to $X \gg R$, and considering small power angle δ, $\sin \delta \approx \delta$, $\cos \delta \approx 1$:

$$P_A = \frac{V_A V_B}{X} \delta, \qquad Q_A = \frac{V_A - V_B}{X} V_A$$

- The Active power is controlled by power angle δ
- The Reactive power is controlled by voltage difference

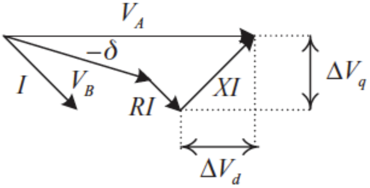

Figure 3 Phasor diagram.

In case of low switching frequencies (e.g. few hundreds of Hz), LC trap filters are used:

- series connected LC elements are tuned to a specific resonance frequency
- at series resonance frequency impedance of the filter is at minimum (zero)
- this represents low impedance path for current harmonic at that frequency

Resonant frequency is calculated as:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Several series LC elements, tuned at different frequencies can be connected in parallel

Impedance of an LC circuit, ignoring resistances and damping, is:

$$Z_{LC} = \frac{s^2LC + 1}{sC}$$

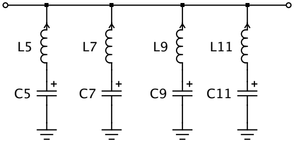

Figure 4 LC trap filters

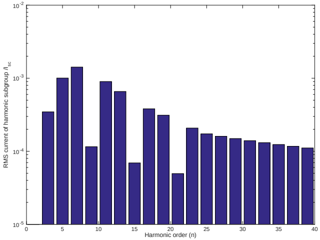

Figure 5 Example: BDEW grid codes.

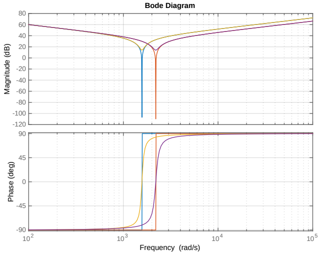

Figure 6 LC trap filter impedance (250Hz and 350Hz).

LCL filter is a low pass filter, offering good high frequency attenuation.

Basic set of expressions related to ideal LCL filter (resistances are neglected):

$$v_c(s) = sL_c i_c(s) + v_f(s)$$
$$v_g(s) = v_f(s) - sL_g i_g(s)$$
$$v_f(s) = i_f(s)/sC_f$$
$$i_c(s) = i_f(s) + i_g(s)$$

Transfer function of interest is $i_g(s)/v_c(s)$, and after setting $v_g(s) = 0$ we have:

$$H_{LCL}(s) = \frac{1}{L_c C_f L_g s^3 + (L_c + L_g)s}$$

Resonant frequency is defined with:

$$\omega_r = \sqrt{\frac{1}{L_{eq}C_f}} \quad \Rightarrow \quad L_{eq} = \frac{L_c L_g}{L_c + L_g} \quad \Rightarrow \quad \omega_r = \sqrt{\frac{L_c + L_g}{L_c C_f L_g}}$$

- Frequency response show high peak at resonant frequency
- $-60dB$ attenuation slope at higher frequencies (above resonant frequency)

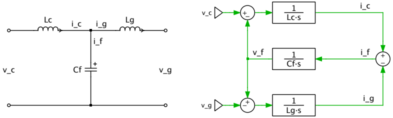

Figure 7 Single-phase ideal LCL filter and model.

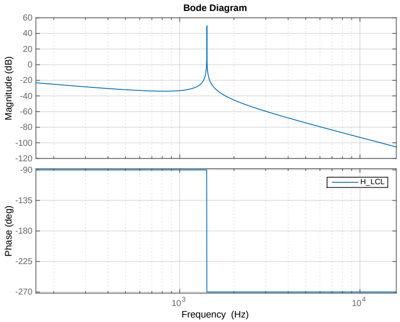

Figure 8 Frequency response - ideal LCL filter.

LCL FILTER - DESIGN CONSIDERATIONS

Certain limits are also usually applied during the design:

- capacitor value is limited by decrease of power factor at rated power (typically less then 5%)
- total value of inductance should be less than 0.1 p.u. to limit AC voltage drop
- the resonant frequency should be in the range between tens time the grid frequency and one half the switching frequency
- resonance problems should be avoided by some means
- **Passive Damping** is simple approach but introduces additional losses
- **Active Damping** solves the problem through a control modifications (lossless)

Design of an LCL filter must consider:

- desired or allowed current ripple on the converter side of the filter
- L_c is designed to limit converter output current ripple
- harmonic limits imposed on the grid side (standards, grid codes)
- L_g is designed considering high frequency harmonics
- the amount of installed reactive power of the LCL filter should be minimized
- the resonant frequency should be carefully selected, considering control bandwidth
- C_f is designed considering these conditions

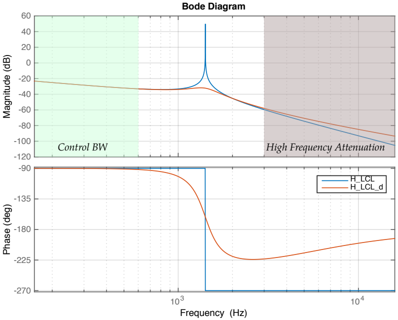

Figure 9 Frequency response - ideal and damped LCL filter.

LCL FILTER - DESIGN I: C_f

To design C_f we have to consider reactive power variations due to LCL filter

We can define base values as:

$$Z_b = \frac{V_{LL}^2}{P_n}, \qquad C_b = \frac{1}{\omega_g Z_b}, \qquad L_b = \frac{Z_b}{\omega_g}$$

where:

- V_{LL} is grid line-to-line RMS voltage
- P_n is the rated active power
- $\omega_g = 2\pi f_g$ is the grid angular frequency

Filter capacitance is related to the base value as percentage of it:

$$C_f = xC_b$$

Setting maximum power variation seen by the grid, e.g. $x = 5\%$, we have:

$$C_f = 0.05C_b$$

Different percentage values can be used as well ($x \in (1 \div 5)\%$).

LCL FILTER - DESIGN I: L_c

To design L_c we have to consider worst case current ripple

We will assume short circuit condition for filter capacitor C_f at ripple frequency

Maximum current ripple a the output of a 3-phase 2-level VSI can be determined as:

$$\Delta i_{L_{c-MAX}} = \frac{V_{DC}}{6 L_c f_{sw}}$$

Setting, for example, desired current ripple as 10%:

$$\Delta i_{L_{c-MAX}} = 0.1 I_{L_{c-peak}}$$

where peak and rms current values are determined from rated voltage and power:

$$I_{L_{c-MAX}} = \frac{P_n}{\sqrt{3} V_{ll}} \quad \Rightarrow \quad I_{L_{c-peak}} = \sqrt{2} I_{L_{c-MAX}}$$

Converter side LCL filter inductance minimum L_c value can be calculated as:

$$L_{c-MIN} = \frac{V_{DC}}{6 \Delta i_{L_{c-MAX}} f_{sw}}$$

Selected L_c value can be related to grid side L_g through scaling factor r as:

$$L_g = r L_c$$

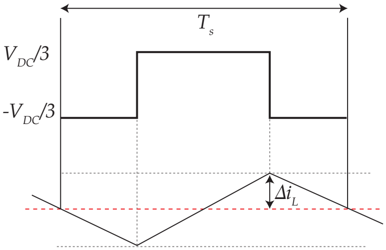

Figure 10 VSI output current ripple.

To design L_g we have to consider:

- harmonic attenuation
- choice of the resonant frequency

Attenuation of high frequency components of interest can be checked against the standards

Neglecting losses, grid current harmonic attenuation can be defined as:

$$\frac{i_g(h_{sw})}{i_c(h_{sw})} = \frac{1}{|1 + r(1 - L_g x C_b \omega_{sw}^2)|} = k_a$$

With L_c already selected, k_a represents desired attenuation at switching frequency ω_{sw}:

Considering desired attenuation k_a, grid side inductance L_g can be calculated as

$$L_g = \frac{\sqrt{\frac{1}{k_a^2}} + 1}{C_f w_{sw}^2}$$

where $C_f = xC_b$, and it may have to be varied to adjust resonant frequency

In practice, grid inductance constitutes the part of L_g

Also choice of $L_c = L_g$ provides good compromise (as discussed on the next slide)

LCL FILTER - DESIGN II

Alternative way to consider problem of design of inductances L_c and L_g (C_f is designed as in Design I).

Considering that at low frequencies, the LCL filter behaves as an inductance, defined by:

$$L_{dc} = L_c + L_g$$

There is a relation between L_c and L_g that minimizes the voltage drop at fundamental frequency and maximizes filtering ability.

If we express L_c and L_g, as percentage of L_{dc} with $\alpha \in (0 \div 1)$

$$L_c = \alpha L_{dc} \qquad L_g = (1-\alpha)L_{dc}$$

The L_{eq} that sets the resonant frequency can be redefined as:

$$L_{eq} = \frac{L_c L_g}{L_c + L_g} \quad \Rightarrow \quad L_{eq} = \alpha(1-\alpha)L_{dc}$$

Minimum of L_{eq} is achieved for $\alpha = 1/2$ or both inductances being equal $L_c = L_g$:

$$\frac{dL_{eq}}{d\alpha} = 0 \quad \Rightarrow \quad \alpha = 1/2 \quad \Rightarrow \quad L_c = L_g$$

By choosing L_{dc} to be 0.1 p.u. of the base impedance L_b at fundamental frequency ω_g, L_c and L_g are easily determined

Zero impedance at the LCL filter resonant frequency may lead to control instability

To reduce high peak gain at resonant frequency, damping resistance R_f is included.

Basic set of equations is now modified as:

$$v_c(s) = sL_c i_c(s) + v_f(s)$$
$$v_g(s) = v_f(s) - sL_g i_g(s)$$
$$v_f(s) = (R_f + 1/sC_f) i_f(s)$$
$$i_c(s) = i_f(s) + i_g(s)$$

Transfer function linking filter output $i_g(s)$ and filer input $v_c(s)$ is:

$$H_{LCL-d}(s) = \frac{C_f R_f s + 1}{L_c C_f L_g s^3 + C_f (L_c + L_g) R_f s^2 + (L_c + L_g) s}$$

Damping resistance R_f attenuates the high peak gain, but also reduces filter effectiveness ($-40dB$ instead of $-60dB$ of ideal LCL) and increases losses.

The value of resistor R_f is often selected as one third of the impedance of the filter capacitor C_f at the resonant frequency:

$$R_f = \frac{1}{3\omega_{res} C_f}$$

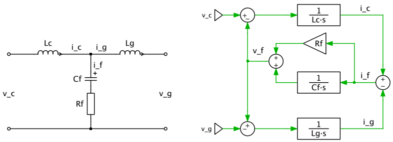

Figure 11 Single-phase damped LCL filter and model.

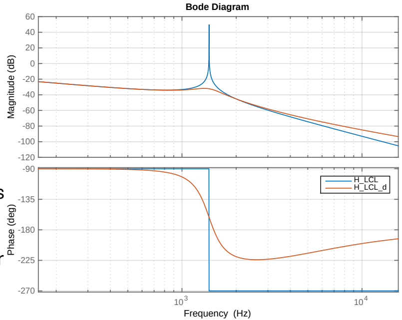

Figure 12 Frequency response - damped LCL filter.

Input data are:

- $V_{LL} = 400$ - line to line RMS voltage [V]
- $P_n = 3500$ - rated active power [W]
- $V_{DC} = 650$ - rated dc-link voltage [V]
- $w_g = 2pi50$ - grid angular frequency [Hz]
- $f_{sw} = 10000$ - switching frequency [Hz]
- $x = 0.05$ - maximum power factor variation seen by the grid [%] (design 1)
- $k_a = 0.2$ - desired harmonic current attenuation [%] (design 1)
- $\Delta i = 0.1$ - allowed current ripple [A] (design 1)
- $k_l = 0.1$ - total DC inductance as percentage of base value [%] (design 2)

Resulting designs can be summarized as:

LCL	Design 1	Design 2
L_c	$15.16\,mH$	$7.28\,mH$
L_g	$0.44\,mH$	$7.28\,mH$
C_f	$3.48\,\mu F$	$3.48\,\mu F$
f_r	$4.14\,kHz$	$1.41\,kHz$
R_f	$3.68\,\Omega$	$10.76\,\Omega$

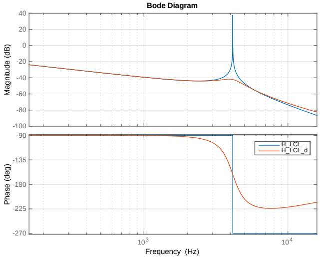

Figure 13 Frequency response - Design 1.

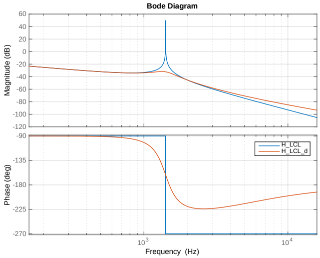

Figure 14 Frequency response - Design 2.

LCL FILTER - ACTIVE DAMPING (I)

Rather than using passive damping with real resistor, it is possible to achieve similar effect my means of control and use of *virtual resistor*

Control structure can be modified, taking into account:

- $v_f(s)$ - capacitor voltage as feedback variable
- $i_f(s)$ - capacitor current as feedback variable

From the basic set of expressions related to ideal LCL filter (resistances are neglected) and after setting $v_g(s) = 0$:

$$v_c(s) = sL_c i_c(s) + v_f(s) \quad \Rightarrow \quad v_c(s) = sL_c(i_f(s) + i_g(s)) + v_f(s)$$
$$v_g(s) = v_f(s) - sL_g i_g(s) \quad \Rightarrow \quad i_g(s) = v_f(s)/sL_g$$
$$v_f(s) = i_f(s)/sC_f \quad \Rightarrow \quad i_f(s) = sC_f v_f(s)$$
$$v_f^d(s) = i_f(s)(R_f + 1/sC_f) \quad \Rightarrow \quad i_f(s) = v_f^d(s)/(R_f + 1/sC_f)$$
$$i_c(s) = i_f(s) + i_g(s)$$

We can derive two transfer functions of interest: $v_f(s)/v_c(s)$, and $i_f(s)/v_c(s)$ for ideal LCL case as:

$$H_{v_f}(s) = \frac{v_f(s)}{v_c(s)} = \frac{1}{L_c C_f}\frac{1}{s^2 + \omega_r^2}, \qquad H_{i_f}(s) = \frac{i_f(s)}{v_c(s)} = \frac{i_f(s)}{v_f(s)}\frac{v_f(s)}{v_c(s)} = \frac{1}{L_c}\frac{s}{s^2 + \omega_r^2}$$

For LCL case with damping resistor we have:

$$H_{v_f}^d(s) = \frac{v_f^d(s)}{v_c(s)} = \frac{1}{L_c C_f}\frac{1 + sR_f C_f}{s^2 + \omega_r^2(1 + sR_f C_f)}, \qquad H_{i_f}^d(s) = \frac{i_f(s)}{v_c(s)} = \frac{i_f(s)}{v_f^d(s)}\frac{v_f^d(s)}{v_c(s)} = \frac{1}{L_c}\frac{s}{s^2 + \omega_r^2(1 + sR_f C_f)}$$

LCL FILTER - ACTIVE DAMPING (II)

Virtual resistor can be placed inside the control structure with ideal LCL filter, to emulate presence of a real resistor

We will considering capacitor voltage $v_f(s)$ as feedback variable, and act on the converter output voltage:

In an ideal LCL case, without damping resistor we have:

$$H_{v_f}(s) = \frac{v_f(s)}{v_c(s)} = \frac{1}{L_c C_f} \frac{1}{s^2 + \omega_r^2} = \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g} = H_1(s), \qquad \frac{i_f(s)}{v_f(s)} = sC_f = H_2(s)$$

For case with damping resistor we have:

$$H_{v_f}^d(s) = \frac{v_f^d(s)}{v_c(s)} = \frac{1}{L_c C_f} \frac{1 + sR_f C_f}{s^2 + \omega_r^2(1 + sR_f C_f)} = \frac{L_g(1 + sR_f C_f)}{s^2 L_c C_f L_g + (L_c + L_g)(1 + sR_f C_f)} = H_3(s)$$

Ideally, we would like to achieve $H_3(s)$ without real R_f but with R_v (virtual)

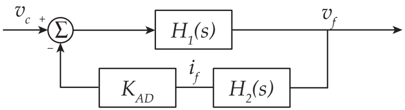

Figure 15 Active Damping control Loop with capacitor voltage/current as feedback.

LCL FILTER - ACTIVE DAMPING (III)

If we measure capacitor voltage $v_f(s)$, calculate capacitor current $i_f(s)$ and add suitable gain as feedback K_{AD} we have:

$$H_{CL}(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)K_{AD}} = \frac{\frac{L_g}{s^2 L_c C_f L_g + L_c + L_g}}{1 + \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g} s C_f K_{AD}} = \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g + s L_g C_f K_{AD}}$$

With K_{AD} selected as:

$$K_{AD} = \frac{L_c + L_g}{L_g} R_v$$

We have

$$H_{CL}(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)K_{AD}} = \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g + s L_g C_f \frac{L_c + L_g}{L_g} R_v} = \frac{L_g}{s^2 L_c C_f L_g + (L_c + L_g)(1 + s R_v C_f)}$$

Control modification achieves closed-loop transfer function similar to targeted $H_3(s)$

Term $R_f C_f$ found in numerator of $H_3(s)$ neglected, as it is much smaller than 1

Implementation must consider both axis of regulator (v_d, v_q)

Recalling definitions for instantaneous active and reactive powers (Clarke with $K = 2/3$):

- $p_{dq} = \frac{3}{2}(v_\alpha i_\alpha + v_\beta i_\beta) = \frac{3}{2}(v_d i_d + v_q i_q)$ - (*instantaneous active power*)
- $q_{dq} = \frac{3}{2}(v_\alpha i_\beta - v_\beta i_\alpha) = \frac{3}{2}(v_d i_q - v_q i_d)$ - (*instantaneous reactive power*)

Current references can be calculated from power references as:

$$\mathbf{i}^*_{\alpha\beta} = \begin{bmatrix} i^*_\alpha \\ i^*_\beta \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} p^*_{\alpha\beta} \\ q^*_{\alpha\beta} \end{bmatrix}$$

$$\mathbf{i}^*_{dq} = \begin{bmatrix} i^*_d \\ i^*_q \end{bmatrix} = \frac{1}{v_d^2 + v_q^2} \begin{bmatrix} v_d & -v_q \\ v_q & v_d \end{bmatrix} \begin{bmatrix} p^*_{dq} \\ q^*_{dq} \end{bmatrix}$$

With d-axis perfectly aligned with grid voltage space vector ($v_q = 0$), we have:

- $p_{dq} = \frac{3}{2} v_d i_d$ - instantaneous active power is proportional to i_d
- $q_{dq} = -\frac{3}{2} v_d i_q$ - instantaneous reactive power is proportional to i_q

Orientation is achieved with help of PLL

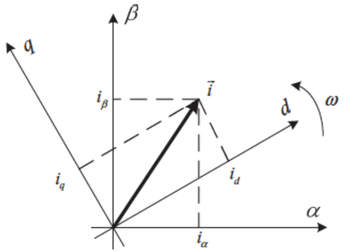

Figure 16 $\alpha\beta$ and dq reference frames.

SENSOR LOCATION

As filter has reactive elements, it may influence desired power factor

Equivalent impedance at the PCC (point of common coupling) will depend on position of voltage and current sensors

With LCL filter installed there are several options regarding sensor location

- a) V_f and i_c sensed
  - $Z_g = R + j(X_2 + X_g - X_c)$
  - $Z_c = R + jX_1$
- b) V_f and i_g sensed
  - $Z_g = R + j(X_2 + X_g)$
  - $Z_c = R + j(X_1 - X_c)$
- c) V_g and i_g sensed
  - $Z_g = R + jX_g$
  - $Z_c = R + j(X_1 - X_2 - X_g - X_c)$
- d) V_g and i_c sensed
  - $Z_g = R + j(X_g - X_c)$
  - $Z_c = R - j(X_1 + X_2)$

Impedance seen at PCC is different for each case

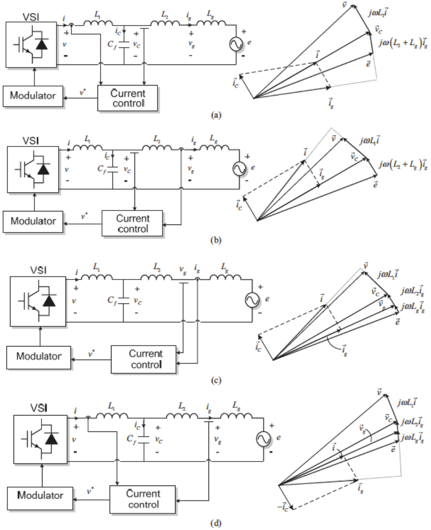

Figure 17 Vector diagram depending on sensor location.

PQ OPEN-LOOP / CLOSED-LOOP CONTROL IN dq FRAME

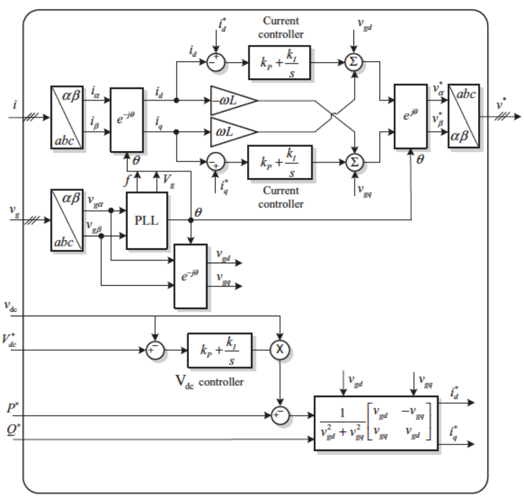

Figure 18 PQ open-loop voltage oriented control in dq frame.

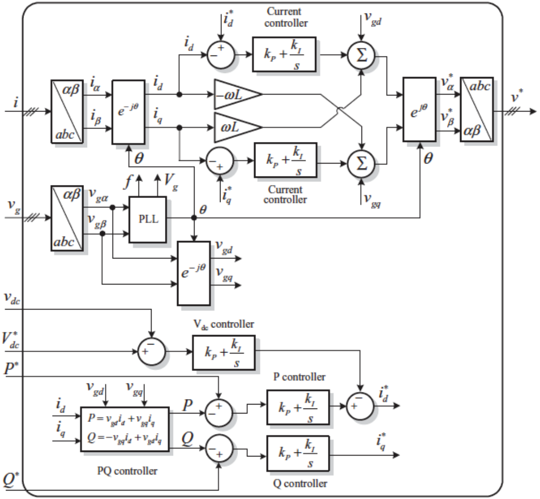

Figure 19 PQ closed-loop voltage oriented control in dq frame.

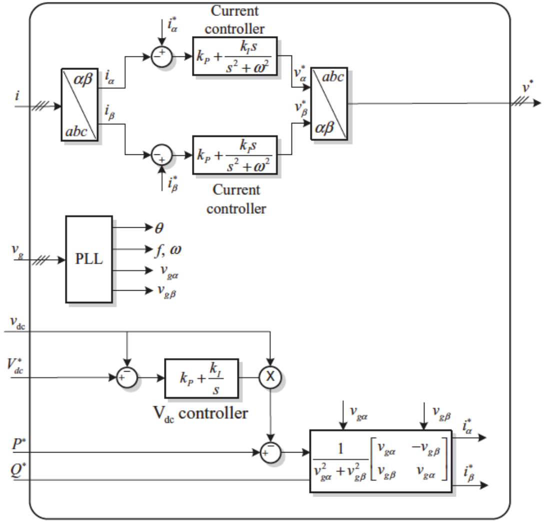

Figure 20 PQ open-loop voltage oriented control in $\alpha\beta$ frame.

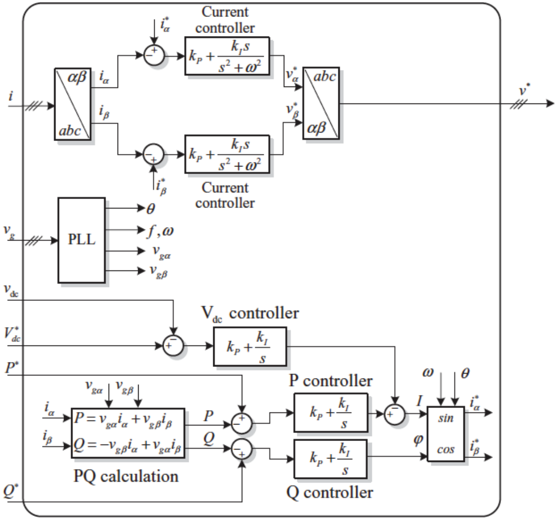

Figure 21 PQ closed-loop voltage oriented control in $\alpha\beta$ frame.

SUMMARY

LCL filter is commonly found in grid connected converters:

- it offers good attenuation of high frequencies
- often results in smaller physical size than simple L filter
- resonance can be damped either passively or actively
- modifications of the control structure are required
- power exchange with the grid can be performed in various ways (more to be covered next week - not PV related)

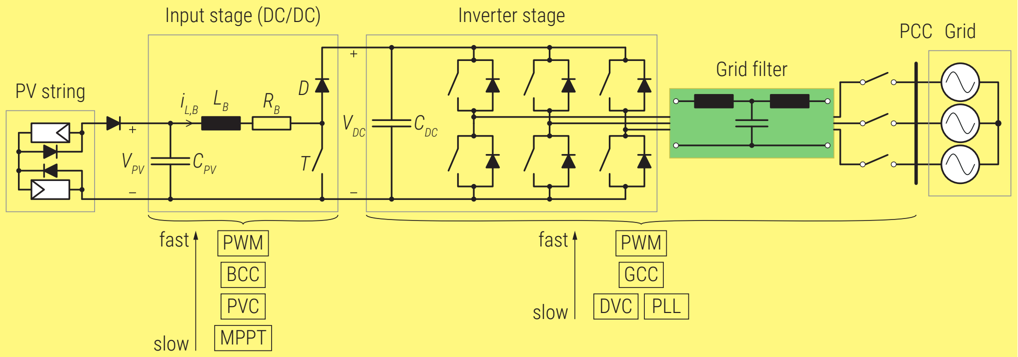

Figure 22 PV double-stage grid connected converter.