

# EE-465 - W6

# SPACE VECTOR

# PWM

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# CARRIER BASED PWM

Carrier Based PWM for 3-phase 2-level converter has been characterized by:

- ▶ Fundamental low frequency modulating signals
- ▶ Zero-sequence signal that can be added to modify fundamental signals
- ▶ High frequency carrier signal (e.g. triangular signal)

Modulation index has been defined as:

$$M = \frac{V_m}{V_{DC}/2}$$

Zero sequence signal injection can improve DC bus utilisation (linear region):

- ▶ SPWM  $M_{max} = 1$
- ▶ THIPWM  $M_{max} = \frac{2}{\sqrt{3}} \approx 1.15$

PWM pulses are obtained as result of comparison of modulating and carrier signal

Resulting PWM pulses are directly applied to the inverter legs

Continuous PWM - two commutations per leg in every switching period

Discontinuous PWM - absence of any commutations in one of the legs

Linear region  $\Rightarrow$  Overmodulation region  $\Rightarrow$  Six-step mode

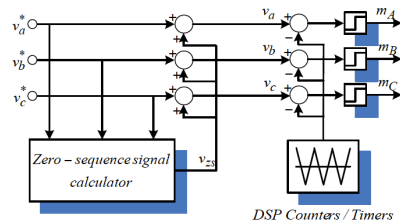


Figure 1 Carrier Based PWM principles.

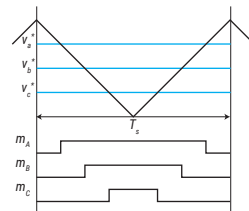


Figure 2 PWM resulting pattern (single shot).

# SPACE VECTORS

To simplify analysis of three-phase systems we can use space vectors defined as

$$\mathbf{v} = \frac{2}{3} (v_a + \alpha v_b + \alpha^2 v_c)$$

Where:

- ▶  $v_a, v_b, v_c$  are balanced sinusoidal set of signals (e.g. voltages)
- ▶ the operator  $\alpha$  is defined as  $\alpha = e^{j2\pi/3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

Three-phase abc system is represented in another set of variables  $\alpha\beta 0$ :

$$\mathbf{v}_{\alpha\beta 0} = [T_{\alpha\beta 0}] \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

In balanced three-phase system ( $v_a + v_b + v_c = 0$ ), we can omit zero-sequence component

Clarke Transformation is used to represent variables in terms of  $\alpha\beta$  components:

$$\mathbf{v}_{\alpha\beta} = [T_{\alpha\beta}] \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

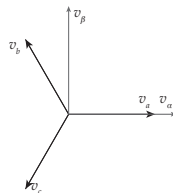


Figure 3  $\alpha\beta$  plane.

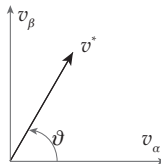


Figure 4 Resulting space vector in  $\alpha\beta$  plane.

# SPACE VECTORS OF A 2-LEVEL 3-PHASE INVERTER (I)

Considering that:

- ▶ there are 3 converter legs with 6 switches in total
- ▶ two switches of the same leg cannot conduct simultaneously

There is only eight possible switching states of the 2-level 3-phase converter

$$n = 2^3 = 8$$

The switching state of each inverter leg can be expressed binary:

- ▶ 1 when the upper device is ON and the lower device is OFF
- ▶ 0 when the upper device is OFF and the lower device is ON

Possible switching combinations and **inverter leg voltages** are:

	A	B	C	$v_A$	$v_B$	$v_C$
$V_0$	0	0	0	0	0	0
$V_1$	1	0	0	$V_{DC}$	0	0
$V_2$	1	1	0	$V_{DC}$	$V_{DC}$	0
$V_3$	0	1	0	0	$V_{DC}$	0
$V_4$	0	1	1	0	$V_{DC}$	$V_{DC}$
$V_5$	0	0	1	0	0	$V_{DC}$
$V_6$	1	0	1	$V_{DC}$	0	$V_{DC}$
$V_7$	1	1	1	$V_{DC}$	$V_{DC}$	$V_{DC}$

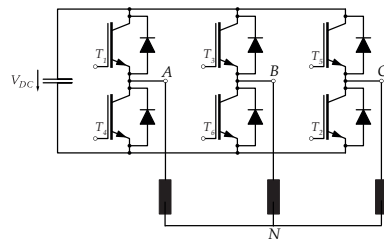


Figure 5 2-level 3-phase inverter.

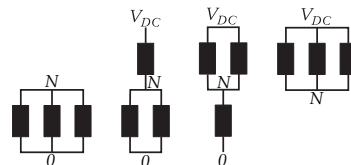


Figure 6 Possible load connections.

# SPACE VECTORS OF A 2-LEVEL 3-PHASE INVERTER (II)

Different switching combinations have different effect on load:

- Configurations 000 and 111 effectively short circuit the load
- Other six configurations actively drive current in the load

Each of the switching configurations produces instantaneous phase voltage

- Assuming per phase load impedance of  $Z$
- Simple voltage divider with  $Z$  and  $Z/2$  can be seen

From here **instantaneous phase voltages** can be easily calculated as:

	A	B	C	$v_a$	$v_b$	$v_c$
$V_0$	0	0	0	0	0	0
$V_1$	1	0	0	$2V_{DC}/3$	$-V_{DC}/3$	$-V_{DC}/3$
$V_2$	1	1	0	$V_{DC}/3$	$V_{DC}/3$	$-2V_{DC}/3$
$V_3$	0	1	0	$-V_{DC}/3$	$2V_{DC}/3$	$-V_{DC}/3$
$V_4$	0	1	1	$-2V_{DC}/3$	$V_{DC}/3$	$V_{DC}/3$
$V_5$	0	0	1	$-V_{DC}/3$	$-V_{DC}/3$	$2V_{DC}/3$
$V_6$	1	0	1	$V_{DC}/3$	$-2V_{DC}/3$	$V_{DC}/3$
$V_7$	1	1	1	0	0	0

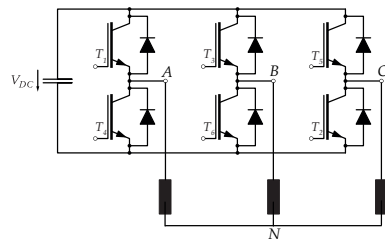


Figure 7 2-level 3-phase inverter.

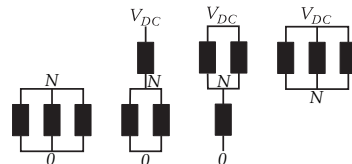


Figure 8 Possible load connections.

# SPACE VECTORS OF A 2-LEVEL 3-PHASE INVERTER (III)

Using definition for the space vectors:

$$\mathbf{V} = \frac{2}{3} (V_A + \alpha V_B + \alpha^2 V_C)$$

We can determine **inverter space vectors** as:

- ▶  $\mathbf{V}_0 = \frac{2}{3}(0 + 0 + 0) = 0$
- ▶  $\mathbf{V}_1 = \frac{2}{3}(V_{DC} + 0 + 0) = \frac{2}{3}V_{DC}$
- ▶  $\mathbf{V}_2 = \frac{2}{3}(V_{DC} + V_{DC} \cdot e^{j2\pi/3} + 0) = \frac{2}{3}V_{DC} \cdot e^{j\pi/3}$
- ▶  $\mathbf{V}_3 = \frac{2}{3}(0 + V_{DC} \cdot e^{j2\pi/3} + 0) = \frac{2}{3}V_{DC} \cdot e^{j2\pi/3}$
- ▶  $\mathbf{V}_4 = \frac{2}{3}(0 + V_{DC} \cdot e^{j2\pi/3} + V_{DC} \cdot e^{j4\pi/3}) = \frac{2}{3}V_{DC} \cdot e^{j3\pi/3}$
- ▶  $\mathbf{V}_5 = \frac{2}{3}(0 + 0 + V_{DC} \cdot e^{j4\pi/3}) = \frac{2}{3}V_{DC} \cdot e^{j4\pi/3}$
- ▶  $\mathbf{V}_6 = \frac{2}{3}(V_{DC} + 0 + V_{DC} \cdot e^{j4\pi/3}) = \frac{2}{3}V_{DC} \cdot e^{j5\pi/3}$
- ▶  $\mathbf{V}_7 = \frac{2}{3}(V_{DC} + V_{DC} \cdot e^{j2\pi/3} + V_{DC} \cdot e^{j4\pi/3}) = 0$

There are:

- ▶ Two zero space vectors:  $V_0$  and  $V_7$
- ▶ Six active space vectors:  $V_1, V_2, V_3, V_4, V_5, V_6$

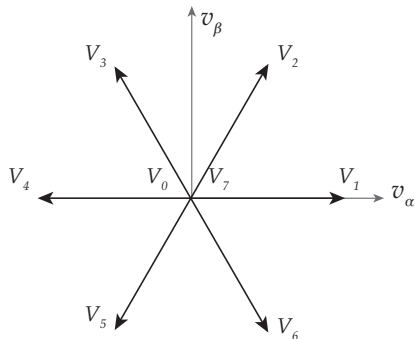


Figure 9 Space vectors of a 2-level 3-phase inverter.

# REFERENCE SPACE VECTORS AND SECTORS

We can easily identify:

- ▶ Hexagon connecting tips of active space vectors
- ▶  $\alpha\beta$  plane being split into six sectors
- ▶ Each sector spans 60 deg in  $\alpha\beta$  plane
- ▶ There are active space vectors at vertices of each sector
- ▶ Two zero space vectors at the origin

Considering desired output voltage and fundamental set of modulating signals:

$$v_a^* = M \frac{V_{DC}}{2} \cos(\omega t)$$

$$v_b^* = M \frac{V_{DC}}{2} \cos(\omega t - 2\pi/3)$$

$$v_c^* = M \frac{V_{DC}}{2} \cos(\omega t - 4\pi/3)$$

Similar to inverter space vectors, we define reference space vector as:

$$\mathbf{v}_{\alpha\beta}^* = [T_{\alpha\beta}] \begin{bmatrix} v_a^* \\ v_b^* \\ v_c^* \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_a^* \\ v_b^* \\ v_c^* \end{bmatrix}$$

$$\mathbf{v}_{\alpha\beta}^* = M \frac{V_{DC}}{2} e^{j\omega t} = M \frac{V_{DC}}{2} e^{j\theta}$$

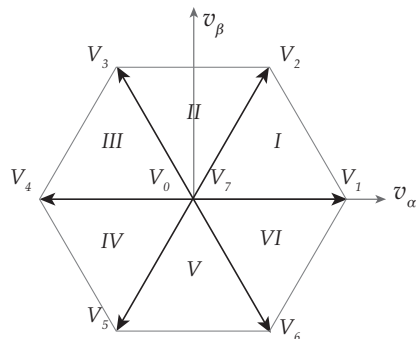


Figure 10 Space vectors and sectors in  $\alpha\beta$  plane

# NORMALIZATION

Having everything transformed into  $\alpha\beta$  plane, we have:

- ▶ eight stationary space vector of an inverter
- ▶ rotating reference space vector ( $\theta = \omega t$ )

To simplify further calculations, we will normalize active space vectors with  $\frac{V_{DC}}{2}$

- ▶  $\mathbf{V}_1 = \frac{4}{3}$
- ▶  $\mathbf{V}_2 = \frac{4}{3} \cdot e^{j\pi/3}$
- ▶  $\mathbf{V}_3 = \frac{4}{3} \cdot e^{j2\pi/3}$
- ▶  $\mathbf{V}_4 = \frac{4}{3} \cdot e^{j3\pi/3}$
- ▶  $\mathbf{V}_5 = \frac{4}{3} \cdot e^{j4\pi/3}$
- ▶  $\mathbf{V}_6 = \frac{4}{3} \cdot e^{j5\pi/3}$

Similarly, reference space vector can be normalized:

$$\mathbf{v}_{\alpha\beta}^* = M e^{j\theta}$$

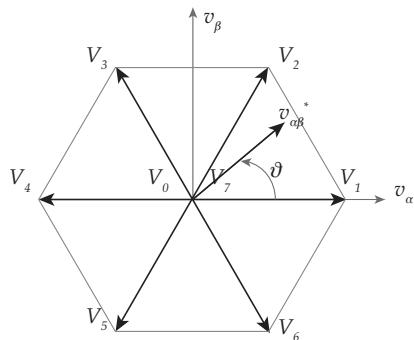


Figure 11 Reference space vector in  $\alpha\beta$  plane

For the analysis it is enough to consider reference space vector in sector  $s = 1$

Reference space vector being defined with:

$$\mathbf{v}_{\alpha\beta}^* = M e^{j\theta}$$

can have in general:

- ▶ Any magnitude, defined by  $M$
- ▶ Any angular position, defined with  $\theta = \omega t$

We can arbitrarily define desired switching frequency and thus, switching period  $T_s$

**Reference space vector represents desired inverter output voltage**

To realize reference space vector over the period  $T_s$  we can use active space vectors

Normally two adjacent active space vectors are used, and problem can be described as:

$$\mathbf{v}_{\alpha\beta}^* \cdot T_s = \mathbf{V}_1 \cdot T_1 + \mathbf{V}_2 \cdot T_2$$

Our task is to find the application times  $(T_1, T_2)$  of active space vectors

We will average reference space vector with two inverter active vectors

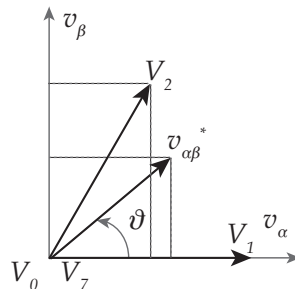


Figure 12 Reference space vector in sector  $s = 1$ .

# APPLICATION TIMES (I)

To calculate application times, we can decompose complex equation into two real ones:

$$\mathbf{v}_{\alpha\beta}^* \cdot T_s = \mathbf{V}_1 \cdot T_1 + \mathbf{V}_2 \cdot T_2$$

This can be easily done by finding vector projections on the  $\alpha\beta$  axes:

$$\alpha \rightarrow M \cos(\theta) T_s = \frac{4}{3} \cdot T_1 + \frac{4}{3} \cos(\pi/3) \cdot T_2$$

$$\beta \rightarrow M \sin(\theta) T_s = 0 \cdot T_1 + \frac{4}{3} \sin(\pi/3) \cdot T_2$$

Application times can be easily determined as:

$$T_1 = \frac{M}{\frac{4}{3} \sin(\pi/3)} \sin(\pi/3 - \theta) \cdot T_s$$

$$T_2 = \frac{M}{\frac{4}{3} \sin(\pi/3)} \sin(\theta) \cdot T_s$$

or after further simplification as:

$$T_1 = M \frac{\sqrt{3}}{2} \sin(\pi/3 - \theta) \cdot T_s$$

$$T_2 = M \frac{\sqrt{3}}{2} \sin(\theta) \cdot T_s$$

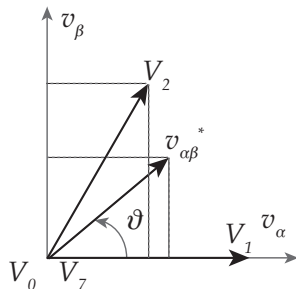


Figure 13 Reference space vector in sector  $s = 1$ .

## APPLICATION TIMES (II)

Calculated application times would normally be smaller than the switching period  $T_s$

The remaining time is allocated to zero-space vectors (ZSV):

$$T_{ZSV} = T_s - T_1 - T_2$$

While this time can be split in many ways, normally it is equally shared:

$$T_0 = T_7 = \frac{T_{ZSV}}{2}$$

Resulting:

$$T_s = T_1 + T_2 + T_0 + T_7$$

Another normalization can be obtained by dividing calculated times with  $T_s$ :

$$\delta_1 = \frac{T_1}{T_s}, \quad \delta_2 = \frac{T_2}{T_s}, \quad \delta_0 = \frac{T_0}{T_s}, \quad \delta_7 = \frac{T_7}{T_s}, \quad 1 = \frac{T_s}{T_s}$$

This allow to operate with duty cycles instead of application times:

$$\delta_1 = M \frac{\sqrt{3}}{2} \sin\left(\frac{\pi}{3} - \theta\right)$$

$$\delta_2 = M \frac{\sqrt{3}}{2} \sin(\theta)$$

$$\delta_0 = \delta_7 = \frac{1}{2} - M \frac{\sqrt{3}}{4} \cos\left(\frac{\pi}{6} - \theta\right)$$

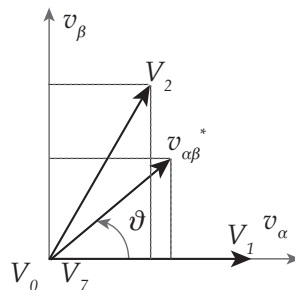


Figure 14 Reference space vector in sector  $s = 1$ .

## APPLICATION TIMES (III)

With duty cycles calculated according to:

$$\delta_1 = M \frac{\sqrt{3}}{2} \sin\left(\frac{\pi}{3} - \theta\right)$$

$$\delta_2 = M \frac{\sqrt{3}}{2} \sin(\theta)$$

$$\delta_0 = \delta_7 = \frac{1}{2} - M \frac{\sqrt{3}}{4} \cos\left(\frac{\pi}{6} - \theta\right)$$

We can evaluate their values, taking sector 1 as an example:

- ▶  $\delta_2$  at the beginning of the sector is zero since  $\mathbf{v}_{\alpha\beta}^*$  is aligned with  $V_1$ , ( $\theta = 0$ )
- ▶  $\delta_1$  at the end of the sector is zero since  $\mathbf{v}_{\alpha\beta}^*$  is aligned with  $V_2$ , ( $\theta = \pi/3$ )
- ▶ Increasing the modulation index  $M$  reduced the duty cycles of zero space vectors
- ▶ At some  $M$ , duty cycles of zero space vectors would become zero
- ▶ DC bus utilisation will be covered soon

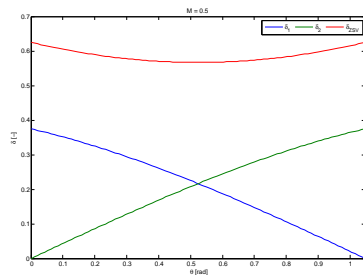


Figure 15 Duty cycles in sector  $s = 1$ ,  $M = 0.5$ .

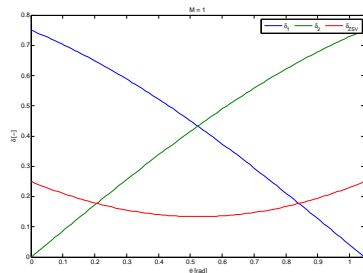


Figure 16 Duty cycles in sector  $s = 1$ ,  $M = 1$ .

# SWITCHING PATTERN SYNTHESIS (I)

So far we have determined (focusing on sector  $s = 1$ ):

- ▶ We can average reference space vector with two space vectors  $V_1$  and  $V_2$
- ▶ Remaining time is allocated to zero space vectors  $V_0$  and  $V_7$
- ▶  $n = 4$  inverter space vectors would be applied over the switching period

The question is now: In which order we should apply these space vectors?

With  $n = 4$  space vectors, we have  $n! = 24$  combinations

- ▶  $V_0 - V_1 - V_2 - V_7$
- ▶  $V_0 - V_1 - V_7 - V_2$
- ▶ ...
- ▶  $V_7 - V_2 - V_1 - V_0$

Normally, symmetrical switching sequence is selected for implementation:

- ▶ Aim is to minimize number of commutations (e.g. two per leg)
- ▶ Symmetry exist with respect to half of the switching period
- ▶ Sequence start and finish with zero space vector  $V_0$
- ▶ Zero space vector  $V_7$  is placed in the middle of the sequence
- ▶ OK  $\rightarrow V_0 - V_1 - V_2 - V_7 - V_2 - V_1 - V_0$
- ▶ NOT OK  $\rightarrow V_0 - V_2 - V_1 - V_7 - V_1 - V_2 - V_0$

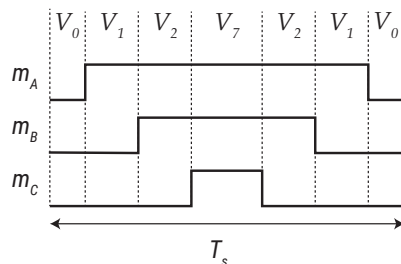


Figure 17 Optimal switching pattern in  $s = 1$ .

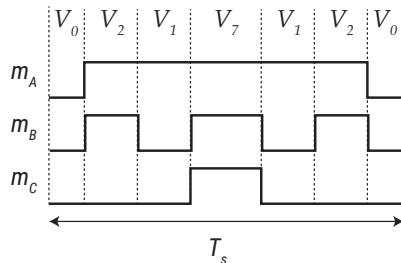


Figure 18 Suboptimal switching pattern in  $s = 1$ .

## SWITCHING PATTERN SYNTHESIS (II)

Analysing sequences in all sectors, it can be determined, that for odd sectors:

- ▶  $s = 1 \rightarrow \mathbf{V}_0 - \mathbf{V}_1 - \mathbf{V}_2 - \mathbf{V}_7 - \mathbf{V}_2 - \mathbf{V}_1 - \mathbf{V}_0$
- ▶  $s = 3 \rightarrow \mathbf{V}_0 - \mathbf{V}_3 - \mathbf{V}_4 - \mathbf{V}_7 - \mathbf{V}_4 - \mathbf{V}_3 - \mathbf{V}_0$
- ▶  $s = 5 \rightarrow \mathbf{V}_0 - \mathbf{V}_5 - \mathbf{V}_6 - \mathbf{V}_7 - \mathbf{V}_6 - \mathbf{V}_5 - \mathbf{V}_0$
- ▶ Imagine that sector starts with space vector  $\mathbf{V}_X$  and ends with vector  $\mathbf{V}_Y$
- ▶ Sequence is always  $\mathbf{V}_0 - \mathbf{V}_X - \mathbf{V}_Y - \mathbf{V}_7 - \mathbf{V}_Y - \mathbf{V}_X - \mathbf{V}_0$

While for the even sectors:

- ▶  $s = 2 \rightarrow \mathbf{V}_0 - \mathbf{V}_3 - \mathbf{V}_2 - \mathbf{V}_7 - \mathbf{V}_2 - \mathbf{V}_3 - \mathbf{V}_0$
- ▶  $s = 4 \rightarrow \mathbf{V}_0 - \mathbf{V}_5 - \mathbf{V}_4 - \mathbf{V}_7 - \mathbf{V}_4 - \mathbf{V}_5 - \mathbf{V}_0$
- ▶  $s = 6 \rightarrow \mathbf{V}_0 - \mathbf{V}_1 - \mathbf{V}_6 - \mathbf{V}_7 - \mathbf{V}_6 - \mathbf{V}_1 - \mathbf{V}_0$
- ▶ Sector still starts with space vector  $\mathbf{V}_X$  and ends with vector  $\mathbf{V}_Y$
- ▶ But the sequence is always  $\mathbf{V}_0 - \mathbf{V}_Y - \mathbf{V}_X - \mathbf{V}_7 - \mathbf{V}_X - \mathbf{V}_Y - \mathbf{V}_0$

For the implementation it is important to distinguish odd from even sectors!

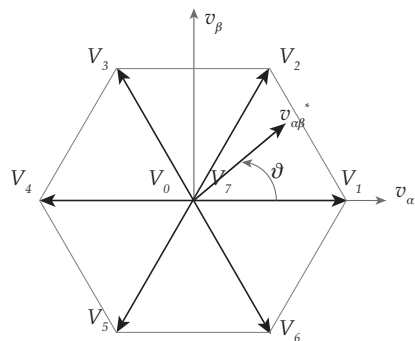
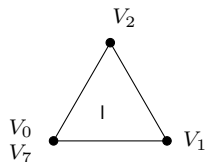


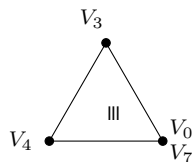
Figure 19 Reference space vector in  $\alpha\beta$  plane

# SWITCHING PATTERN SYNTHESIS (III)

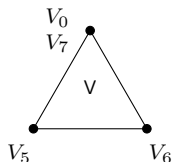
Odd sectors



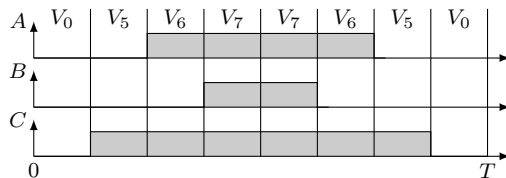
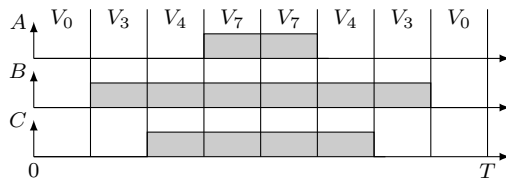
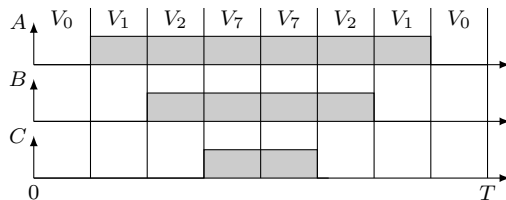
	A	B	C
$V_0$	0	0	0
$V_1$	1	0	0
$V_2$	1	1	0
$V_7$	1	1	1



	A	B	C
$V_0$	0	0	0
$V_3$	0	1	0
$V_4$	0	1	1
$V_7$	1	1	1

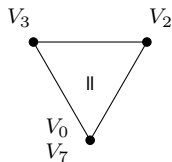


	A	B	C
$V_0$	0	0	0
$V_5$	0	0	1
$V_6$	1	0	1
$V_7$	1	1	1

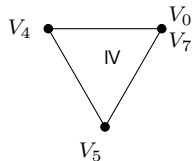


# SWITCHING PATTERN SYNTHESIS (IV)

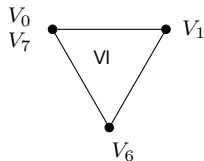
Even sectors



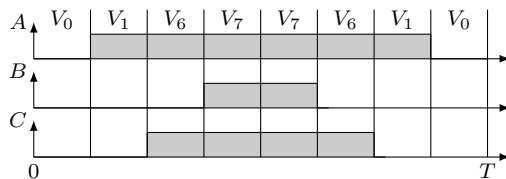
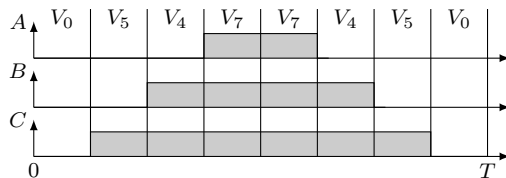
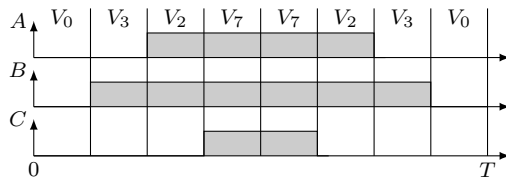
	A	B	C
$V_0$	0	0	0
$V_3$	0	1	0
$V_2$	1	1	0
$V_7$	1	1	1



	A	B	C
$V_0$	0	0	0
$V_5$	0	0	1
$V_4$	0	1	1
$V_7$	1	1	1



	A	B	C
$V_0$	0	0	0
$V_1$	1	0	0
$V_6$	1	0	1
$V_7$	1	1	1



# DC BUS UTILIZATION

DC bus utilization of the SVPWM can be easily determined from the  $\alpha\beta$  plane:

- ▶ increasing the  $M$  brings tip of the reference space vector closer to the hexagon
- ▶ **linear modulation** region is defined by inscribed circle inside the hexagon
- ▶ further increase of  $M$  would take the tip outside the hexagon - **overmodulation**
- ▶ at limit, reference space vector (circle) will touch hexagon at  $\theta = \pi/6$  (sector 1)
- ▶ at limit, time of application of zero space vectors will be zero
- ▶ in the six step mode, only active space vectors are used

Simple trigonometry yields:

$$M_{max-SVPWM} = \frac{4}{3} \cos(\pi/6) = \frac{4}{3} \frac{\sqrt{3}}{2} = 1.1547$$

DC bus utilisation of the SVPWM is identical to that of harmonic injection CB-PWM

- ▶ SPWM  $M_{max} = 1$
- ▶ THIPWM  $M_{max} = \frac{2}{\sqrt{3}} = 1.1547$
- ▶ SVPWM  $M_{max} = \frac{2}{\sqrt{3}} = 1.1547$

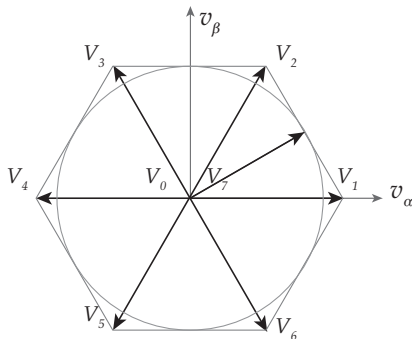


Figure 20 Inscribed circle in the  $\alpha\beta$  plane

# GENERALIZATION

Presentation so far has been focused on sector  $s = 1$

It is easy to generalize duty cycle calculations to take into account different sectors

Assuming that sector  $s$  has been somehow determined:

$$\delta_1 = M \frac{\sqrt{3}}{2} \sin\left(s \frac{\pi}{3} - \theta\right)$$

$$\delta_2 = M \frac{\sqrt{3}}{2} \sin\left(\theta - (s-1) \frac{\pi}{3}\right)$$

$$\delta_0 = \delta_7 = \frac{1}{2} - M \frac{\sqrt{3}}{4} \cos\left((2s-1) \frac{\pi}{6} - \theta\right)$$

Sector can be easily determined from the argument of the reference space vector:

$$\mathbf{v}_{\alpha\beta}^* = M e^{j\theta}$$

- ▶  $s = 1$  if  $(\theta > 0 \text{ and } \theta < \pi/3)$
- ▶  $s = 2$  if  $(\theta > \pi/3 \text{ and } \theta < 2\pi/3)$
- ▶  $s = 3$  if  $(\theta > 2\pi/3 \text{ and } \theta < \pi)$
- ▶ etc

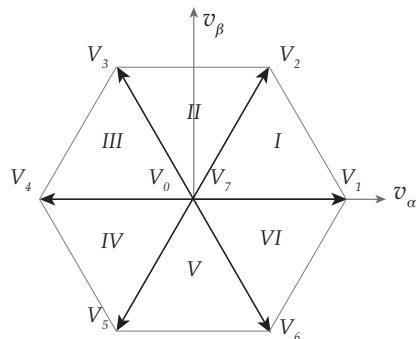


Figure 21 Space vectors and sectors in  $\alpha\beta$  plane

# SVPWM VERSUS CARRIER BASED PWM (I)

If we analyse fundamental modulating signals (SPWM) over the fundamental period

$$v_a^* = M \cos(\theta)$$

$$v_b^* = M \cos\left(\theta - \frac{2\pi}{3}\right)$$

$$v_c^* = M \cos\left(\theta - \frac{4\pi}{3}\right)$$

One can conclude

- ▶ every  $\theta = \pi/3$  order of modulating signals changes!
- ▶ e.g. for  $0 < \theta < \pi/3$  we have  $v_c^* < v_b^* < v_a^*$
- ▶ e.g. for  $\pi/3 < \theta < 2\pi/3$  we have  $v_b^* < v_a^* < v_c^*$
- ▶ resulting PWM is having duty of pulses in the same order
- ▶ this is identical to sector  $s = 1$  of the SVPWM

Injection of zero-sequence signal does not change order of modulating signals

How much are then duty cycles applied to each leg?

Where are the space vectors of the inverter?

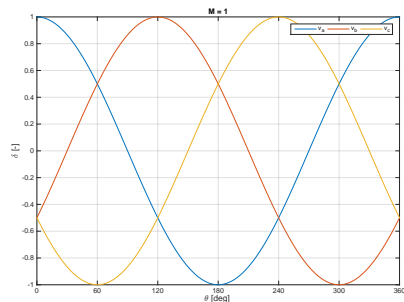


Figure 22 CBPWM - fundamental sinusoidal signals

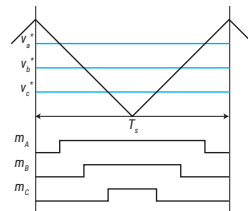


Figure 23 CBPWM -  $v_c^* < v_b^* < v_a^*$ .

# SVPWM VERSUS CARRIER BASED PWM (II)

Duty cycles of SPWM can be easily related to fundamental modulating signals:

$$\delta_A = \frac{1}{2}(1 + v_a^*)$$

$$\delta_B = \frac{1}{2}(1 + v_b^*)$$

$$\delta_C = \frac{1}{2}(1 + v_c^*)$$

From CBPWM switching pattern that corresponds to case  $0 < \theta < \pi/3$  or  $v_c^* < v_b^* < v_a^*$

- ▶ these duty cycles are directly determined without considering inverter space vectors
- ▶ duty cycles of inverter legs are related as:  $\delta_C < \delta_B < \delta_A$
- ▶ this is identical to the one discussed for SVPWM and sector  $s = 1$
- ▶ we can easily verify that switching pattern is:

$$\mathbf{V}_0 - \mathbf{V}_1 - \mathbf{V}_2 - \mathbf{V}_7 - \mathbf{V}_2 - \mathbf{V}_1 - \mathbf{V}_0$$

To calculate duty cycles of these space vectors we need to solve:

$$\delta_7 = \delta_C$$

$$\delta_2 = \delta_B - \delta_C$$

$$\delta_1 = \delta_A - \delta_B$$

$$\delta_0 = 1 - \delta_A$$

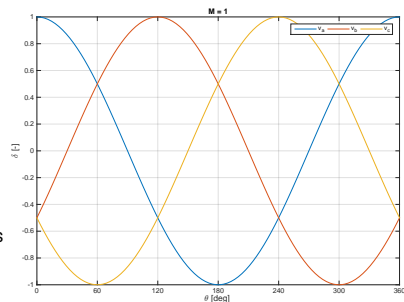


Figure 24 CBPWM - fundamental sinusoidal signals

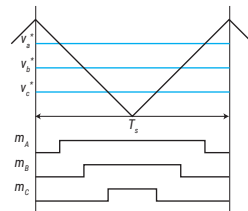


Figure 25 CBPWM -  $v_c^* < v_b^* < v_a^*$ .

# SVPWM VERSUS CARRIER BASED PWM (III)

Duty cycles of space vector activated by CBPWM - SPWM:

$$\delta_7 = \frac{1}{2}(1 + M \cos(\theta - 4\pi/3))$$

$$\delta_2 = M \frac{\sqrt{3}}{2} \sin(\theta)$$

$$\delta_1 = M \frac{\sqrt{3}}{2} \sin(\pi/3 - \theta)$$

$$\delta_0 = \frac{1}{2}(1 - M \cos(\theta))$$

Few important observations:

- ▶ SPWM duty cycles of active space vectors  $\delta_1$  and  $\delta_2$  are identical as for SVPWM
- ▶ SPWM duty cycles of zero space vectors  $\delta_0$  and  $\delta_7$  are not the same
- ▶ for SVPWM we made a choice for  $\delta_0 = \delta_7 = T_{ZSV}/2$
- ▶ however, for both SPWM and SVPWM it is still valid  $\delta_0 + \delta_7 = 1 - \delta_1 - \delta_2$

The same applies for harmonic injection CBPWM - THIPWM

Injection of zero-sequence signal only modifies duty cycles of zero space vectors

Triangular (min-max) zero sequence signal is analogue equivalent of SVPWM

This can be easily checked with basic trigonometric calculations...

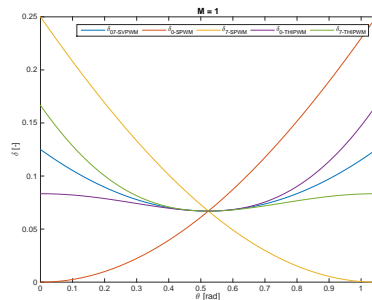


Figure 26 ZSV duty cycles in sector 1

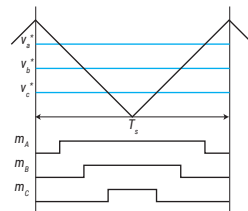


Figure 27 CBPWM -  $v_c^* < v_b^* < v_a^*$ .

# SVPWM IMPLEMENTATION

Implementation steps of the SVPWM could be summarized as:

1. determine the sector where the reference space vector is in the  $\alpha\beta$  plane
2. calculate duty cycles for active space vectors
3. calculate duty cycles for zero space vectors
4. assemble the switching pattern based on the current sector
5. send the PWM signals out

Regarding step 4, and implementation of modulating signals  $m_A, m_B, m_C$ .

In sector  $s = 1$ , and similarly in other odd sectors (use correct vectors!)

- ▶  $s = 1$ , leg signal  $m_A$  has a duty cycle of  $\delta_A = \delta_7 + \delta_2 + \delta_1$
- ▶  $s = 1$ , leg signal  $m_B$  has a duty cycle of  $\delta_B = \delta_7 + \delta_2$
- ▶  $s = 1$ , leg signal  $m_C$  has a duty cycle of  $\delta_C = \delta_7$

In sector  $s = 2$ , and similarly in other even sectors (use correct vectors!)

- ▶  $s = 2$ , leg signal  $m_A$  has a duty cycle of  $\delta_A = \delta_7 + \delta_2$
- ▶  $s = 2$ , leg signal  $m_B$  has a duty cycle of  $\delta_B = \delta_7 + \delta_2 + \delta_3$
- ▶  $s = 2$ , leg signal  $m_C$  has a duty cycle of  $\delta_C = \delta_7$

Regarding step 5, it may required that you use carrier for actual PLECS implementation

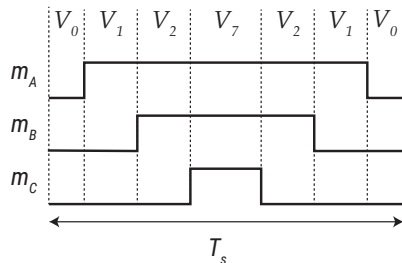


Figure 28 Switching pattern in  $s = 1$ .

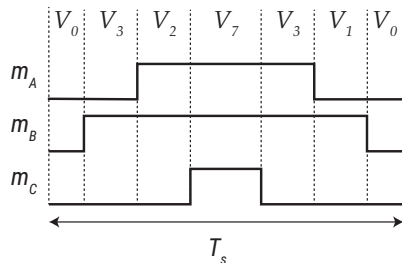


Figure 29 Switching pattern in  $s = 2$ .

# SVPWM VERSUS CARRIER-BASED PWM

## Carrier-Based PWM:

- ▶ implemented in  $abc$  frame
- ▶ simple implementation
- ▶ provides duty cycles for inverter legs
- ▶ degree of freedom - zero sequence signal
- ▶ the same DC bus utilization
- ▶ DPWM can be achieved with suitable zero sequence signal

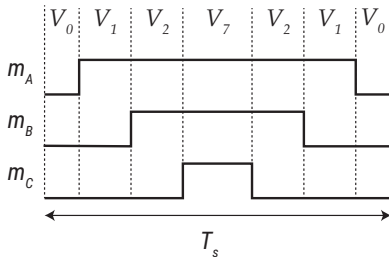


Figure 30 Switching pattern: SVPWM, SPWM, THIPWM.

## Space Vector PWM:

- ▶ implemented in  $\alpha\beta$  frame
- ▶ slightly more involved implementation
- ▶ provides duty cycles for switching configuration (space vectors)
- ▶ degree of freedom - switching pattern, zero space vectors
- ▶ the same DC bus utilization
- ▶ DPWM through switching pattern modifications

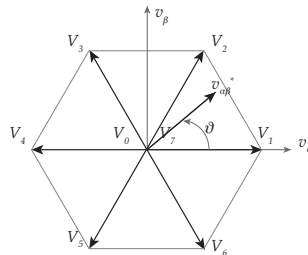


Figure 31 Reference space vector and inverter vectors in  $\alpha\beta$  plane