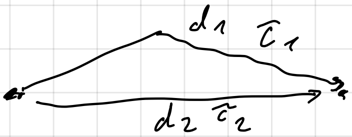
Wireless Receivers

Fading:

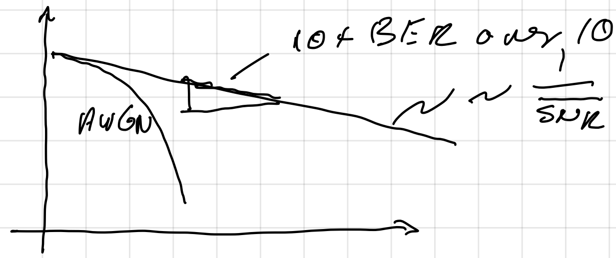


$$\frac{1}{N}\sum_{n=1}^{N} \underbrace{s(t-\tau_n)}_{s(t)}\, e^{j\phi_n} = s(t) \underbrace{\sum_{n=1}^{N} e^{j\phi_n}}_{\underline{h}}$$

$h : \mathcal{CN}(0,1)$ $\qquad h_r : \mathcal{N}(0, 1/2)$

$\qquad\qquad\qquad h_i : \mathcal{N}(0, 1/2)$

BER:



Optimum Receiver

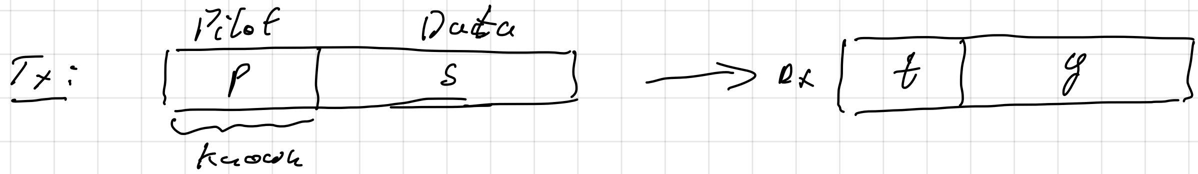
$\underline{y = hs + w}$ → Recover s

→ Assume we know $\underline{h}$

$$\hat{s} = \arg\min_{s} \left(|y - hs|^2 \right)$$

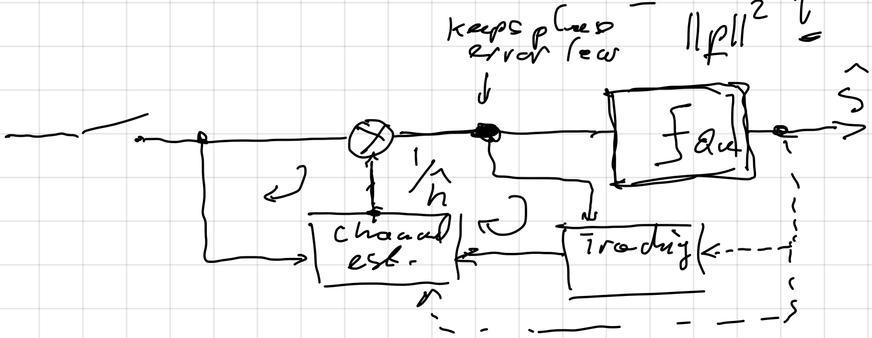
$$\hat{s} = Qu\left(\frac{1}{h} y\right) = Qu\left(\frac{h^*}{|h|^2} y\right) \qquad \frac{1}{h} = \frac{h^*}{|h|^2}$$

Estimate the channel:



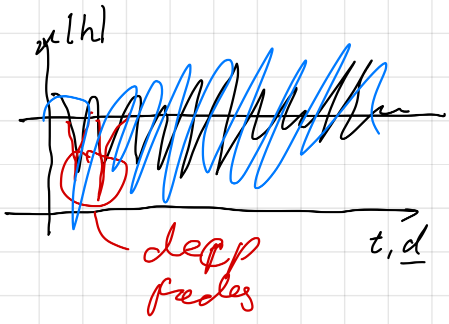
p: pilot symbols → training samples t

$$\hat{h} = \arg\min_{h} |t - hp|^2 \rightarrow \hat{h} = \arg\min_{h} \|\underline{t} - \underline{p} h\|^2 = \frac{\underline{p}^H}{\|\underline{p}\|^2} \underline{t}$$



Diversity Receiver

Basic Idea: Avoid deep fades



→ try to get multiple channels

$Pr(h_1 < \gamma) \to$ deep fade

$Pr(h_2 < \gamma) \to$ deep fade

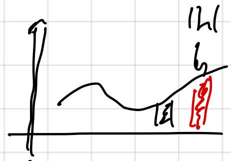
h_1, h_2 independent

$$Pr(|h_1| < \gamma \text{ and } |h_2| < \gamma) = \underbrace{Pr(|h_1| < \gamma)}_{<1} \underbrace{Pr(|h_2| < \gamma)}_{<1}$$

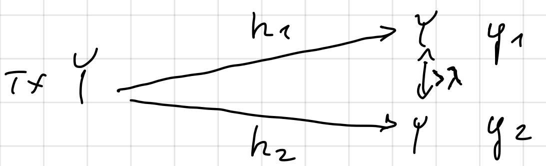
Multiple channel realizations reduces the risk of a deep fade everywhere

⇒ **Diversity**

Forms of diversity:
- Time diversity
- Frequency diversity
- Space → multi-antenna receiver



Example: Space (Rx-Diversity)

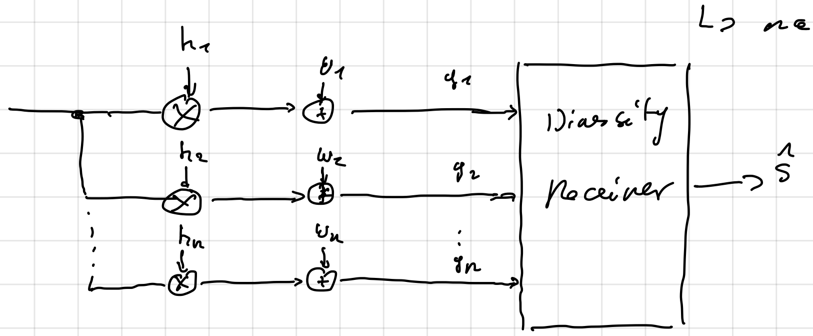


h_1, h_2: independent

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} x + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

In vector notation $\underline{y} = \underline{h} x + \underline{w}$

How to combine y_1 and y_2? → Selection diversity
↳ looses the worse branch
↳ not very optimal



ML Receiver

$y_\ell = h_\ell s + w_\ell$ $\rightarrow$ Collect in vector notation

L: Branches $\quad \underline{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_L \end{bmatrix} \quad \underline{h} = \begin{bmatrix} h_1 \\ \vdots \\ h_L \end{bmatrix} \quad \underline{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_L \end{bmatrix} \in \mathcal{CN}(0, \underline{I}\sigma_n^2)$

i.i.d compl. Gaussian

<u>ML criterion</u>

$$\hat{s} = \arg\max_s \exp\Big(\underbrace{-[\underline{y} - \underline{h}s]^H[\underline{y} - \underline{h}s]}_{\|y-hs\|^2}\Big)$$

$$\mathcal{L}(s) = -\|y\|^2 + 2\,\mathrm{Re}\{s^* h^H y\} - \|hs\|^2$$

- Remove $-\|y\|^2$

$$\rightarrow \arg\min \|h\|^2\|s\|^2 - \underbrace{2\,\mathrm{Re}\{s^* h^H y\}}_{\rightarrow \max}$$

min if $s^* = \overline{y^H h}\alpha$

find α

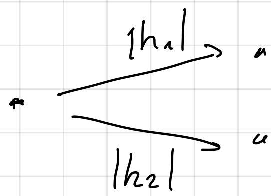
$$\arg\min \underbrace{\|h\|^2\|h\|^2\|y\|^2\alpha^2 - 2\|y\|^2\|h\|^2 \cdot \alpha}_{\frac{d}{d\alpha} \rightarrow 0} \Rightarrow \alpha = \frac{1}{\|h\|^2}$$

$$\hat{s} = \frac{\underline{h}^H}{\|\underline{h}\|^2}\underline{y} \longrightarrow \text{Matched filter}$$

BER:

$$\tilde{s} = \frac{\underline{h}^H}{\|h\|^2} \underbrace{\underline{y}}_{\underline{h}s+\underline{w}} = \underline{s} + \underline{\frac{\underline{h}^H}{\|h\|^2} \underline{w}}$$

$$\underline{y} = \underline{h} s + \underline{w}$$



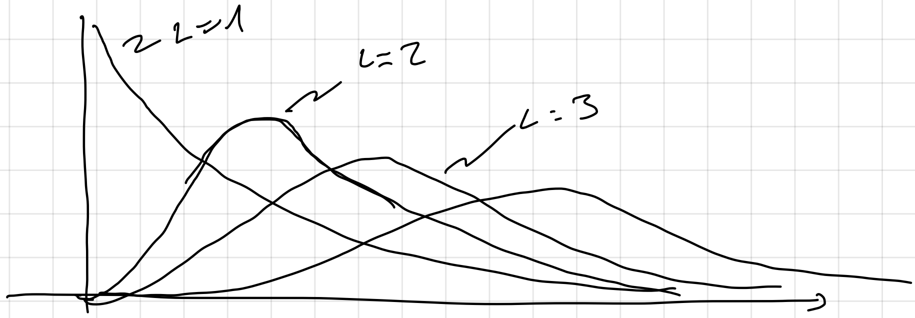
$$\mathcal{E}_h\left\{ Q\left\{ \sqrt{\underline{\|h\|}^2 \, SNR} \right\} \right\}$$

Note $\|\underline{h}\|^2 = \underline{\sum_{\ell=1}^{L} |h_\ell|^2}$

$\uparrow$ Sum of squared Gaussians

Distribution of $\|\underline{h}\|^2$: chi-squared χ_{2L} with $2L$ degrees of freedom

$$f(\|h\|^2) = \frac{1}{(L-1)!} e^{-\|h\|^2} (\|h\|^2)^{L-1}$$

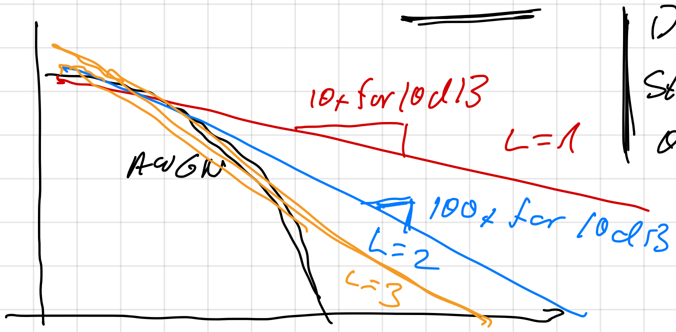


Outage probability:

$$P_r(\|\underline{h}\|^2 SNR < \gamma) = \int_0^{\gamma/SNR} f(\|h\|^2) \, d\|h\|^2$$

$$\approx \frac{1}{L!} \cdot \frac{1}{SNR^L}$$

Outage $\approx$ BER decays with $\underline{\left(\frac{1}{SNR}\right)^L}$



Diversity (order): Steepness of BER/Outage for SNR $\to \infty$