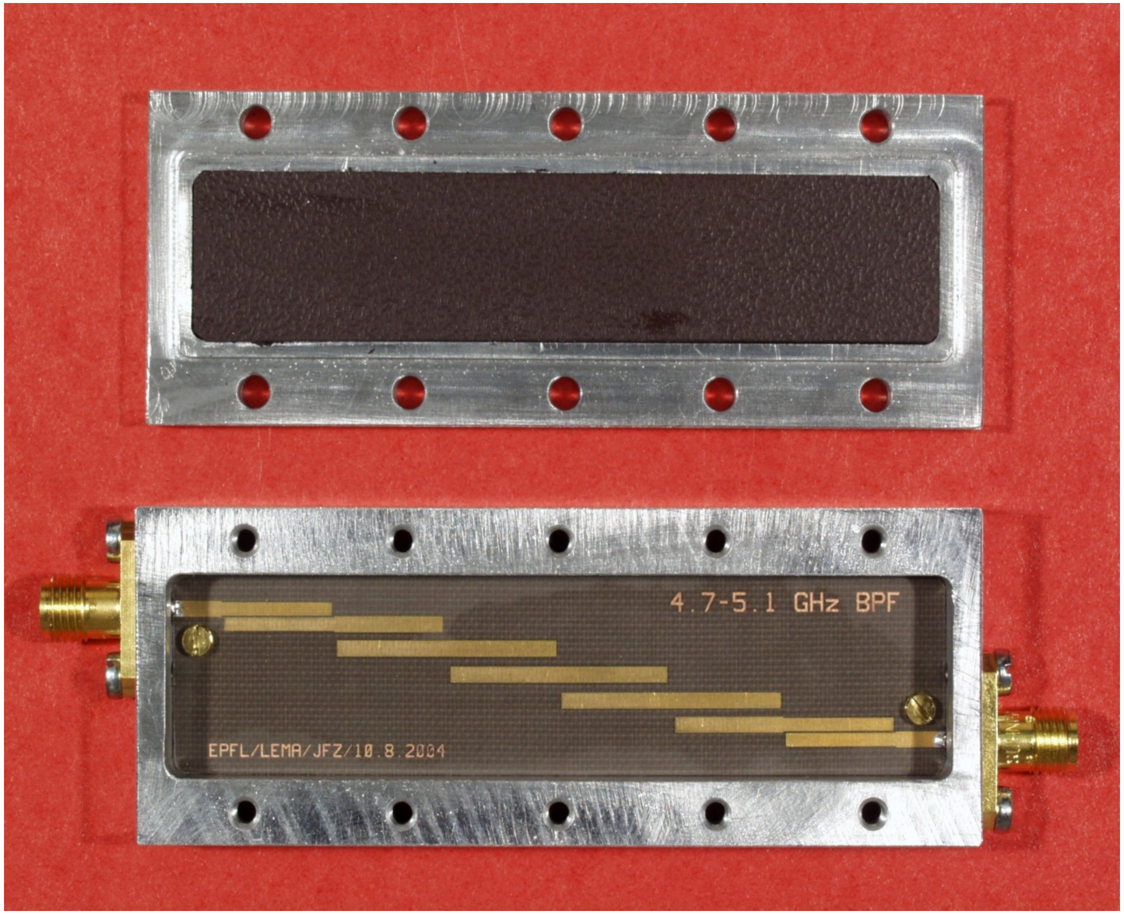
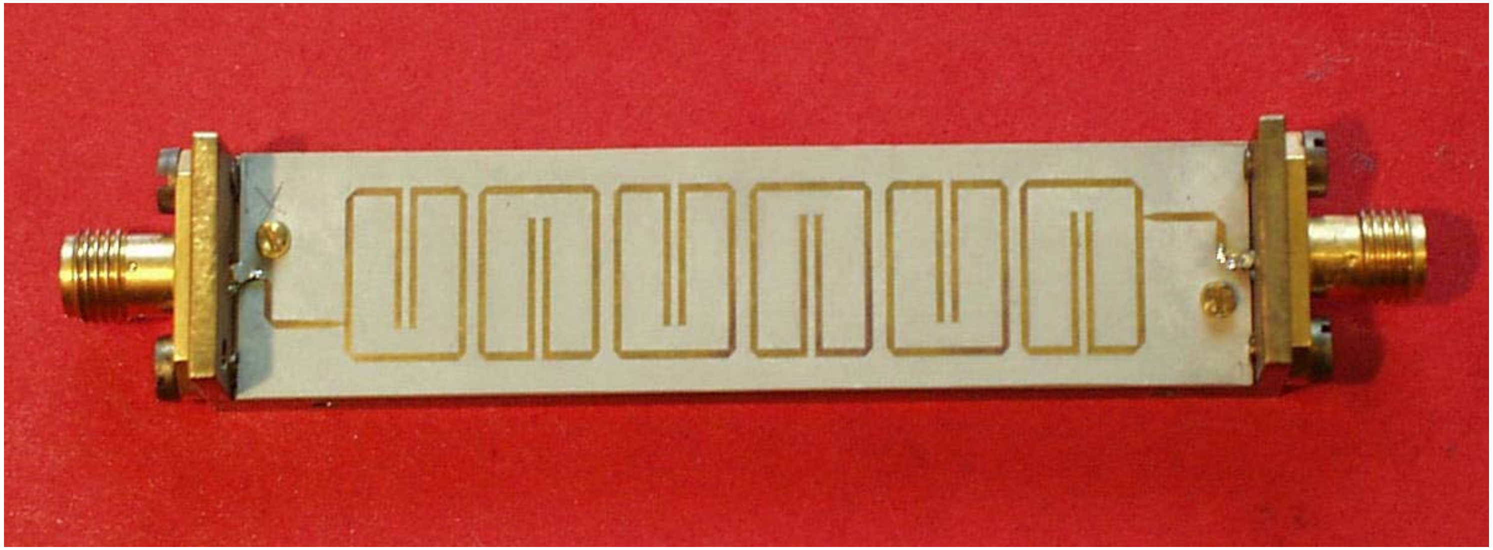
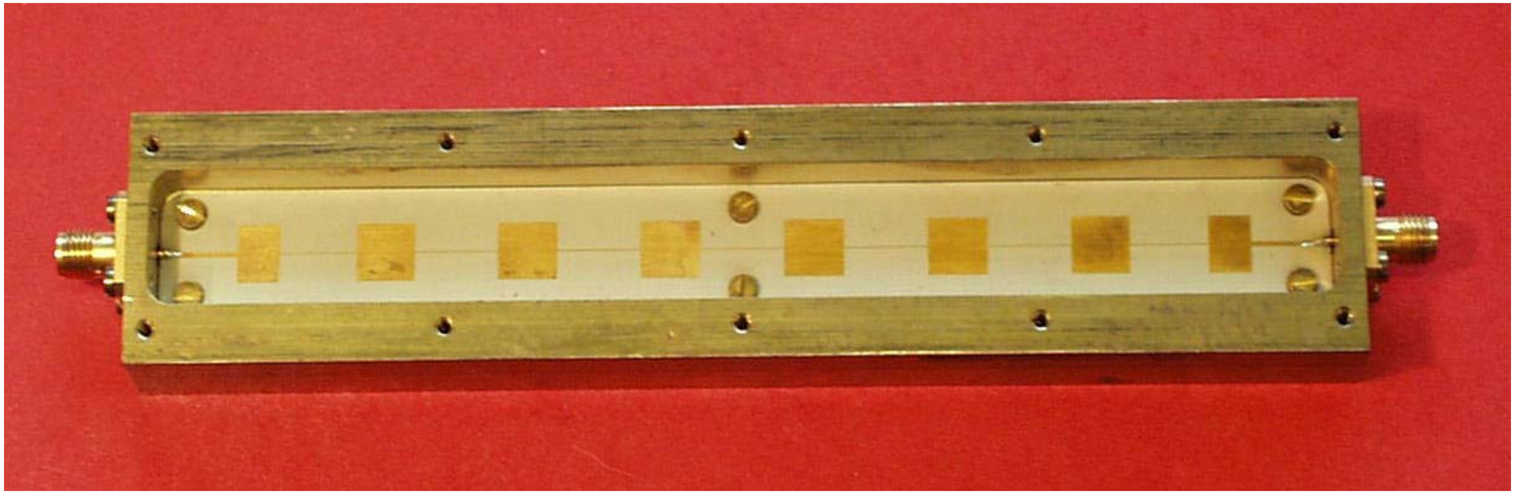
Transmission lines

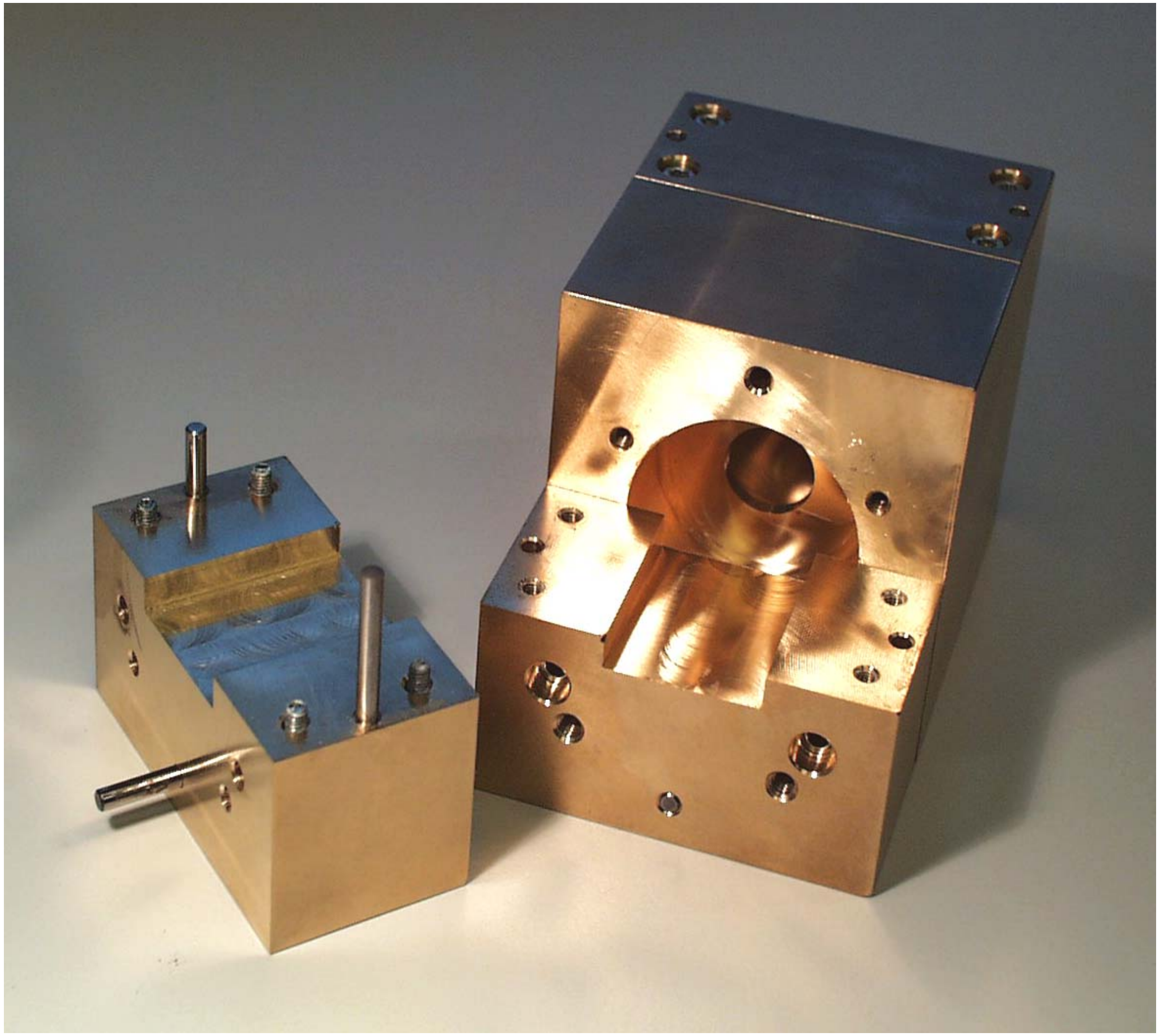
Chapter 3

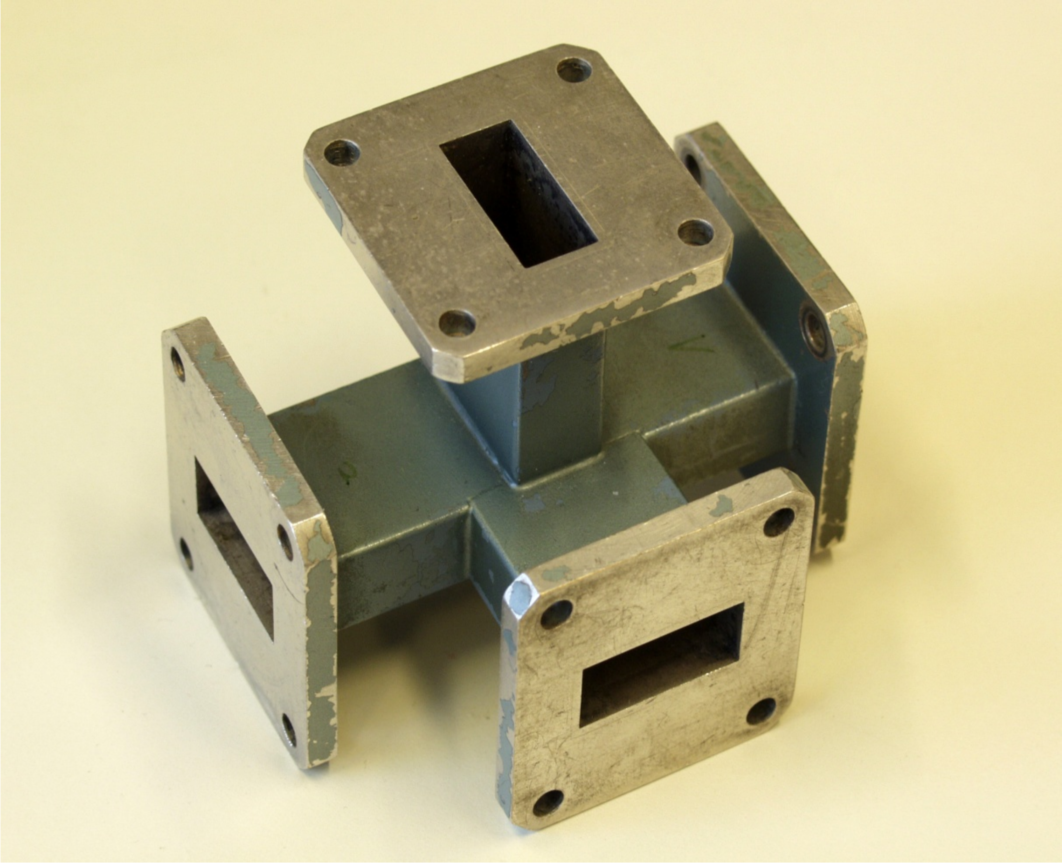

4.7-5.1 GHz BPF
EPFL/LEMA/JFZ/10.8.2004

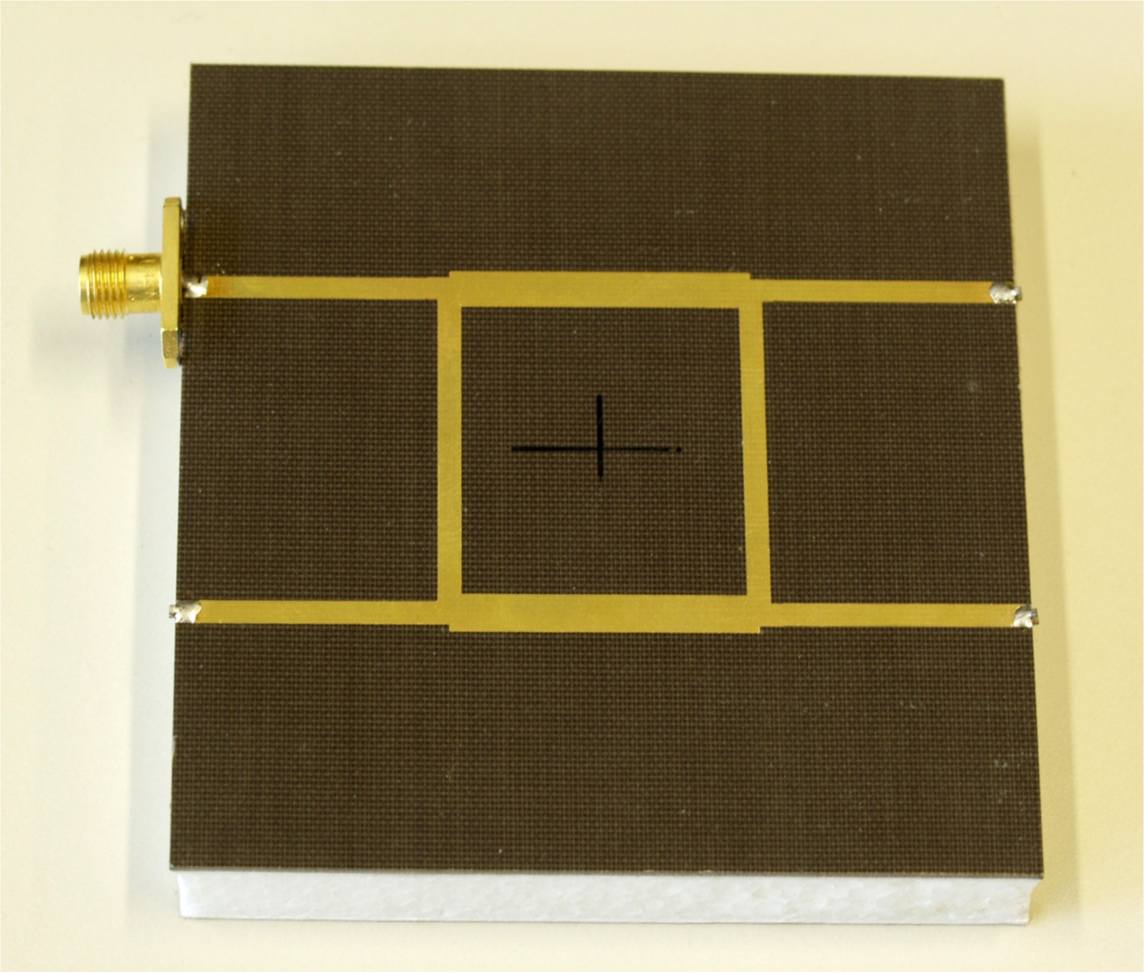


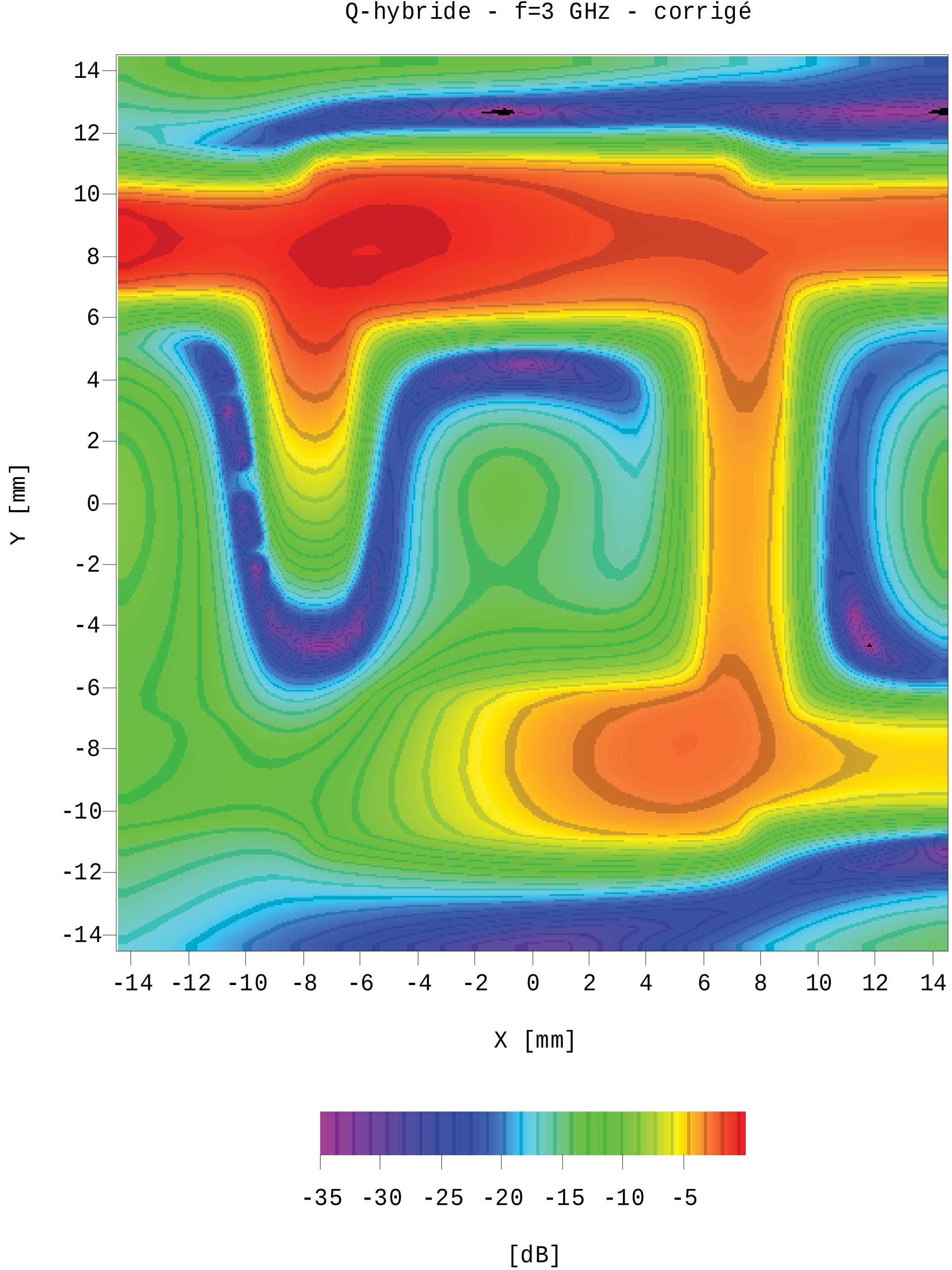


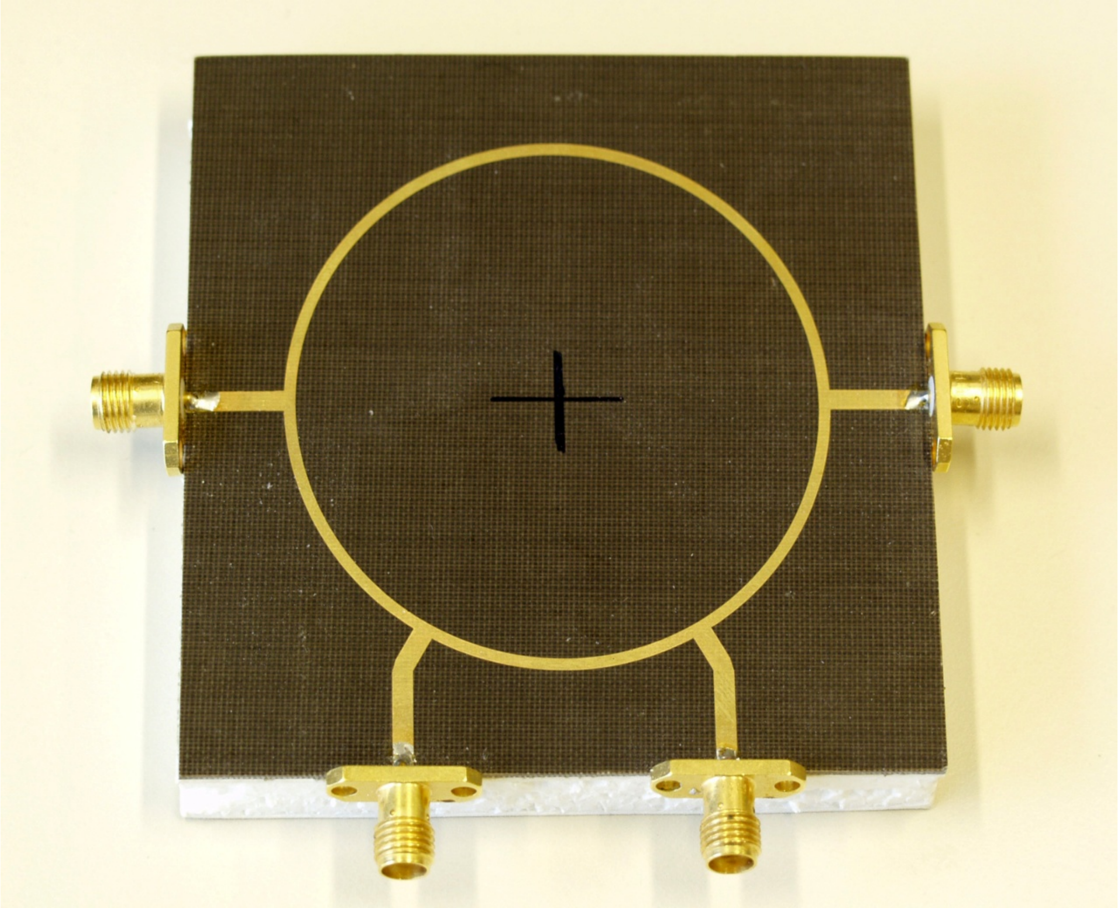
EPFL



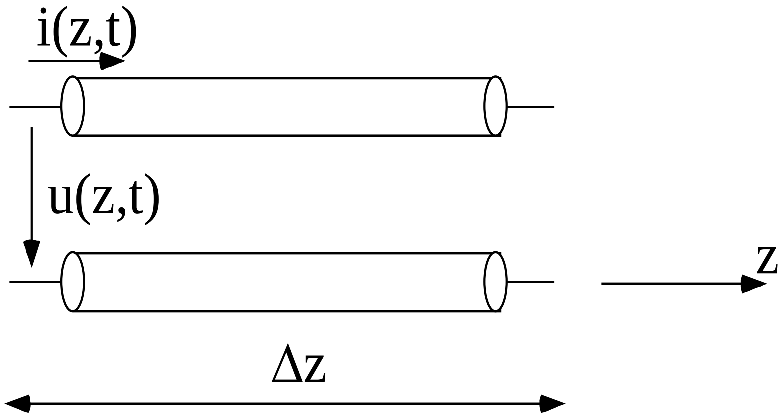


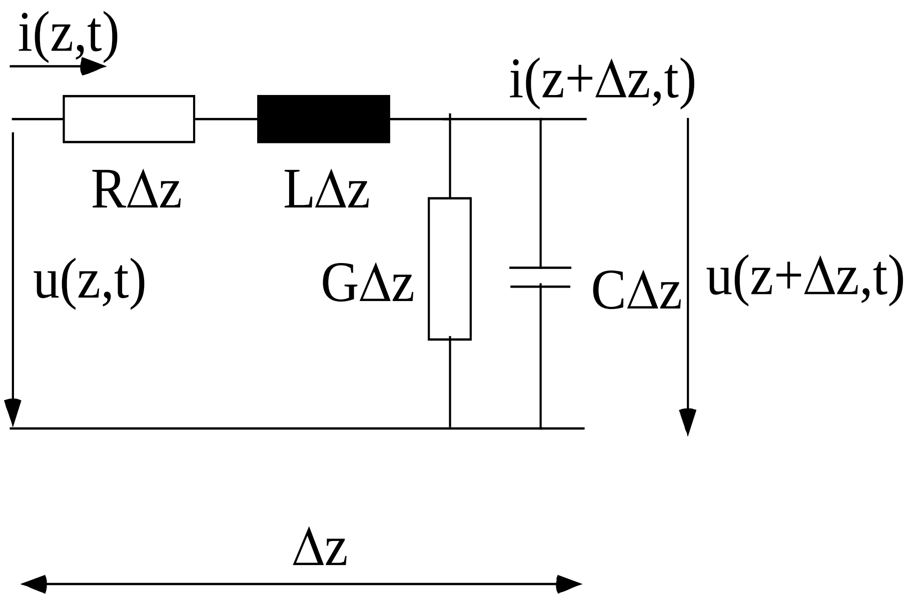



Q-hybride - f=3 GHz - corrigé
Y [mm]
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14
X [mm]
-35 -30 -25 -20 -15 -10 -5
[dB]



Incremental model (1)





Incremental model (2)

Kirchhoff's laws : $v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial u(z,t)}{\partial t}$$

$$\rightarrow$$

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z)$$

Incremental model (3)

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

with

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Incremental model (4)

Solutions :

$$V(z) = V_0^+ \, e^{-\gamma z} + V_0^- \, e^{\gamma z}$$

$$I(z) = I_0^+ \, e^{-\gamma z} + I_0^- \, e^{\gamma z}$$

or :

$$I(z) = \frac{\gamma}{R + j\omega L}\left(V_0^+ \, e^{-\gamma z} - V_0^- \, e^{\gamma z}\right)$$

$$I(z) = \frac{V_0^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$

with $Z_o = \dfrac{R + j\omega L}{\gamma} = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

Incremental model (5)

wave length : $\lambda = 2\pi/\beta$

Phase velocity : $v_\varphi = dz/dt = \omega/\beta = \lambda f$

Lossless transmission lines

$$\gamma = j\beta = j\omega\sqrt{LC}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

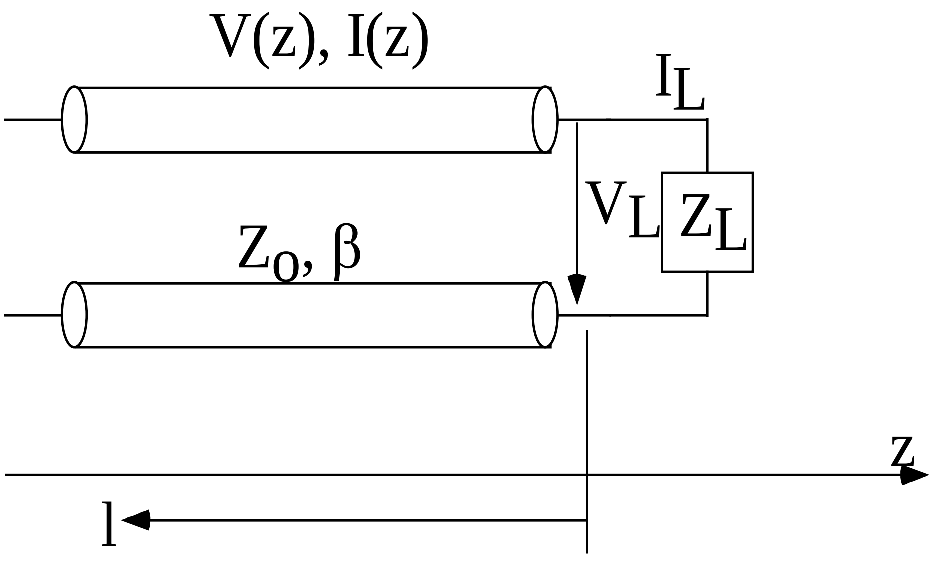
$$V(z) = V_0^+ \,\mathrm{e}^{-j\beta z} + V_0^- \,\mathrm{e}^{j\beta z}$$

$$I(z) = I_0^+ \,\mathrm{e}^{-j\beta z} + I_0^- \,\mathrm{e}^{j\beta z} = \frac{V_0^+}{Z_o}\mathrm{e}^{-j\beta z} - \frac{V_0^-}{Z_o}\mathrm{e}^{j\beta z}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$$

$$v_\varphi = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Terminated lines (1)



$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$$

$$V_0^- = \frac{Z_L - Z_o}{Z_L + Z_o} V_0^+$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Terminated lines (2)

$$V(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$I(z) = \frac{V_0^+}{Z_o}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

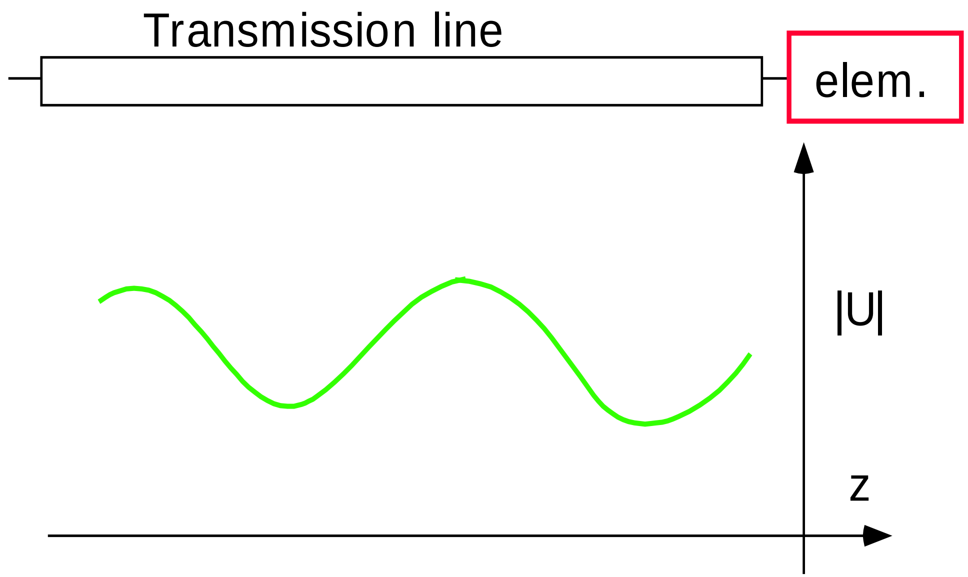
$$P_{av} = \mathrm{Re}\left[V(z) I^*(z)\right] = \frac{\left|V_o^+\right|^2}{Z_o}\mathrm{Re}\left[1 - \Gamma^* e^{-2j\beta z} + \Gamma \ \ e^{2j\beta z} - |\Gamma|^2\right]$$

$$\boxed{P_{av} = \mathrm{Re}\left[V(z) I^*(z)\right] = \frac{\left|V_o^+\right|^2}{Z_o}\left(1 - |\Gamma|^2\right)}$$

Terminated lines (3)

Losses due to mismatch : $RL = -20\log_{10}\left|\Gamma\right|$ dB

Terminated lines : VSWR



$$\left|V(z)\right| = \left|V_0^+\right|\left|1+\Gamma\,\mathrm{e}^{2j\beta z}\right| = \left|V_0^+\right|\left|1+\Gamma\,\mathrm{e}^{-2j\beta l}\right| = \left|V_0^+\right|\left|1+\left|\Gamma\right|\mathrm{e}^{j(\theta-2\beta)l}\right|$$

$$\Gamma(l) = \frac{V_0^-\,\mathrm{e}^{-j\beta l}}{V_0^+\,\mathrm{e}^{j\beta l}} = \Gamma(0)\mathrm{e}^{-2j\beta l}$$

Terminated lines : VSWR

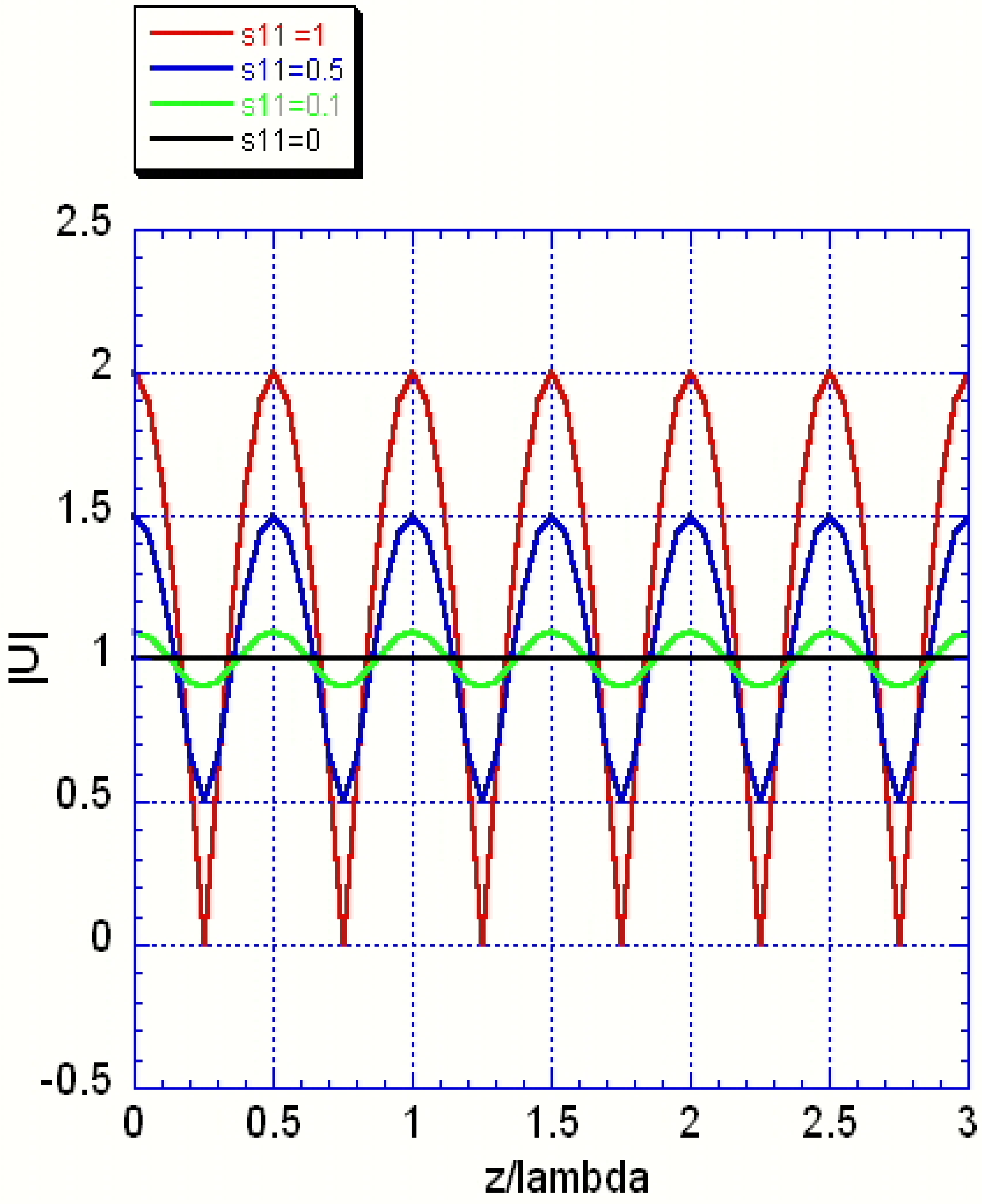
$$\left|V(z)\right| = \left|V_0^+\right|\left|1+\Gamma\,\mathrm{e}^{2j\beta z}\right| = \left|V_0^+\right|\left|1+\Gamma\,\mathrm{e}^{-2j\beta l}\right| = \left|V_0^+\right|\left|1+\left|\Gamma\right|\mathrm{e}^{j(\theta-2\beta)l}\right|$$

$$\Downarrow$$

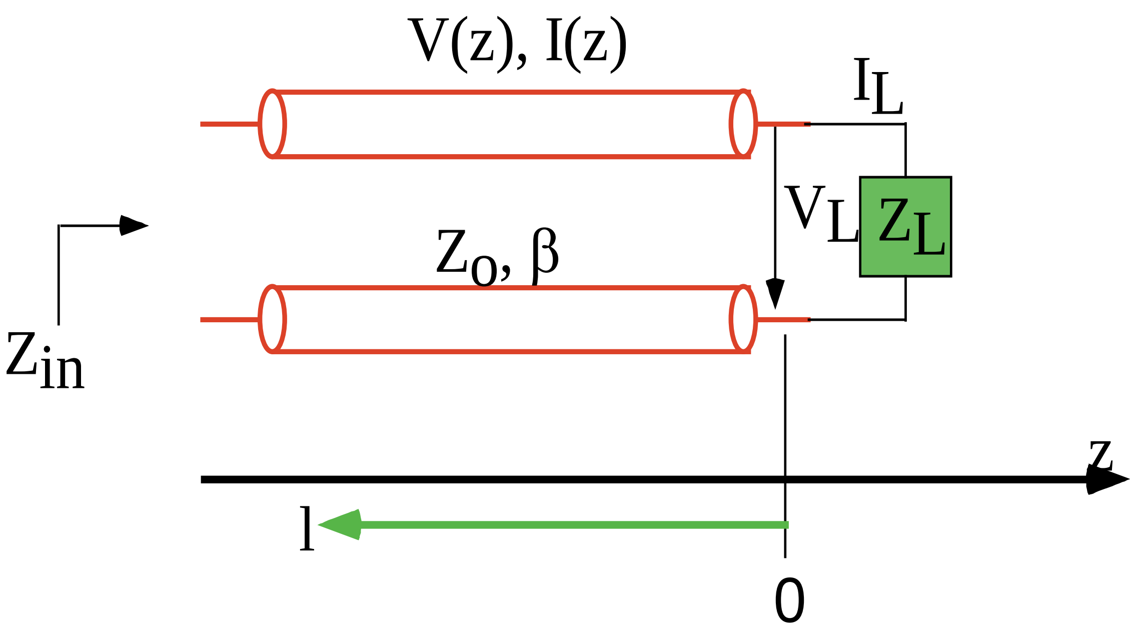
$$V_{\mathrm{max}} = \left|V_0^+\right|\left(1+\left|\Gamma\right|\right) \qquad V_{\mathrm{min}} = \left|V_0^+\right|\left(1-\left|\Gamma\right|\right)$$

$$\Downarrow$$

$$SWR = \frac{V_{\mathrm{max}}}{V_{\mathrm{min}}} = \frac{\left(1+\left|\Gamma\right|\right)}{\left(1-\left|\Gamma\right|\right)}$$


s11 =1
s11=0.5
s11=0.1
s11=0
2.5
2
1.5
1
0.5
0
-0.5
|U|
0
0.5
1
1.5
2
2.5
3
z/lambda

Terminated lines : input impedance



$$Z_{in} = \frac{V(-l)}{I(-l)}$$

$$= Z_o \frac{V_0^+\left(e^{j\beta l} + \Gamma e^{-j\beta l}\right)}{V_0^+\left(e^{j\beta l} - \Gamma e^{-j\beta l}\right)}$$

$$= Z_o \frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}$$

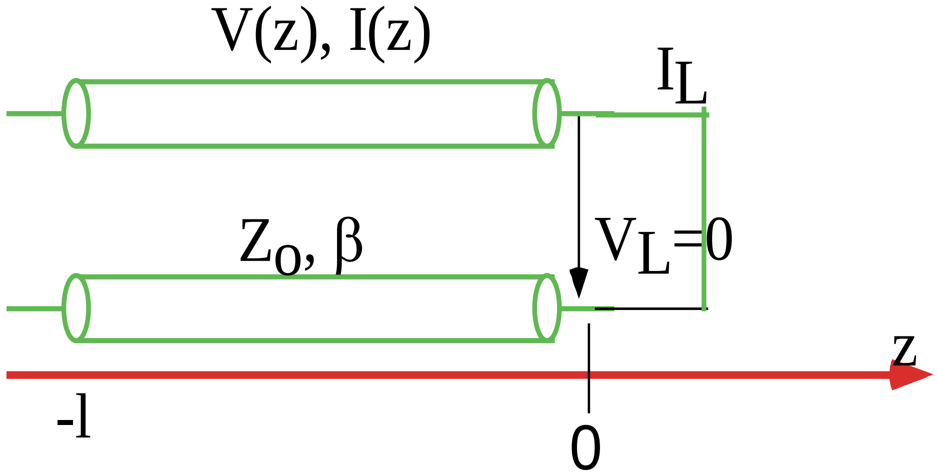
Terminated lines : input impedance

Thus, using $\Gamma = \dfrac{V_0^-}{V_0^+} = \dfrac{Z_L - Z_o}{Z_L + Z_o}$

$$Z_{in} = Z_o \frac{(Z_L + Z_o)e^{j\beta l} + (Z_L - Z_o)e^{-j\beta l}}{(Z_L + Z_o)e^{j\beta l} - (Z_L - Z_o)e^{-j\beta l}} = Z_o \frac{Z_L \cos\beta l + jZ_o \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}$$

$$= Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

Terminated lines : short circuit

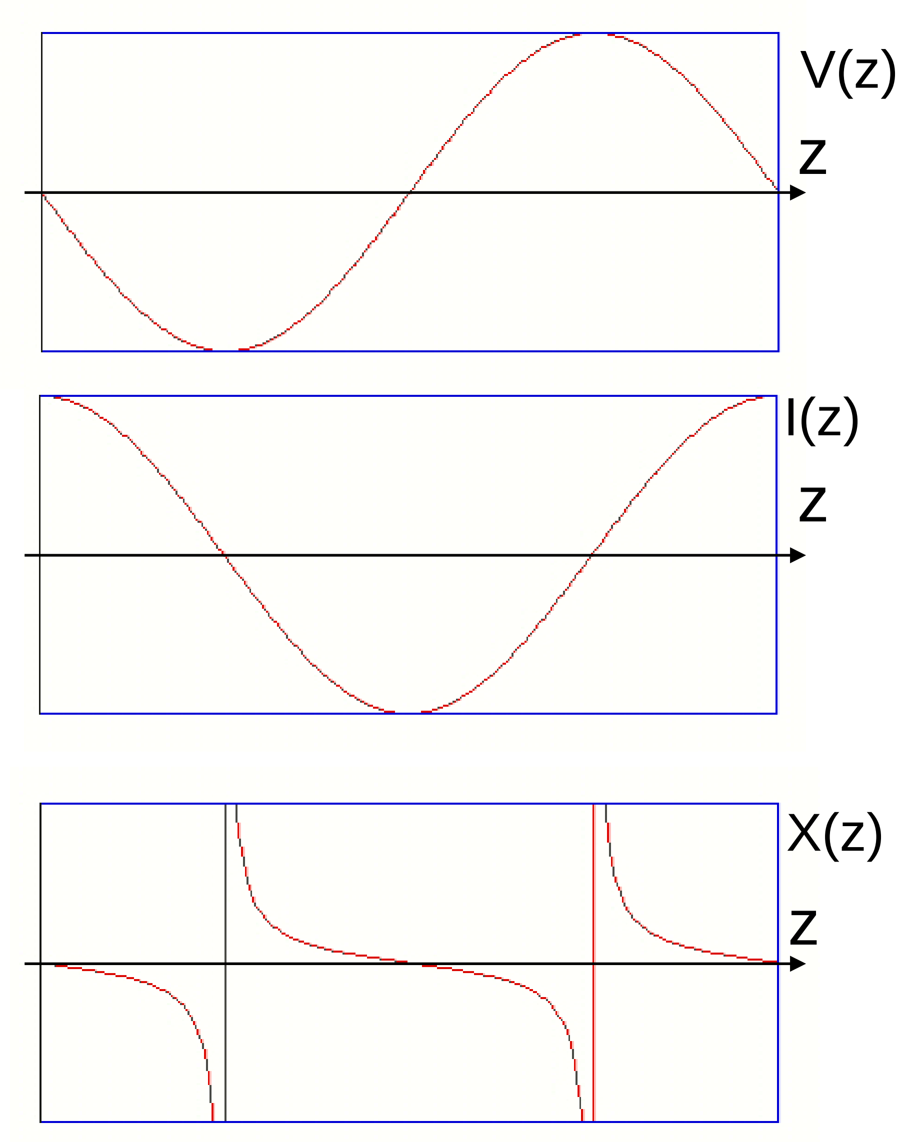


$$V(z) = V_0^+\left(e^{-j\beta z} - e^{j\beta z}\right) = -2jV_0^+ \sin\beta z$$

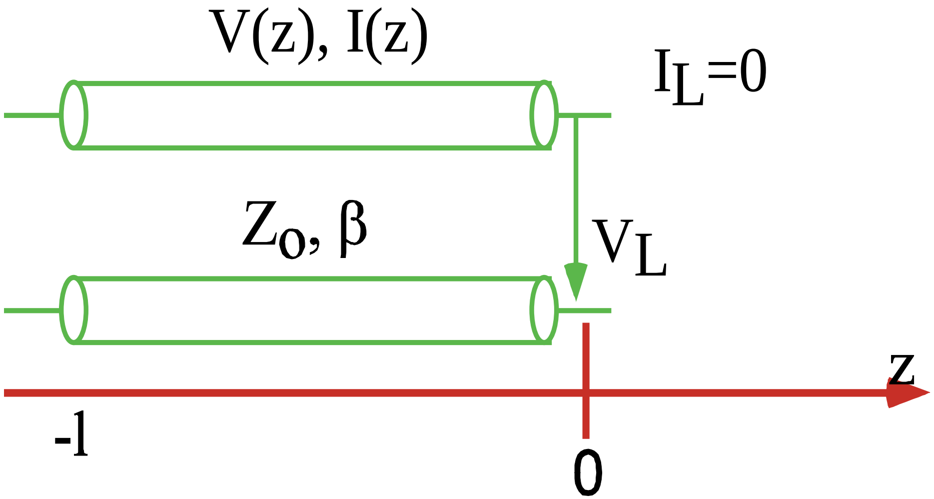
$$I(z) = \frac{V_0^+}{Z_o}\left(e^{-j\beta z} + e^{j\beta z}\right) = \frac{2V_0^+}{Z_o}\cos\beta z$$

$\rightarrow$ $Z_{in} = jZ_o \tan\beta z$

Terminated lines : short circuit



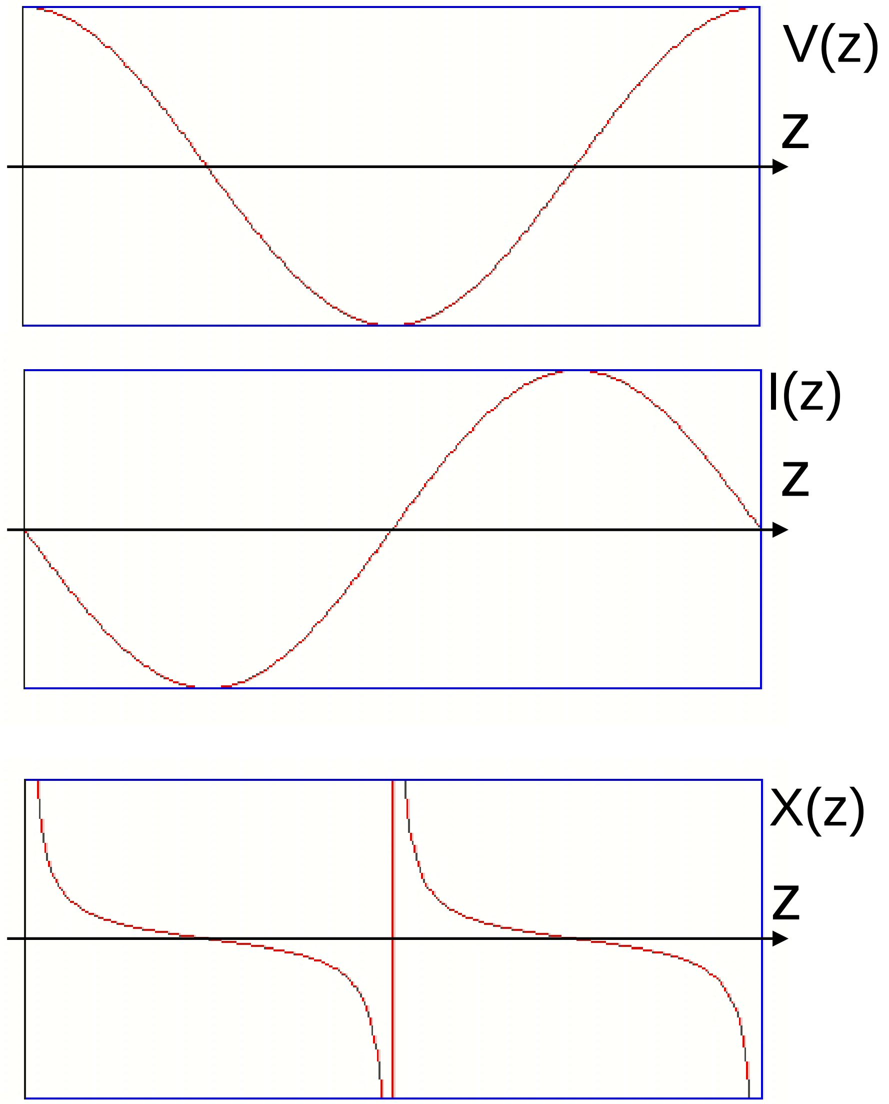
Terminated lines : open circuit



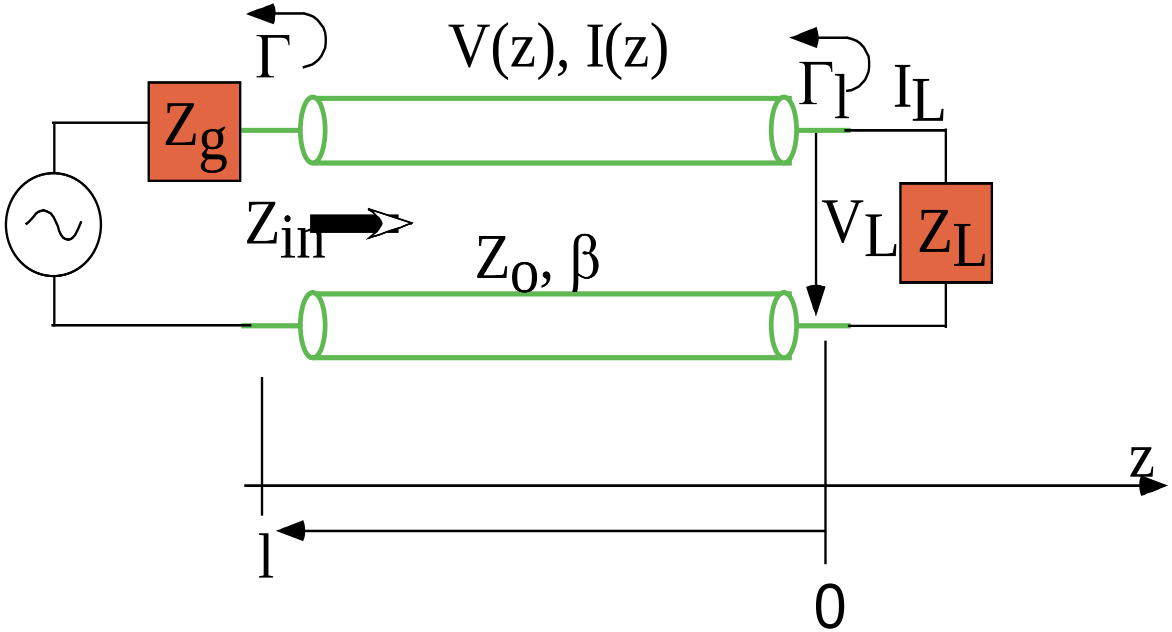
$$V(z) = V_0^+\left(e^{-j\beta z} + e^{j\beta z}\right) = 2V_0^+ \cos\beta z$$

$$I(z) = \frac{V_0^+}{Z_o}\left(e^{-j\beta z} - e^{j\beta z}\right) = \frac{-2jV_0^+}{Z_o}\sin\beta z \quad \rightarrow \quad Z_{in} = -jZ_o \cot\beta z$$

Terminated lines : open circuit



Generator and load mismatches



Generator and load mismatches

Input impedance (at l) looking towards the load :

$$Z_{in} = Z_o \frac{1+\Gamma_l \, e^{-j2\beta l}}{1-\Gamma_l \, e^{-j2\beta l}} = Z_o \frac{Z_l + jZ_o \tan \beta l}{Z_o + jZ_l \tan \beta l}$$

with

$$\Gamma_l = \frac{Z_l - Z_o}{Z_l + Z_o}$$

Generator and load mismatches

Amplitude of the incident wave from the generator :

$$V(-l) = V_g \frac{Z_{in}}{Z_{in} + Z_g} = V_0^+ \left(\mathrm{e}^{j\beta l} + \Gamma_l \, \mathrm{e}^{-j\beta l} \right)$$

Thus $$V_0^+ = V_g \frac{Z_{in}}{Z_{in} + Z_g} \frac{1}{\left(\mathrm{e}^{j\beta l} + \Gamma_l \, \mathrm{e}^{-j\beta l} \right)}$$

and

$$V_0^+ = V_g \frac{Z_{in}}{Z_{in} + Z_g} \frac{1}{\left(\mathrm{e}^{j\beta l} + \Gamma_l \, \mathrm{e}^{-j\beta l} \right)} = V_g \frac{Z_o}{Z_o + Z_g} \frac{\mathrm{e}^{-j\beta l}}{\left(1 - \Gamma_l \Gamma_g \, \mathrm{e}^{-2j\beta l} \right)}$$

Generator and load mismatches

Power delivered to the load

$$P_l = \mathrm{Re}\left[V_{in} I_{in}^{*}\right] = \left|V_{in}\right|^2 \mathrm{Re}\left[\frac{1}{Z_{in}}\right] = \left|V_g\right|^2 \left|\frac{Z_{in}}{Z_{in} + Z_g}\right|^2 \mathrm{Re}\left[\frac{1}{Z_{in}}\right]$$

$$\boxed{P_l = \left|V_g\right|^2 \frac{R_{in}}{\left(R_{in} + R_g\right)^2 + \left(X_{in} + X_g\right)^2}}$$

Generator and load mismatches

If the load is matched ($Z_l = Z_o$)

$$P_l = |V_g|^2 \frac{Z_o}{(Z_o + R_g)^2 + X_g^{\,2}}$$

If the generator is matched to the mismatched line ($Z_g = Z_{in}$)

$$\Gamma = \frac{Z_{in} - Z_g}{Z_{in} + Z_g} = 0,\ \Gamma_g \neq 0,\ \Gamma_l \neq 0$$

$$P_l = |V_g|^2 \frac{R_g}{4\left(R_g^{\,2} + X_g^{\,2}\right)}$$

Generator and load mismatches

optimum power transfer

$$\frac{\partial P_l}{\partial R_{in}} = 0 \rightarrow \frac{1}{\left(R_{in}+R_g\right)^2+\left(X_{in}+X_g\right)^2} - \frac{2R_{in}\left(R_{in}+R_g\right)}{\left[\left(R_{in}+R_g\right)^2+\left(X_{in}+X_g\right)^2\right]^2} = 0$$

$$\boxed{Z_{in} = Z_g^*}$$

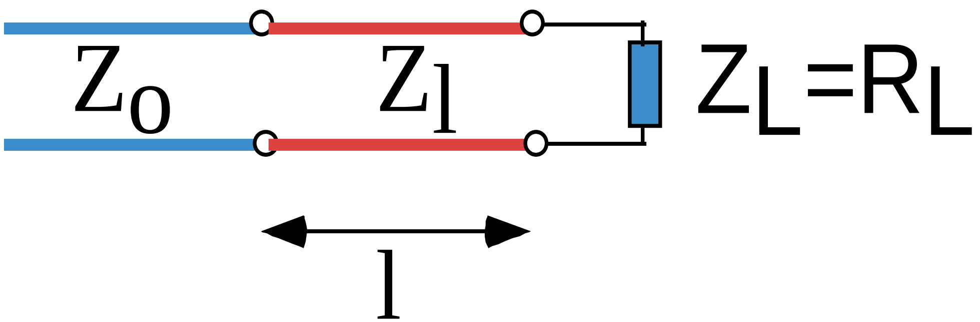
$$\boxed{P_{l\max} = \left|V_g\right|^2 \frac{1}{4R_g} \geq \left|V_g\right|^2 \frac{Z_o}{\left(Z_o+R_g\right)^2+X_g{}^2} \geq \left|V_g\right|^2 \frac{R_g}{4\left(R_g{}^2+X_g{}^2\right)}}$$

Impedance matching

Aim : achieve maximum power transfer

A narrowband matching network can always be found for a passive load

Quarter wave transformer



$$Z_l = \sqrt{Z_o Z_L}$$

$$l = \frac{\lambda_o}{4} \Rightarrow \beta l = \frac{\pi}{2}$$

$$Z_{in} = Z_l \frac{Z_L + jZ_l \tan \beta l}{Z_l + jZ_L \tan \beta l}$$

$$\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{Z_l\left(Z_L - Z_o\right) + j\tan \beta l\left(Z_l^2 - Z_o Z_L\right)}{Z_l\left(Z_L + Z_o\right) + j\tan \beta l\left(Z_l^2 + Z_o Z_L\right)}$$

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o + j2\tan \beta l \sqrt{Z_o Z_L}}$$

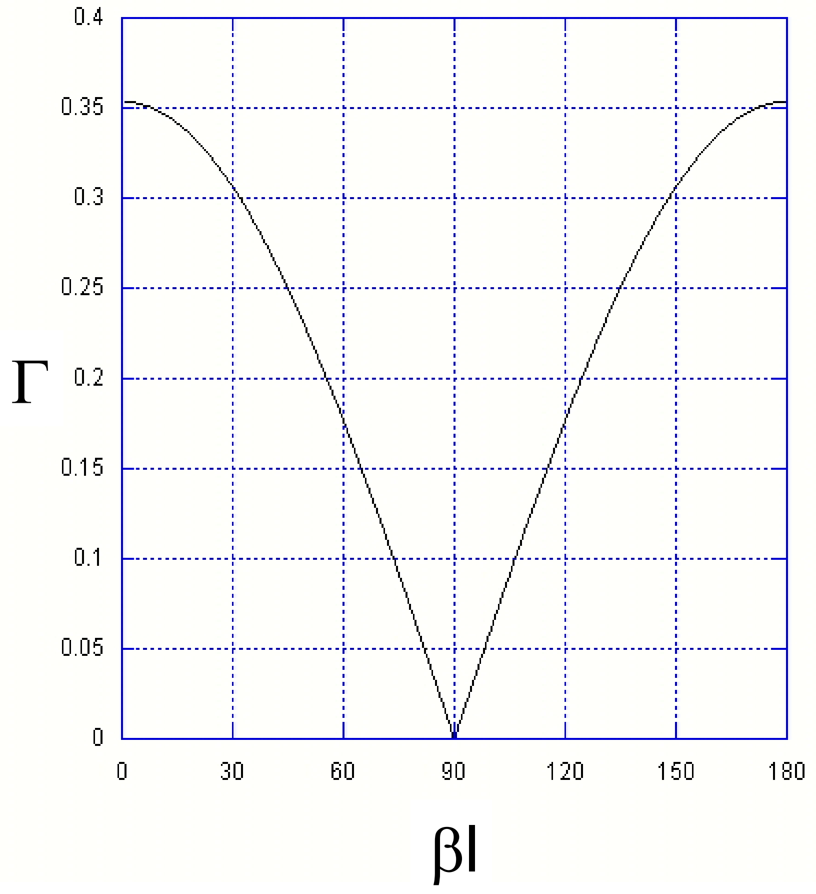
Quarter wave transformer

$$\left|\Gamma\right| = \frac{\left|Z_L - Z_o\right|}{\sqrt{\left(Z_L + Z_o\right)^2 + 4\tan^2 \beta l Z_o Z_L}} = \frac{1}{\sqrt{\dfrac{\left(Z_L + Z_o\right)^2}{\left(Z_L - Z_o\right)^2} + \dfrac{4\tan^2 \beta l Z_o Z_L}{\left(Z_L - Z_o\right)^2}}}$$

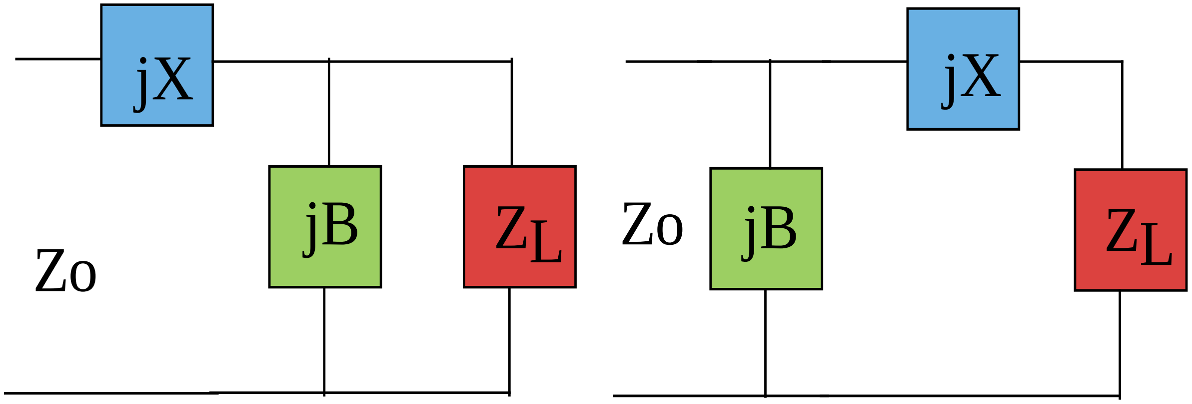
$$= \frac{1}{\sqrt{\dfrac{1 + 4Z_o Z_L}{\left(Z_L - Z_o\right)^2} + \dfrac{4\tan^2 \beta l Z_o Z_L}{\left(Z_L - Z_o\right)^2}}} = \frac{1}{\sqrt{1 + \dfrac{\left[4Z_o Z_L\right] sec^2 \beta l}{\left(Z_L - Z_o\right)^2}}}$$

$$1 + \tan^2 \beta l = sec^2 \beta l$$

Quarter wave transformer



L sections

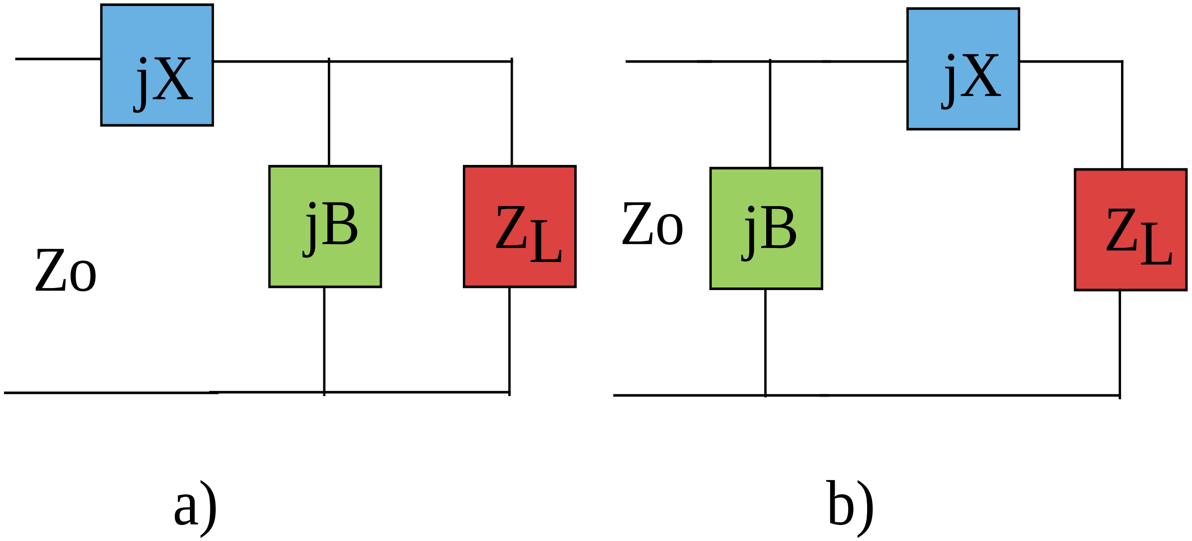


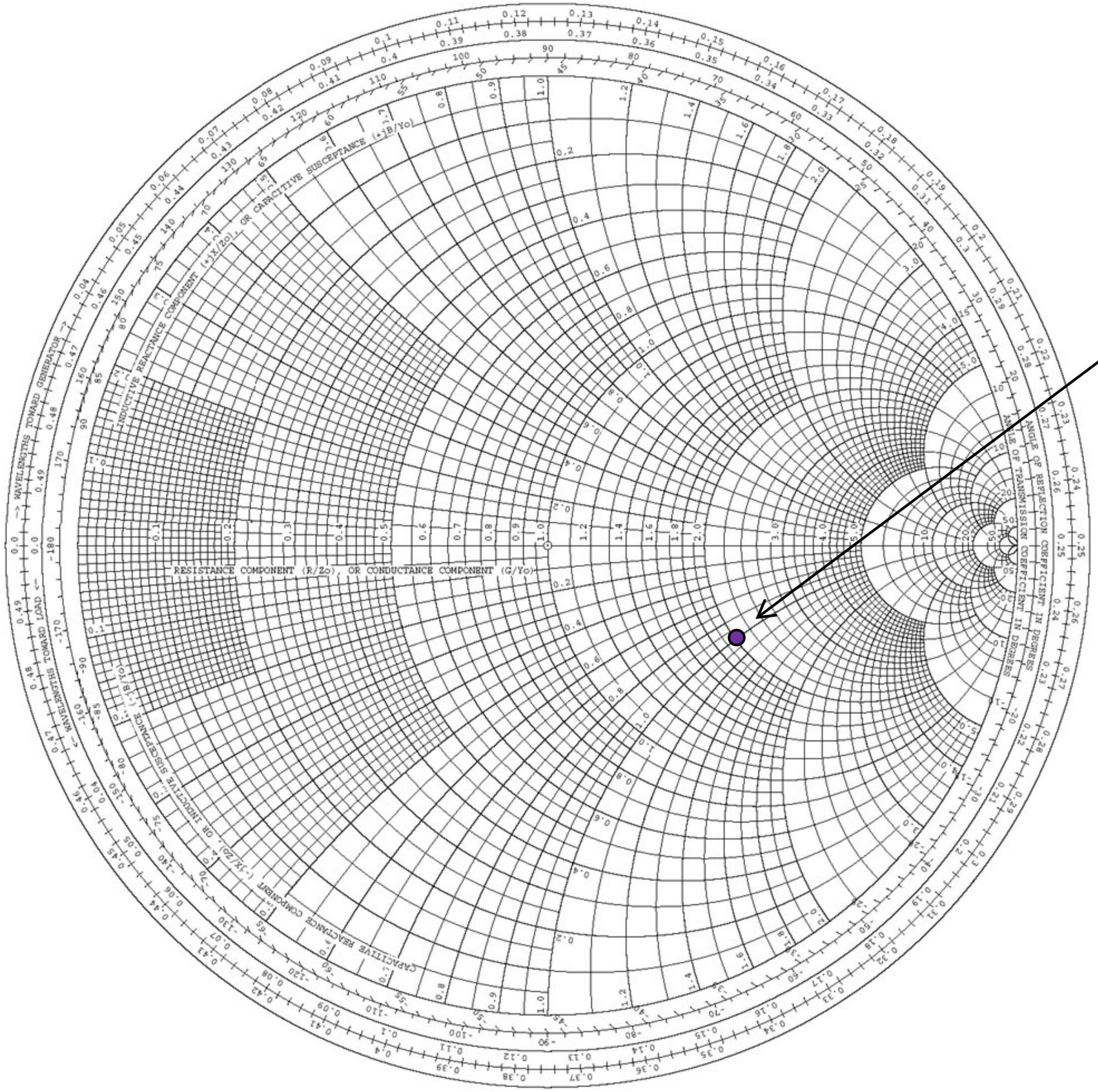
a) b)

Example

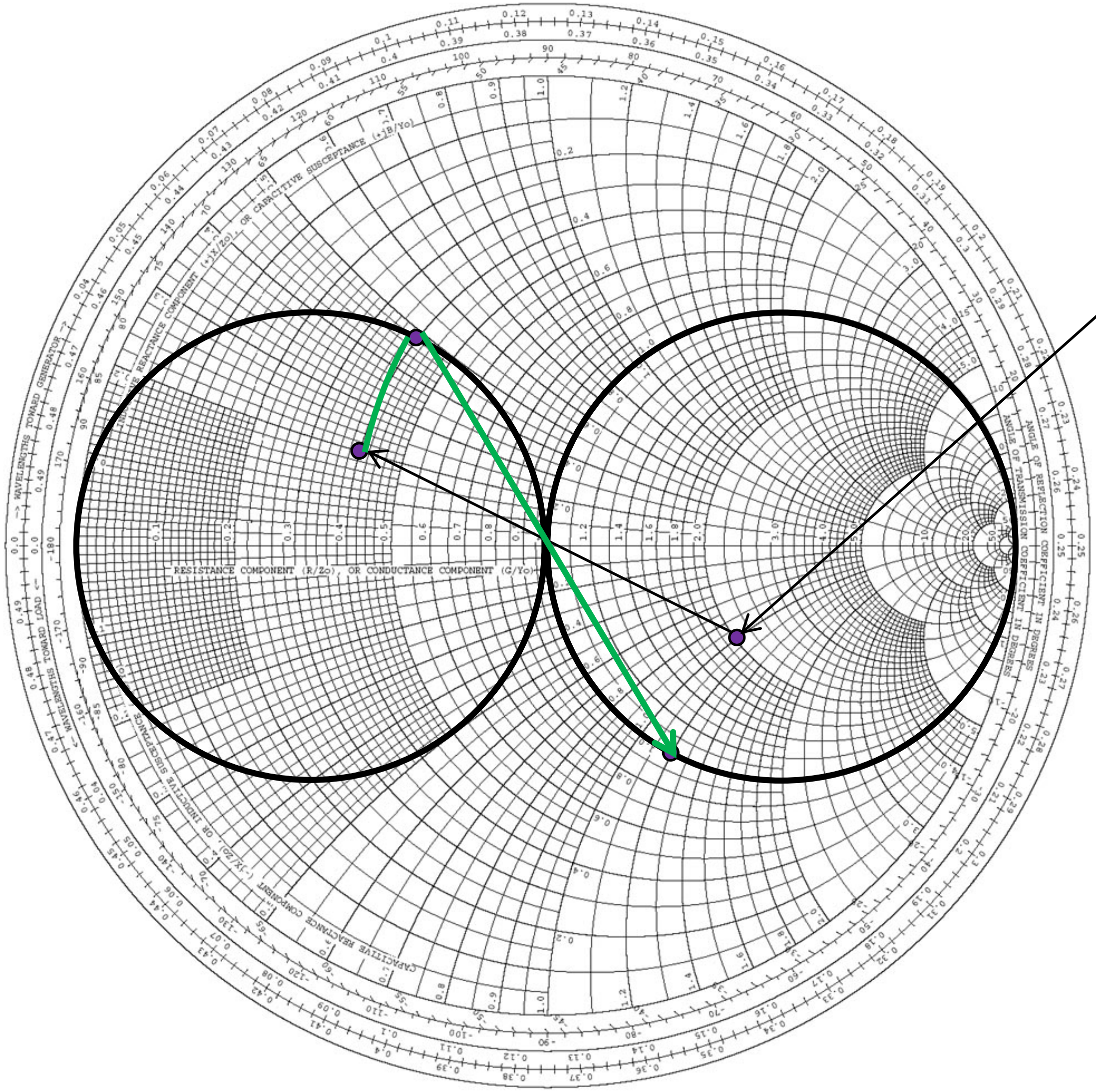
Example: we want to design an L-section matching network to match a series RC load having an impedance $Z_L = 200 - j100\ \Omega$,
to a 100 Ω line, at a frequency of 500 MHz.

L sections



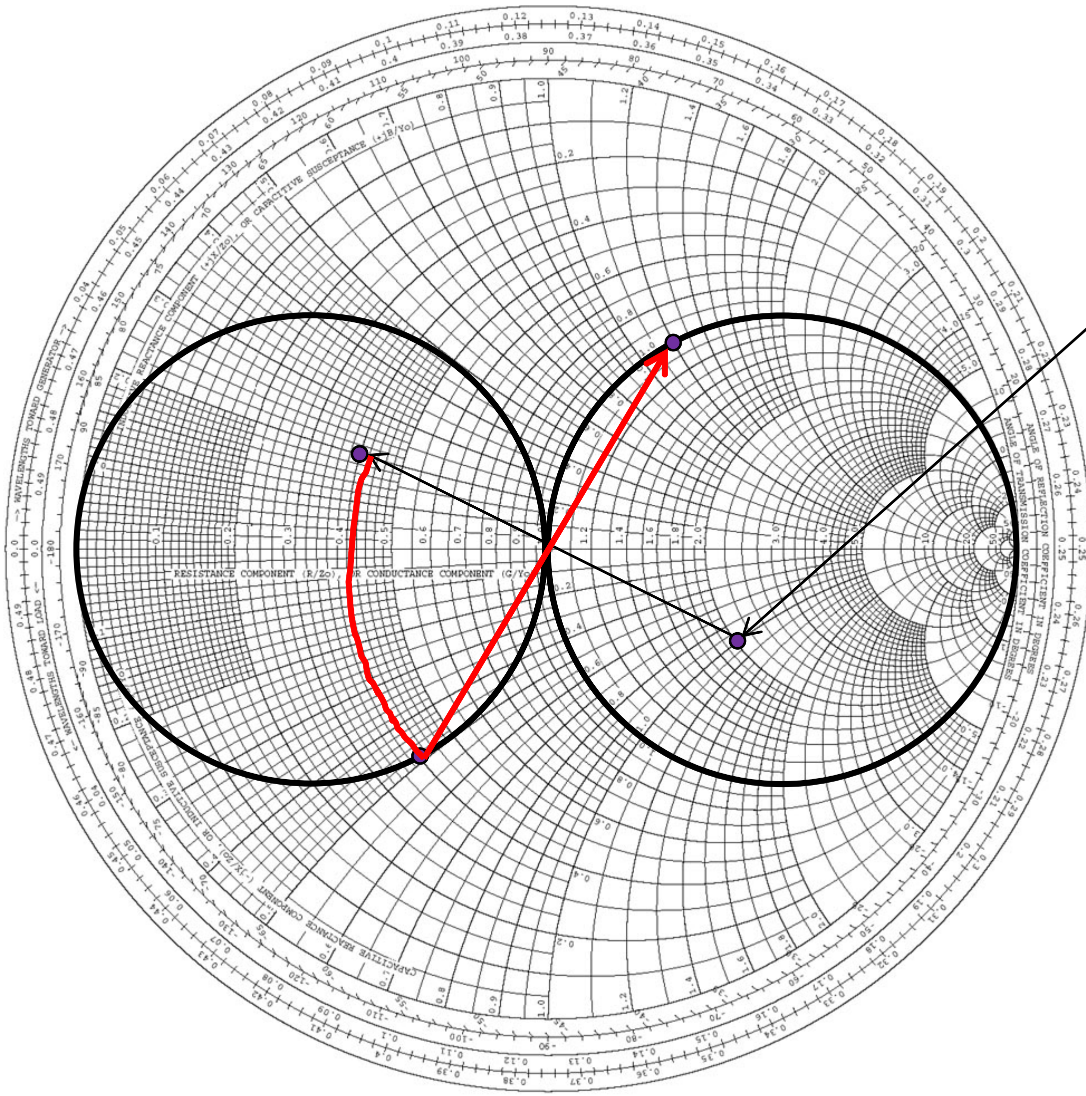


ZL =200-j100 Ω
Z_0=100 Ω
z_l=2-j



ZL =200-j100 Ω
Z_0=100 Ω
z_l=2-j

y_l=0.4+0.2j



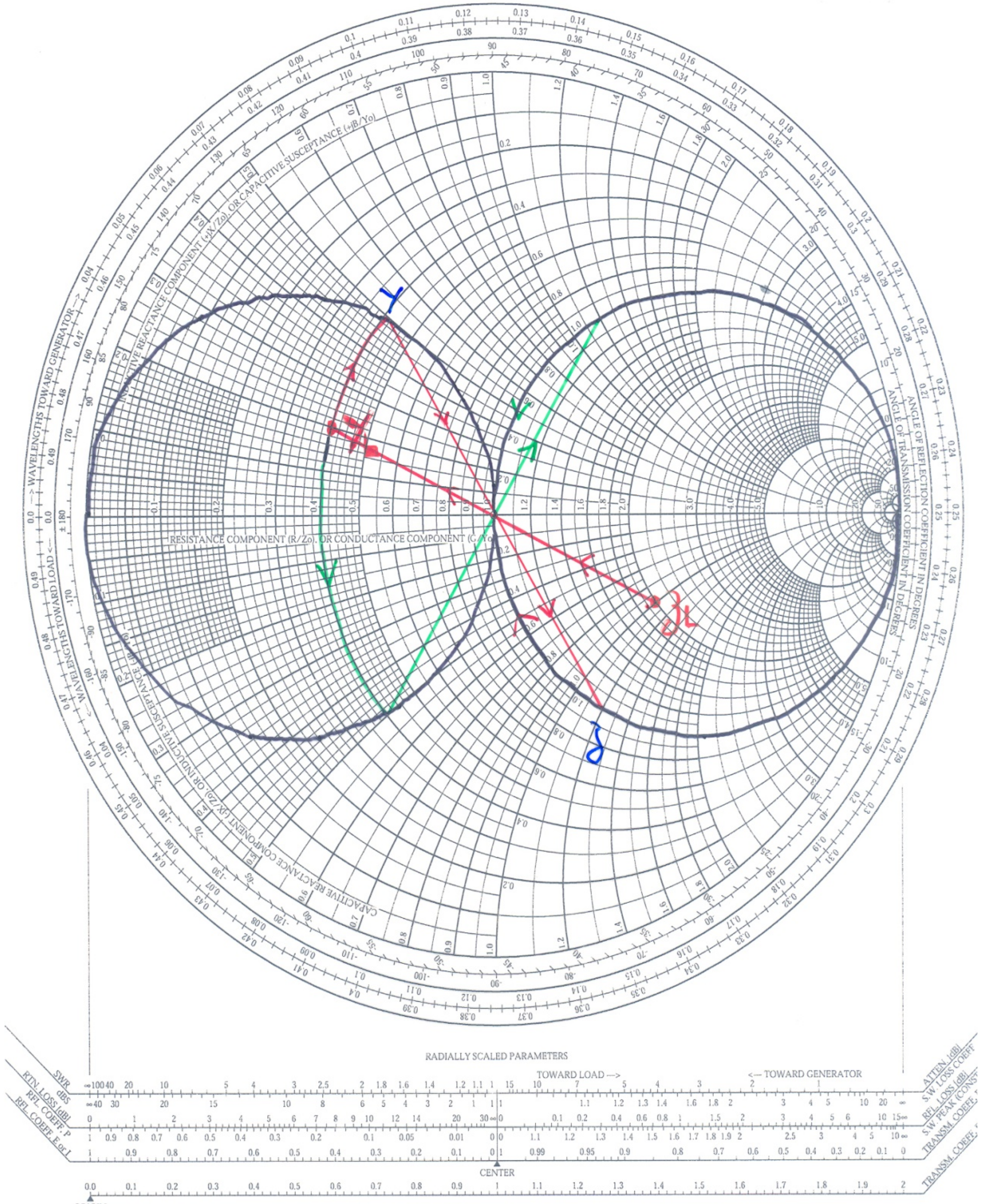
$Z_L = 200 - j100\ \Omega$

$Z_0 = 100\ \Omega$

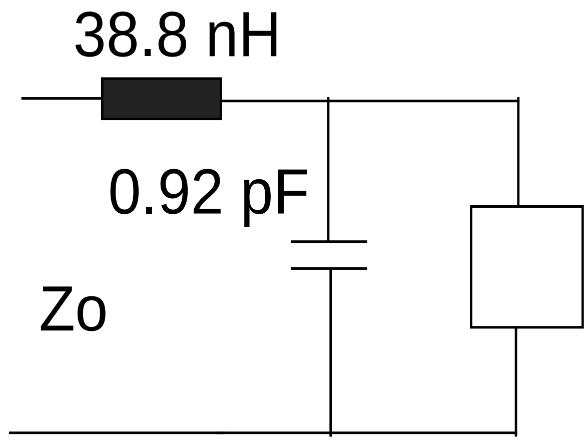
$z_l = 2 - j$

$y_l = 0.4 + 0.2j$

EPFL

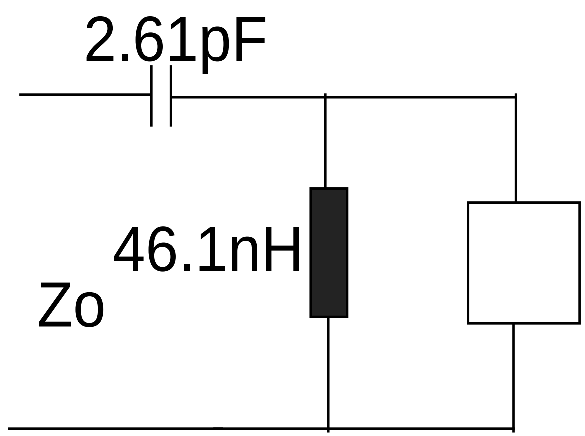

RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF. P
RFL. COEFF. E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST.)
TRANSM. COEFF.
CENTER
ORIGIN
Y
y
z_L

Example



$C = c1/\omega/Z_o = 0.3/\omega/100$

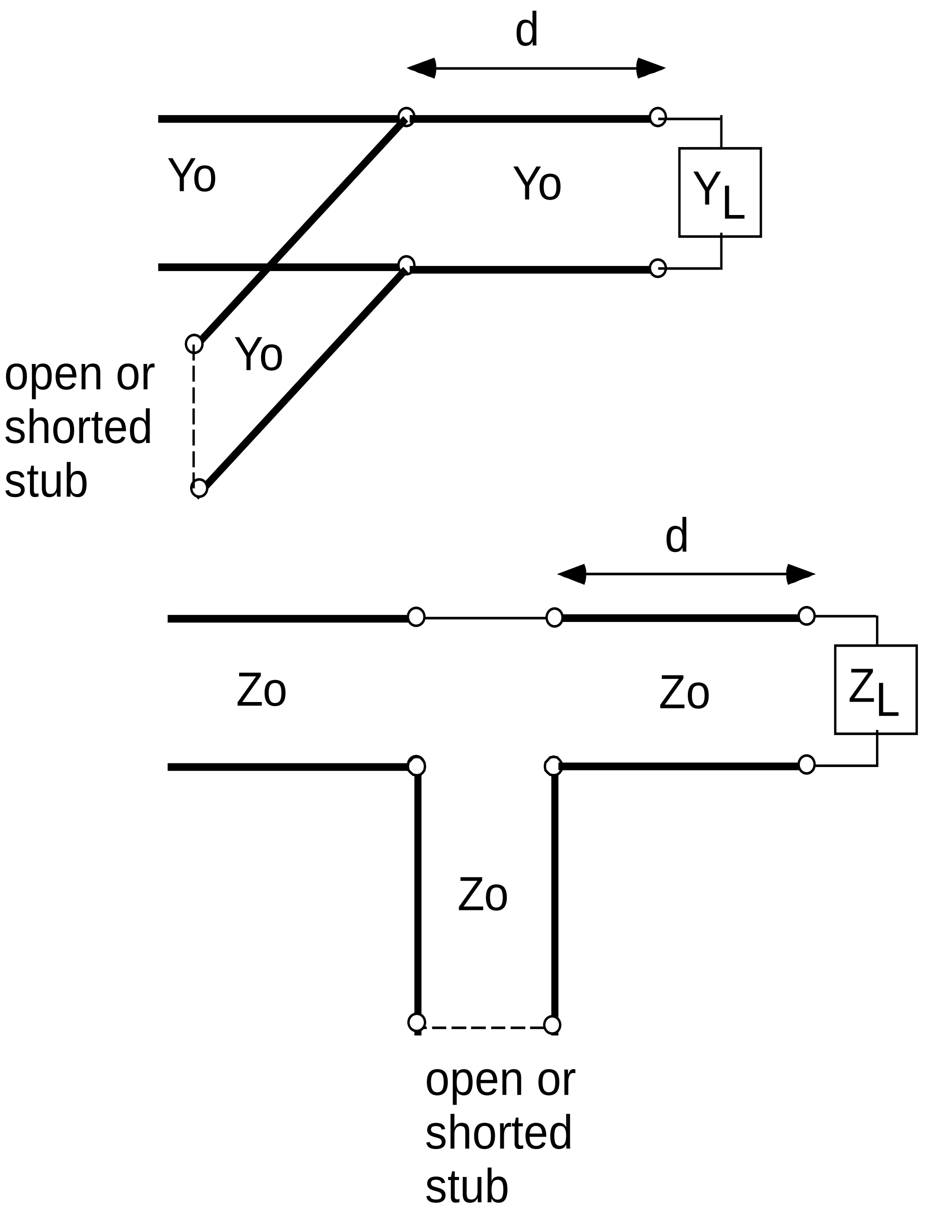
$L = l1 * Z_0/\omega = 1.2 * 100/\omega$



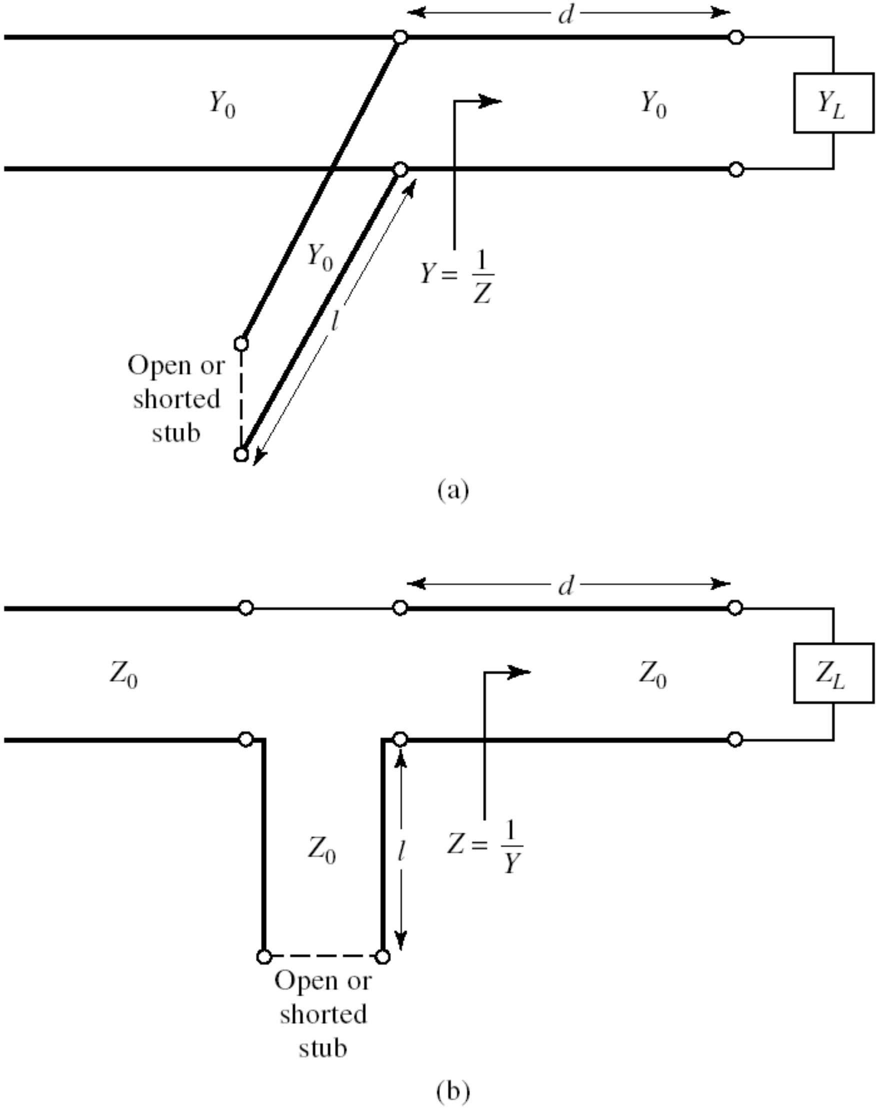
$L = Z_o/\omega/l2 = 100/0.7/\omega$

$C = 1/c2/Z_0/\omega = 1/1.2/100/\omega$

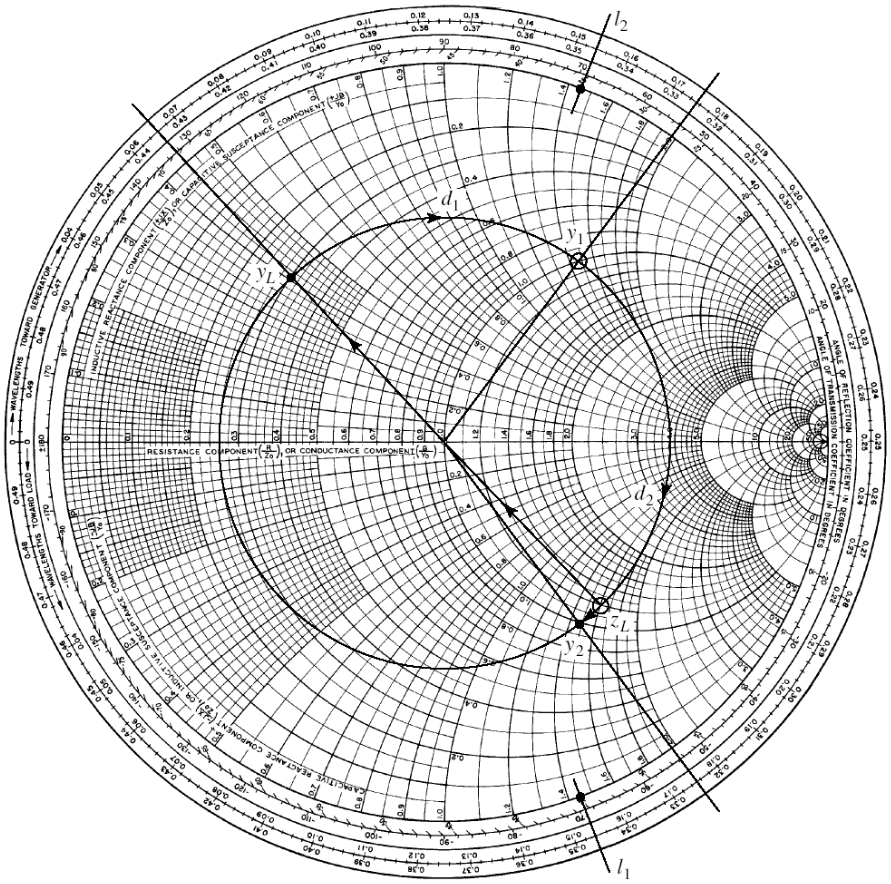
Single-stub tuning



Single-stub tuning

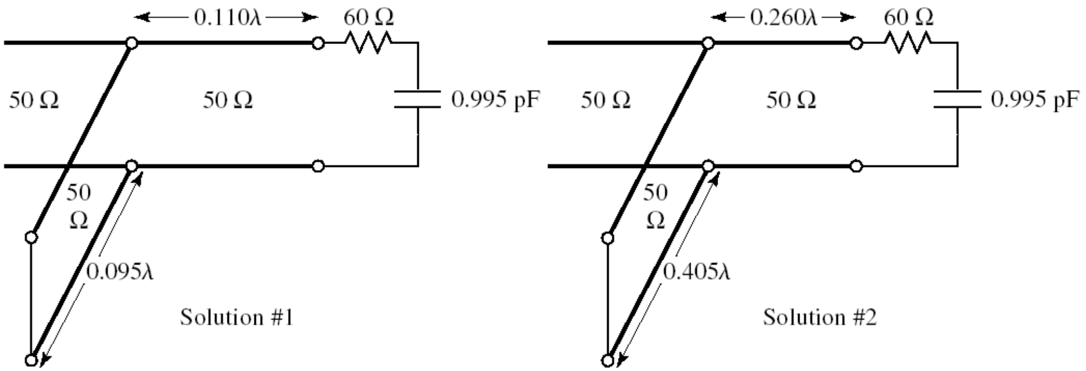


Single-stub tuning

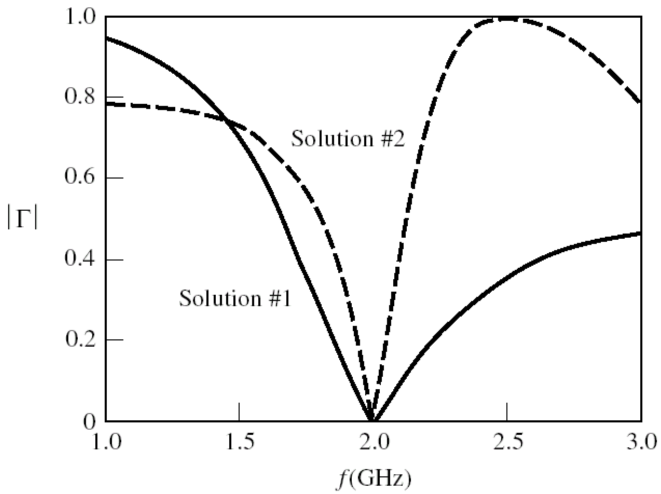


(a)

Single stub tuning



(b)



(c)

Example: Single stub tuning

Example:
match an antenna
Zin=44.8 - j 107 Ω to
Zo= 75 Ω