20/11/2020

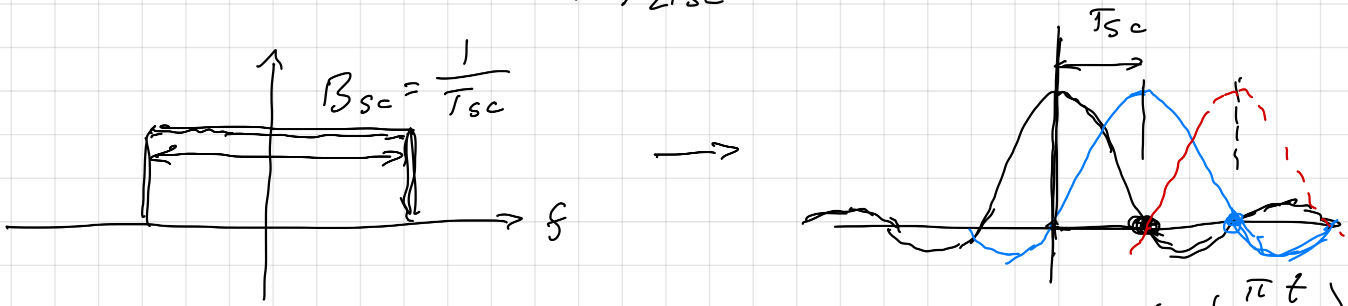
Orthogonal Multiplexing

MC Problem: Carriers should overlap

Is this necessary? NO

Overlap and keep orthogonal - How

Single carrier: we wanted a Band-limited signal to $\pm 1/2T_{sc}$



Symbols overlap in TD, but they fulfill 1st Nyquist criterion

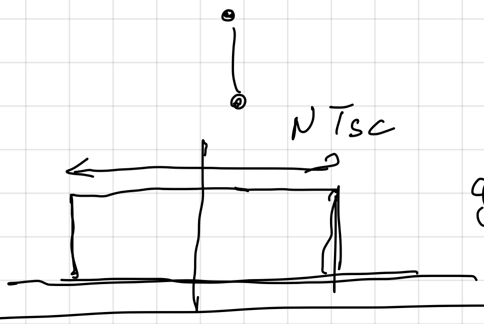
$g(nT_{sc}) = 0$ if $n \neq 0$

$$g(t) = \frac{\sin\left(\frac{\pi t}{T_{sc}}\right)}{\pi t / T_{sc}}$$

B_{sc} = wideband signal

=> Same idea, but Nyquist Pulse in FD with zeros in $n \cdot \frac{B_{sc}}{N} = n \cdot \frac{1}{N T_{sc}}$

$$G(f) = \frac{\sin\left(\frac{\pi f \cdot N}{B_{sc}}\right)}{\pi f N / B_{sc}} = \frac{\sin(\pi N T_{sc} \cdot f)}{\pi N T_{sc} f}$$



$$g(t) = \begin{cases} 1, & |t| < \frac{N T_{sc}}{2} \\ 0, & \text{else} \end{cases}$$

Rectangular TD symbol of duration $N T_{sc}$ ≙ FD symbol with spectral nulls @ $n \cdot \frac{1}{T_{sc} N}$

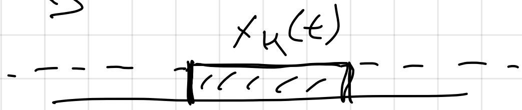
→ can place them $\Delta\omega = \frac{1}{N T_{sc}}$ without interference

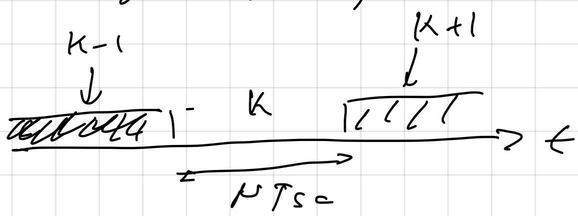
1 Symbol

$$x_K(t) = \sum_{\ell} a_{\ell,K}\, e^{j\frac{2\pi \ell}{NT_{sc}} t}$$

Kth multi-carrier symbol

ℓ: frequency bin (carrier)

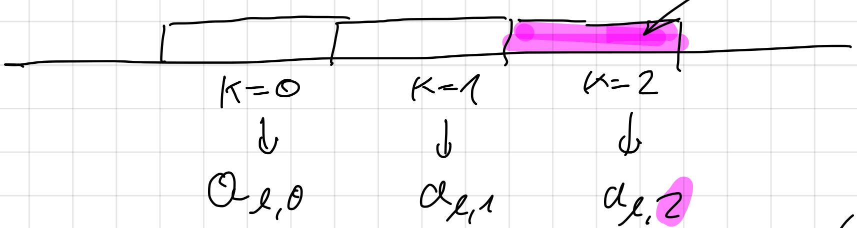




Multiple symbols

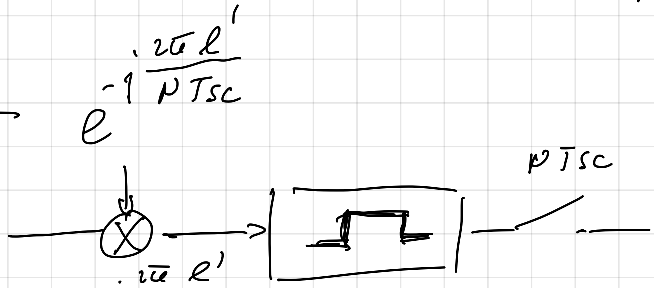
$$x(t) = \sum_{K=-\infty}^{+\infty} x_K\, g(t - KNT_{sc})$$

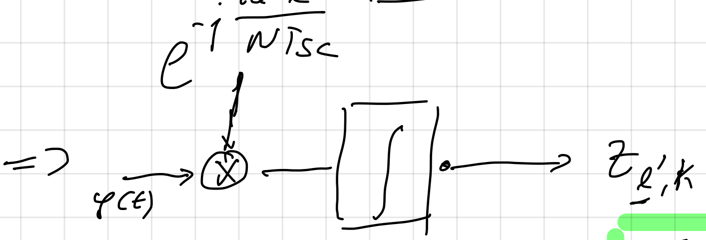
multi-carrier symbol



N symbols

Receiver:





$\times$ N-times

$$\boxed{z_{\ell',K} = \int_0^{NT_{sc}} y(t)\, e^{-j\frac{2\pi \ell'}{NT_{sc}}} dt\, \frac{1}{NT_{sc}}}$$

Fourier Transform of $y(t)$

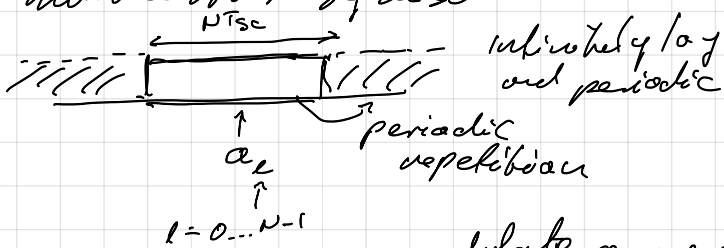
No channel: $y(t) \to x(t)$

$$z_{\ell',K} = \sum a_{\ell,K} \underbrace{\int_0^{NT_{sc}} e^{j\frac{2\pi \ell}{NT_{sc}}\cdots} e^{-j\frac{2\pi \ell'}{NT_{sc}}} dt\, \frac{1}{NT_{sc}}}_{\substack{1 \text{ for } \ell = \ell' \\ 0, \text{ for } \ell \neq \ell'}}$$

Multipath Channel

$$x[t] = \sum_{l=0}^{N-1} a_l e^{i\frac{2\pi}{N}tl}$$

Signal: periodic repetition of one multicarrier symbol



modulate a_l $l=0..N-1$ symbols to N carriers

Multipath channel

$$x[t] \xrightarrow{X(f)} \boxed{h[t]}_{H(f)} \xrightarrow{Y(f)} y[t]$$

Assume $h[t] = 0$ $t \geq \underline{L}-1$ or $t < 0$ = channel has finite length smaller than the mc symbol time

Since $x[t]$ is periodic we consider only $x[t]$ $0 \leq t \leq N-1$ and $x[t] = x[t \bmod N]$. Same holds for $y[n]$

- Conv. with channel: $y[t] = \sum_{\tau=0}^{L-1} h[\tau]\, x[t-\tau]$

$$= \sum_{\tau=0}^{L-1} h[\tau] \sum_{l=0}^{N-1} a_l e^{i\frac{2\pi}{N}l(t-\tau)}$$

$$= \sum_{l=0}^{N-1} a_l \sum_{\tau=0}^{L-1} h[\tau]\, e^{i\frac{2\pi}{N}lt} e^{-i\frac{2\pi}{N}l\tau}$$

$$= \sum_{l=0}^{N-1} a_l e^{i\frac{2\pi}{N}lt} \underbrace{\sum_{\tau=0}^{L-1} h[\tau]\, e^{-i\frac{2\pi}{N}l\tau}}_{H[l] = DFT\{h[\tau]\}}$$

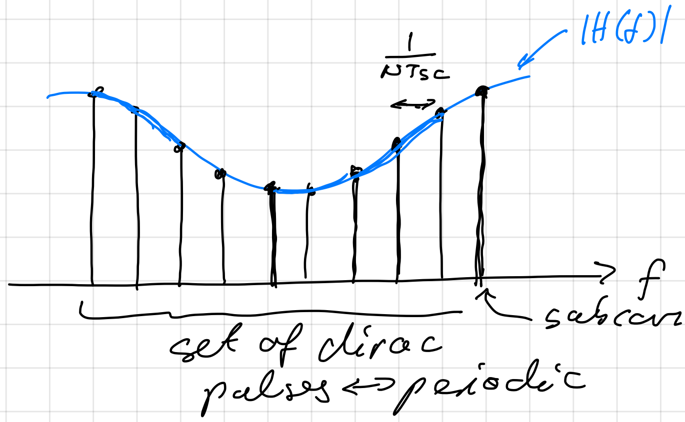
$$= \underbrace{\sum_{l=0}^{N-1} a_l H[l]\, e^{i\frac{2\pi}{N}lt}}_{\text{each carrier is simply mult. with channel at same freq.}}$$

Demodulate:

$$IDFT\left\{\sum_{l=0}^{N-1} a_l H[l]\, e^{i\frac{2\pi}{N}lt}\right\} = \underline{\underline{H[l]\, a_l}}$$

Expected: $IDFT\{a_l\} * h[t] = y(t)$

Why is no ISI at all?

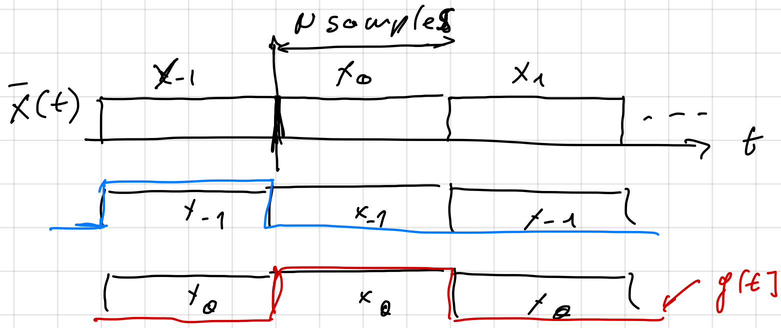


Periodic Multi-Carrier signal spectrum

Finite symbol duration:

$$\bar{x}[t] = \sum_{k=-\infty}^{+\infty} g[t-kN]\, x_k[t] \qquad x_k[t] = \underbrace{\sum_{\ell=0}^{N-1} a_{k,\ell}\, e^{j\frac{2\pi}{N}\ell t}}_{\text{one periodic MC symbol}}$$

$$g(t) = \begin{cases} 1, & 0 \le t \le N-1 \\ 0, & \text{else} \end{cases}$$



Multipath channel: $\bar{y}[t] = \bar{x}[t] * h[\tau]$

- Consider only N received samples $\bar{y}[t]$, $0 \le t \le N-1$

$$\bar{y}[t] = \sum_{\tau=0}^{L-1} h[\tau] \underbrace{\sum_{k=-\infty}^{+\infty} g(t-\tau-kN)\, x_k(t-\tau)}$$

$$\boxed{\begin{array}{l} 0 \le t \le N-1 \\ 0 \le \tau \le L-1 \\ L < N \end{array}}$$

$$\begin{cases} 0, & k>0 \\ 1, & k=0 \wedge t \ge \tau \\ 1, & k=-1 \wedge t < \tau \\ 0, & \text{else} \end{cases} \qquad g(t) = \begin{cases} 1, & 0 \le t \le N-1 \\ 0, & \text{else} \end{cases}$$

$\boxed{\text{ISI is limited to first } L \text{ samples of a MC symbol}}$

$$= \sum_{k=-1}^{0} \sum_{\tau=0}^{L-1} h[\tau]\, g(t-\tau-Nk)\, x_k[t-\tau] = \sum_{\tau=0}^{L-1} h[\tau]\, g(t-\tau)\, x_0[t-\tau]$$

$$+ \underbrace{\sum_{\tau=0}^{L-1} h[\tau]\, g(t-\tau+N)\, x_{-1}[t-\tau]}_{ISI}$$

Two regions: $t \ge L \Rightarrow t-\tau+N > N \Rightarrow g(t-\tau+N) = 0$

↳ No ISI for samples other than first L

$t < L \Rightarrow$ ISI