ICT-Labs:

Quadrature Amplitude Modulation

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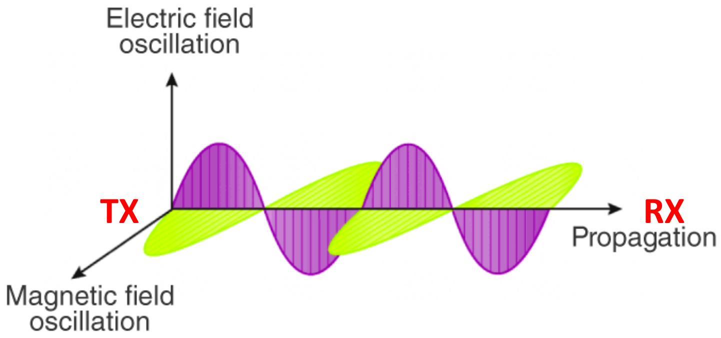
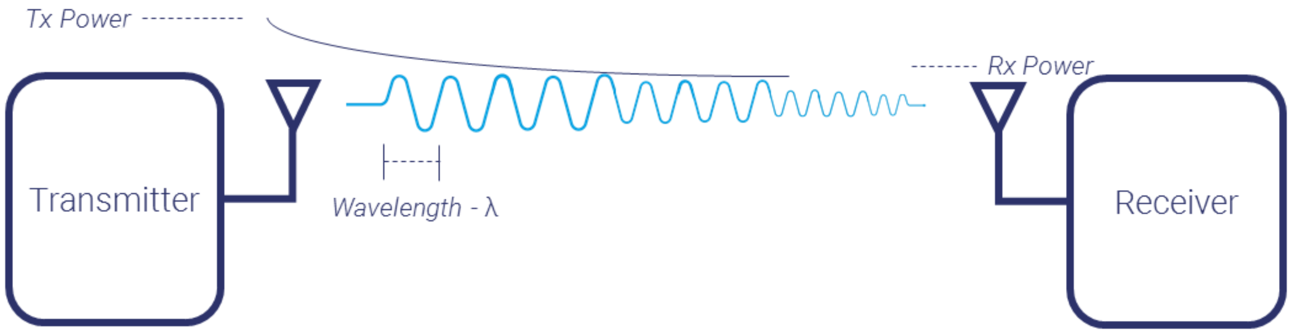
Radio Waves

Wireless communication is enabled by electromagnetic fields

- Rapidly alternate the field in one place (TX)
- **Waves propagate** through the medium and **can be sensed** in another place (RX)

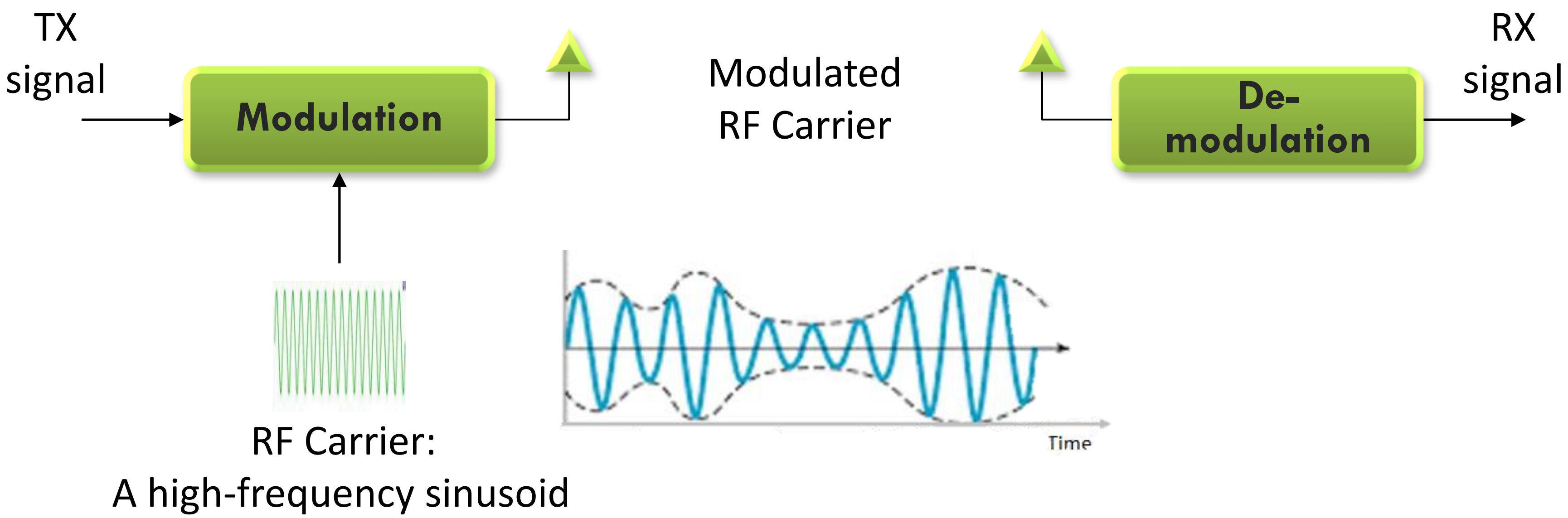


Radio frequency (RF) communications rely on a high frequency that propagates in the EM field.



Modulation and Demodulation

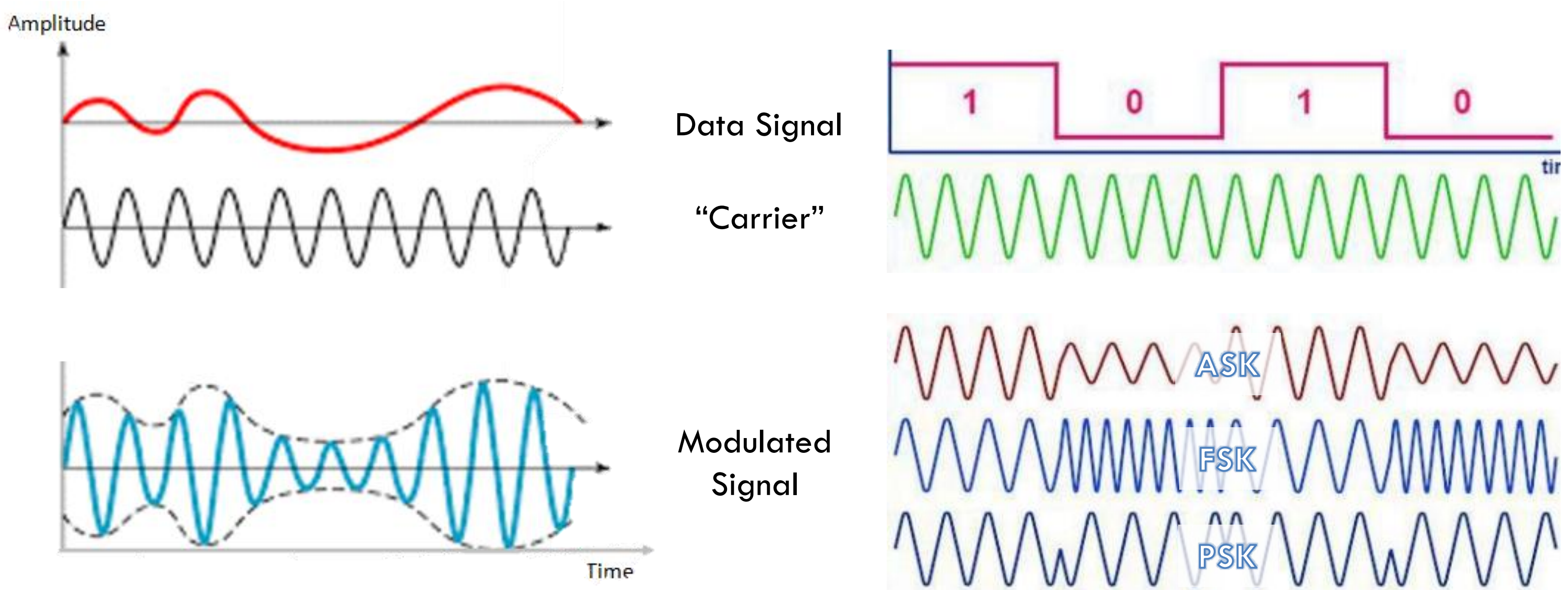
- Modulation: Alter properties of a radio frequency signal (carrier) according to the signal or information we would like to send
- Demodulation: Detect changes in the carrier and translate them back into the corresponding signal or information



Analog vs. Digital Communication

We distinguish between Analog and Digital modulation:

- Analog: a **continuous analog signal** that directly modulates the carrier
- Digital: a **sampled digital (discrete) signal** comprising 0s and 1s that modulate the carrier



Analog Modulation Digital Modulation

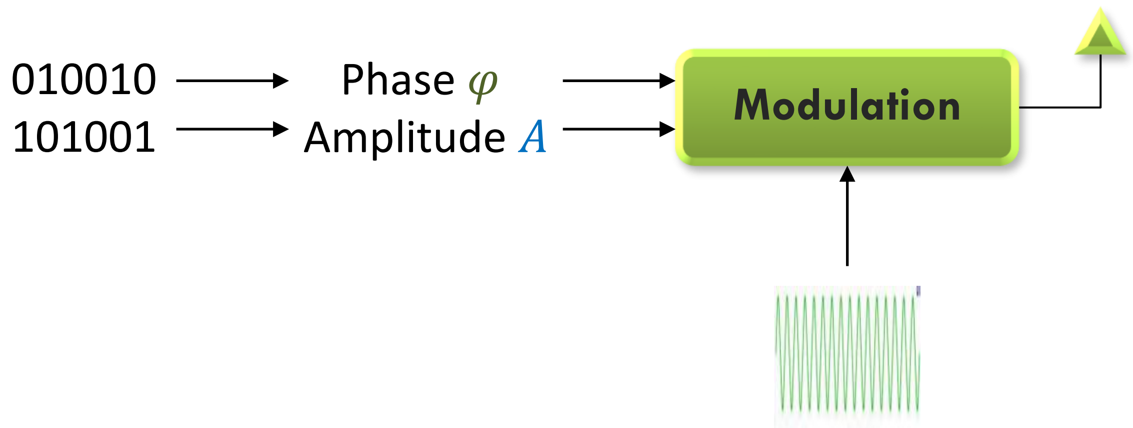
The Modulated Carrier

Linear modulation modulates the phase and the amplitude of the carrier

$$u(t) = A \cdot cos(2\pi \cdot f_0 \cdot t + \varphi)$$

φ, A: Phase and amplitude selected based on data

- Since we can modulate both phase and amplitude as two properties to modulate independently, we have two different “channels” in one carrier



Quadrature Amplitude Modulation (QAM)

- Phase and amplitude are two very different properties
- Modulating them with two different signals or bits is very inconvenient and asymmetric (different properties)

With some mathematical transformation, we can write

- the phase and amplitude modulated carrier

$$u(t) = A \cdot cos(2\pi \cdot f_0 \cdot t + \varphi)$$

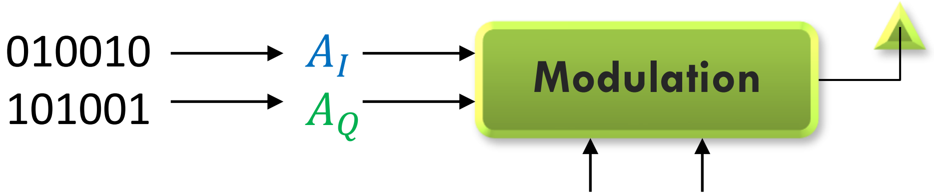
- as the sum of two 90 degree-out-of-phase independently amplitude modulated carriers:

$$u(t) = A_I \cdot cos(2\pi \cdot f_0 \cdot t) + A_Q \cdot sin(2\pi \cdot f_0 \cdot t)$$

A_I, A_Q : “In-phase” and “quadrature” components

QAM Modulator Physical Realization

The QAM modulator can be realized in a very straightforward manner



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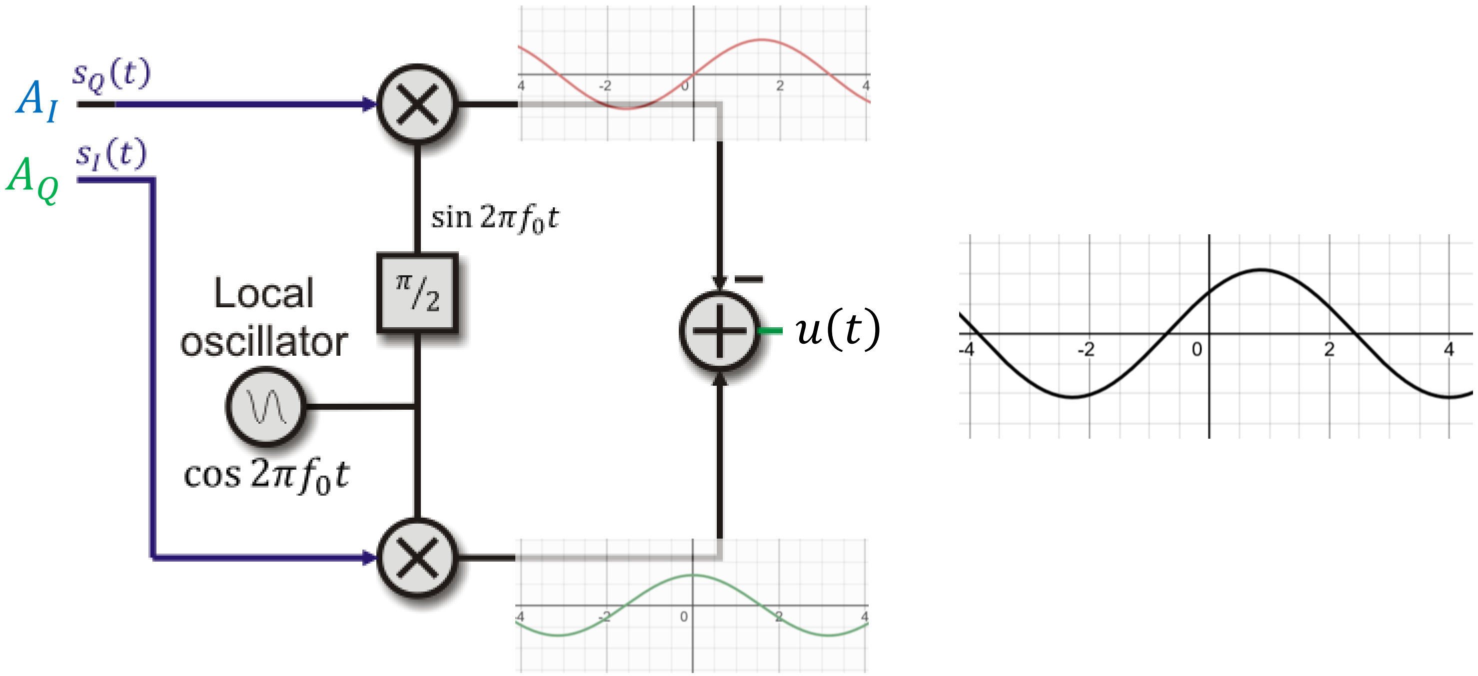
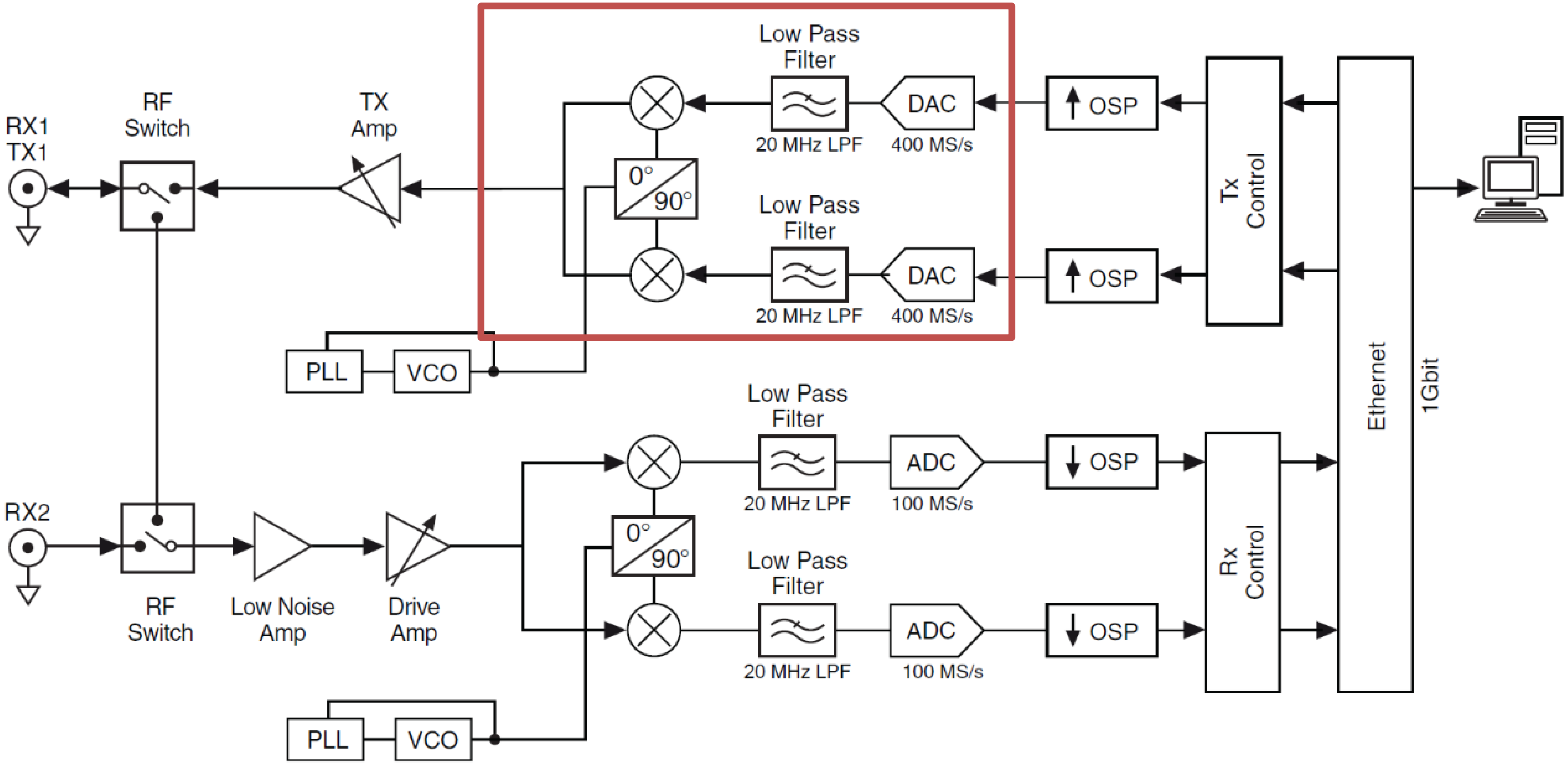


Physical Realization of Transmitter and Receiver



Complex Representation of QAM

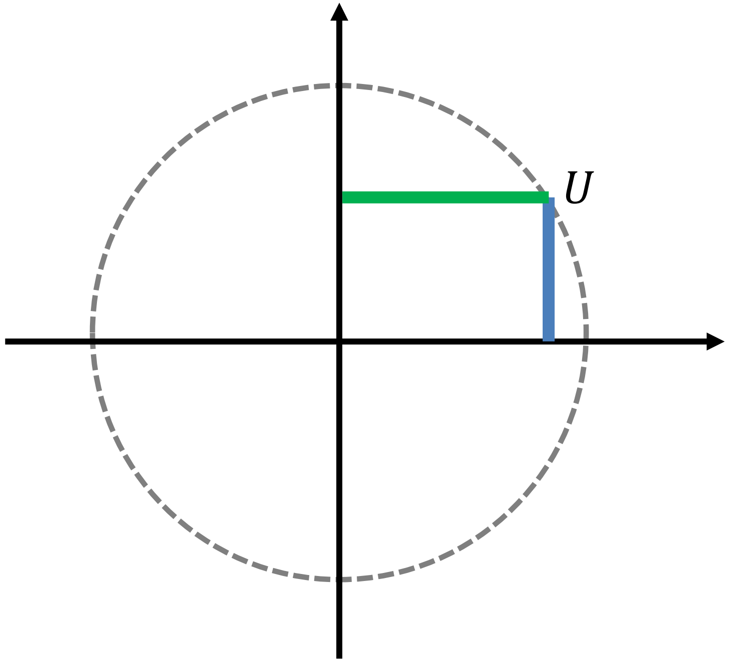
Quadrature and in-phase components of the QAM signal

$$u(t) = A_I \cdot cos(2\pi \cdot f_0 \cdot t) + A_Q \cdot sin(2\pi \cdot f_0 \cdot t)$$

A_I, A_Q : "In-phase" and "quadrature" components

can also be represented by a complex number

$$U = A_I + j \cdot A_Q$$

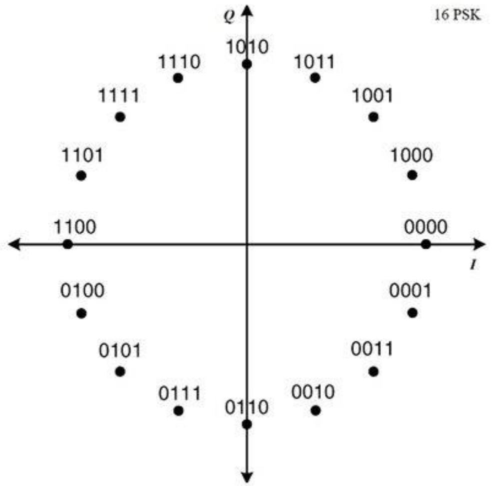


Constellations & QAM Modulation of Digital Data

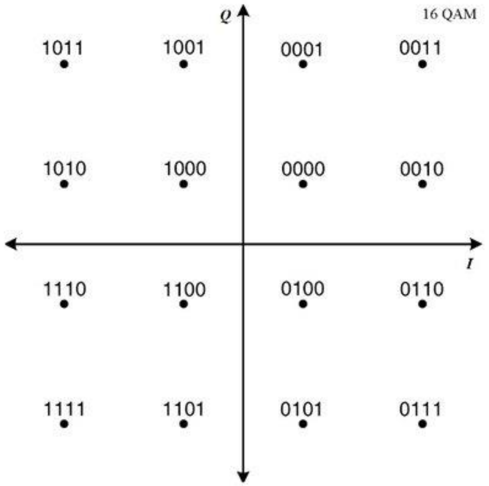
Every point in the plane represents a specific combination of in-phase and quadrature data

Constellations are sets of points on the complex plane

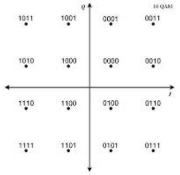
- Each point represents a unique combination of A_I, A_Q
- Each point carries a label that represents a specific combination of bits

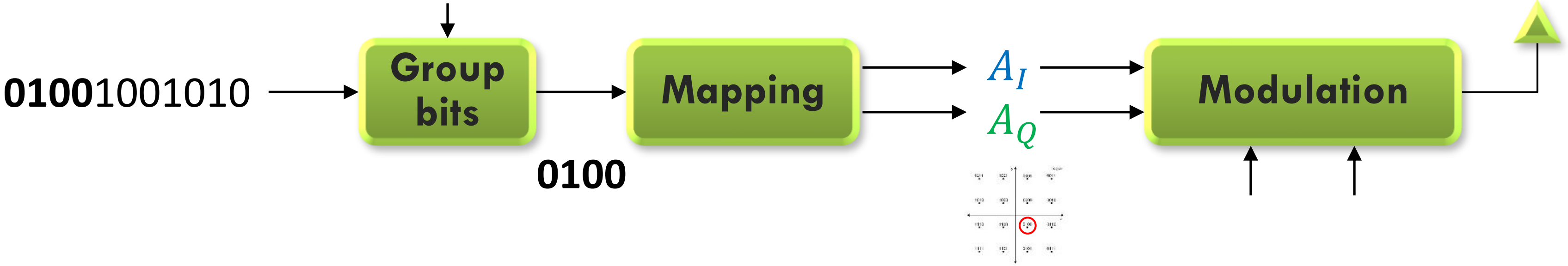


PSK: phase shift keying



QAM: quadrature amplitude modulation

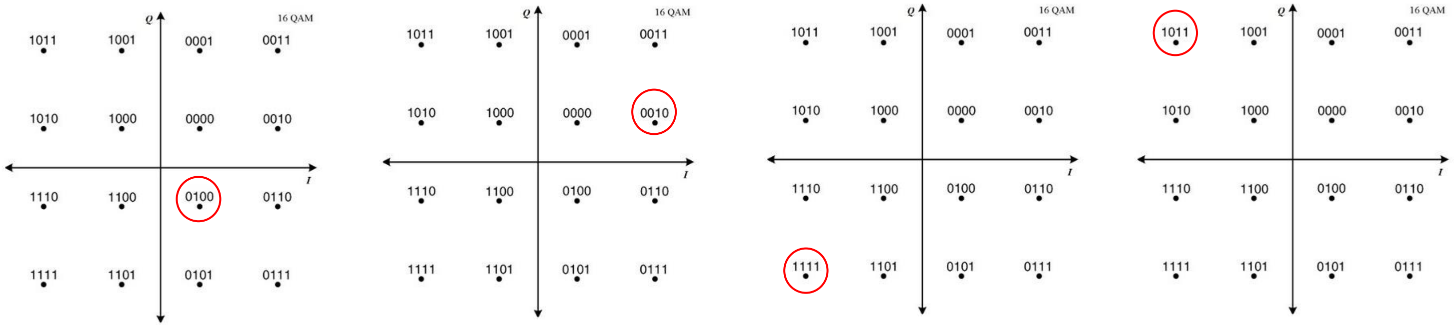




Baseband Signal Generation: Discrete Time to Continuous Time

A digital signal is composed of a discrete succession of values (constellation points)

0100 0010 1111 1011



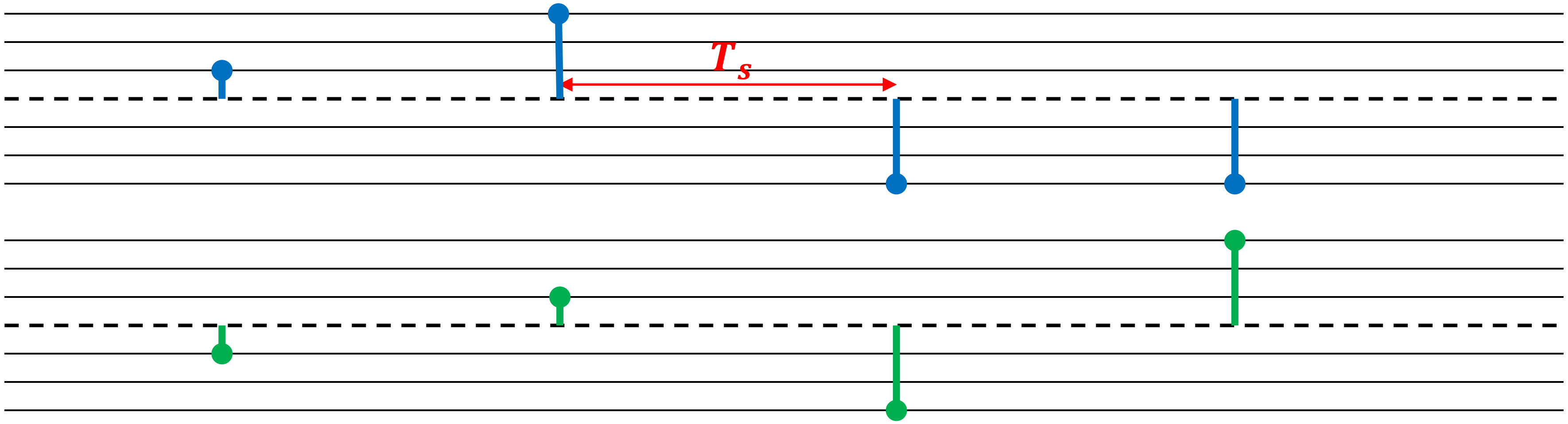
$A_I[1] = +1$
$A_Q[0] = -1$

$A_I[1] = +3$
$A_Q[0] = +1$

$A_I[1] = -3$
$A_Q[0] = -3$

$A_I[1] = -3$
$A_Q[0] = +3$

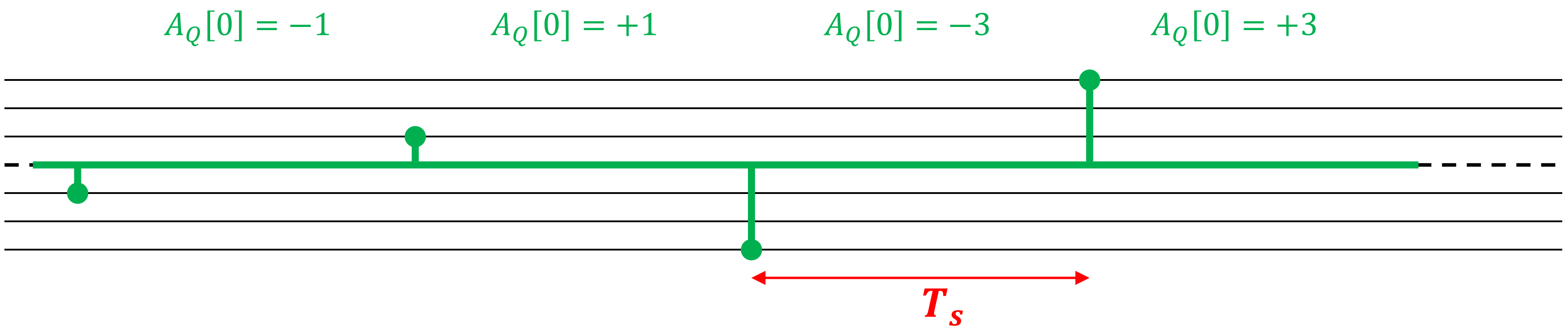
To modulate the continuous carrier, the discrete-time sequence needs to be converted to a continuous time signal with **sampling time T_s (baud rate = 1/T_s)**



Writing a Discrete Time Signal as Continuous Time Signal

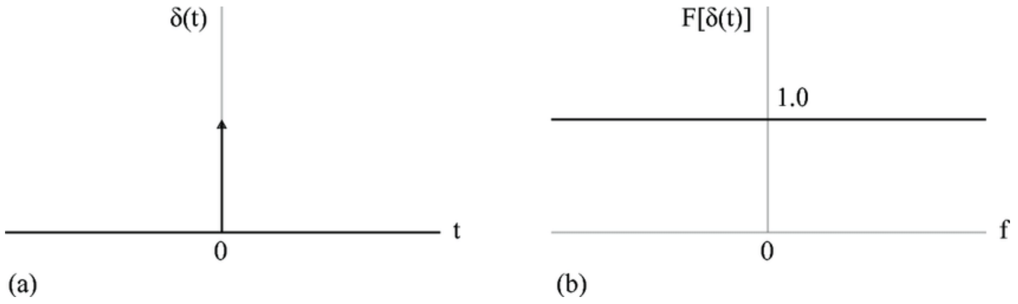
- Look at the in-phase and quadrature signals independently

A discrete time signal is an abstract concept with no physical counterpart,
BUT we can think of a discrete time signal as a series of Dirac pulses



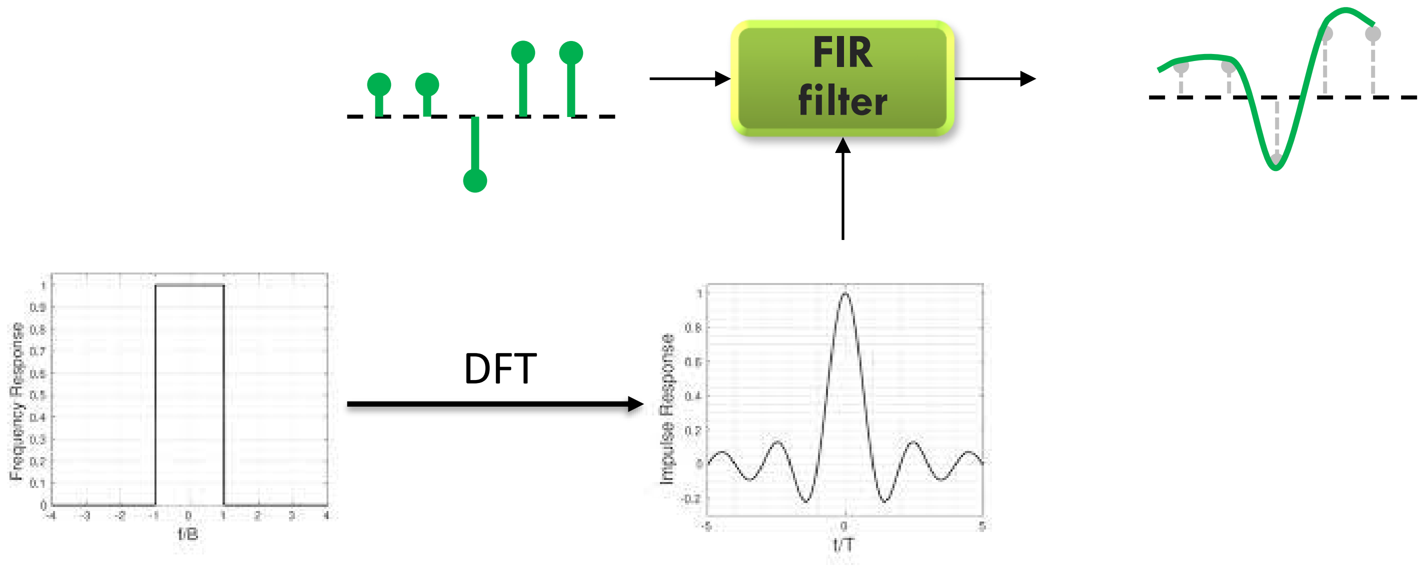
Even if we could generate Dirac pulses, there would be two problems:

- Pulses are infinitely short
- Signal occupies an infinite bandwidth



Limiting the Bandwidth of a Train of Dirac Pulses

To limit the bandwidth of the train of Dirac pulses
apply a filter with the desired bandwidth

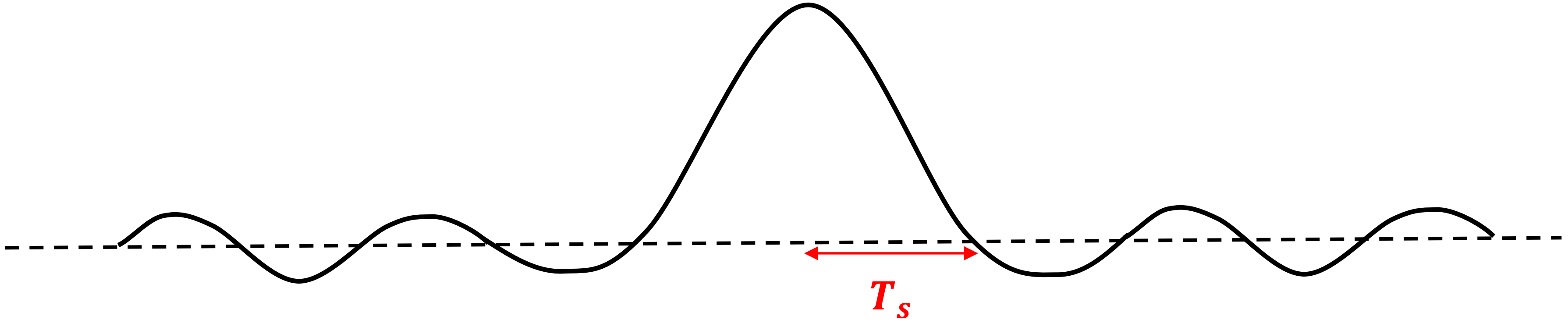


The filter that defines the spectrum is called the pulse shape filter

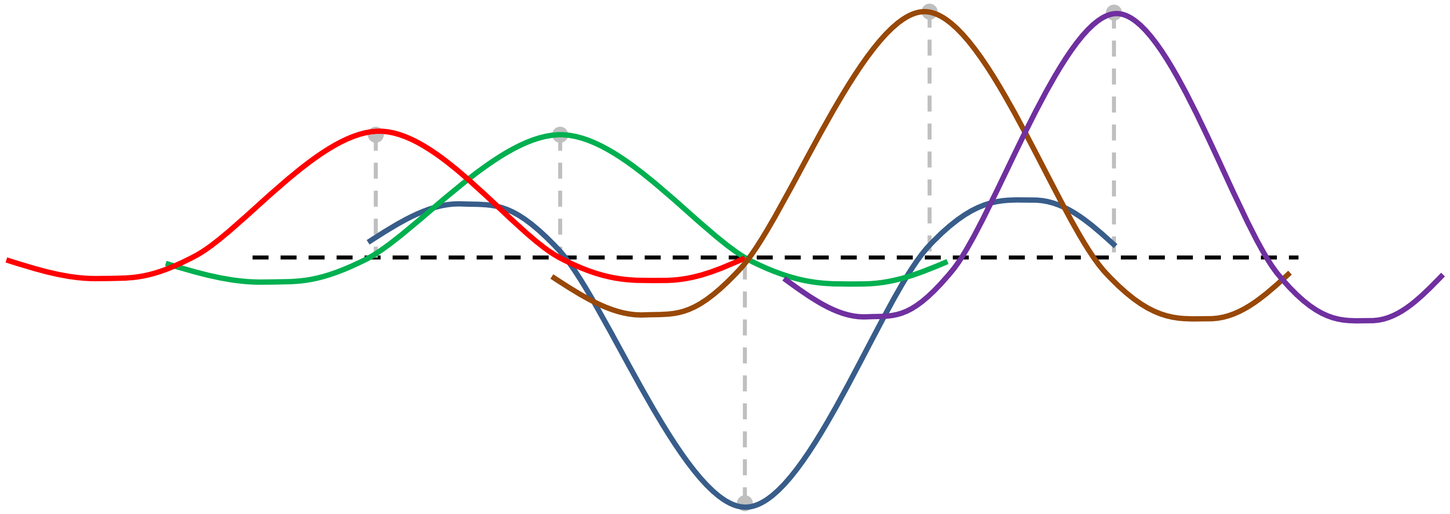
- Fourier transform of the filter impulse response defines the spectrum of the signal after the filter.

Signals with (infinitely) Long Pulse Shapes

The pulse shape duration is typically longer than a symbol period



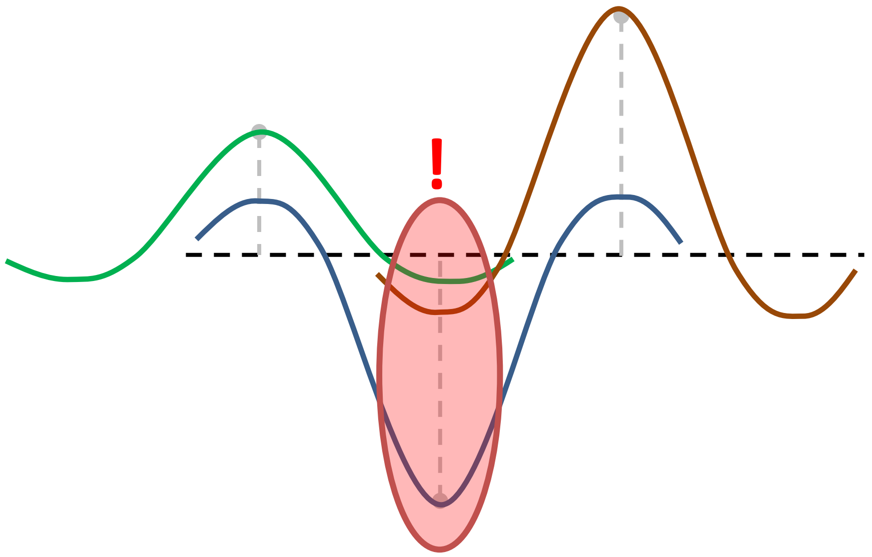
Subsequent pulses overlap: we observe only the sum of the subsequent modulated pulse shape

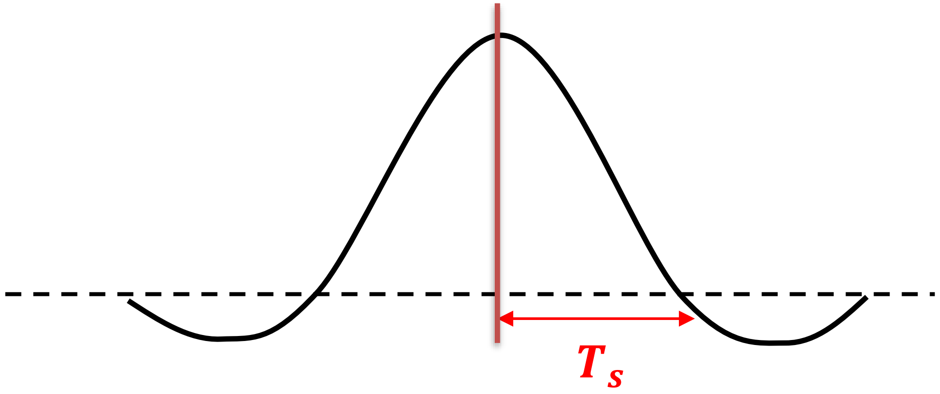


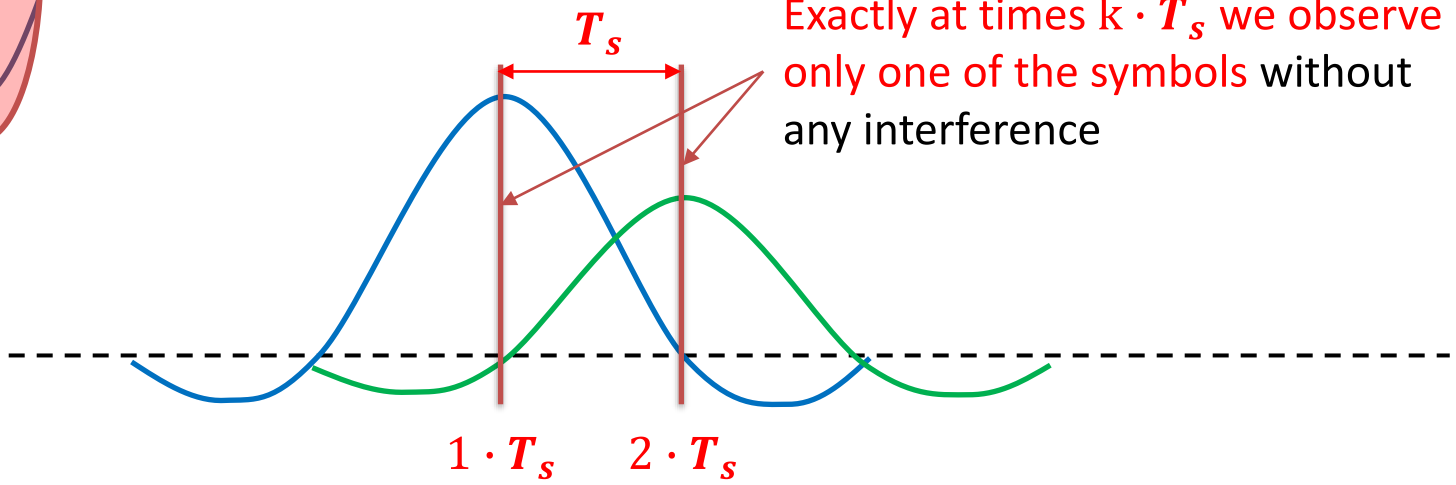
Avoiding Interference between Symbols

To be able to distinguish the individual symbols we use a **pulse shape that has a zero every period T_s**

- The pulse shape fulfills the Nyquist ISI criterion







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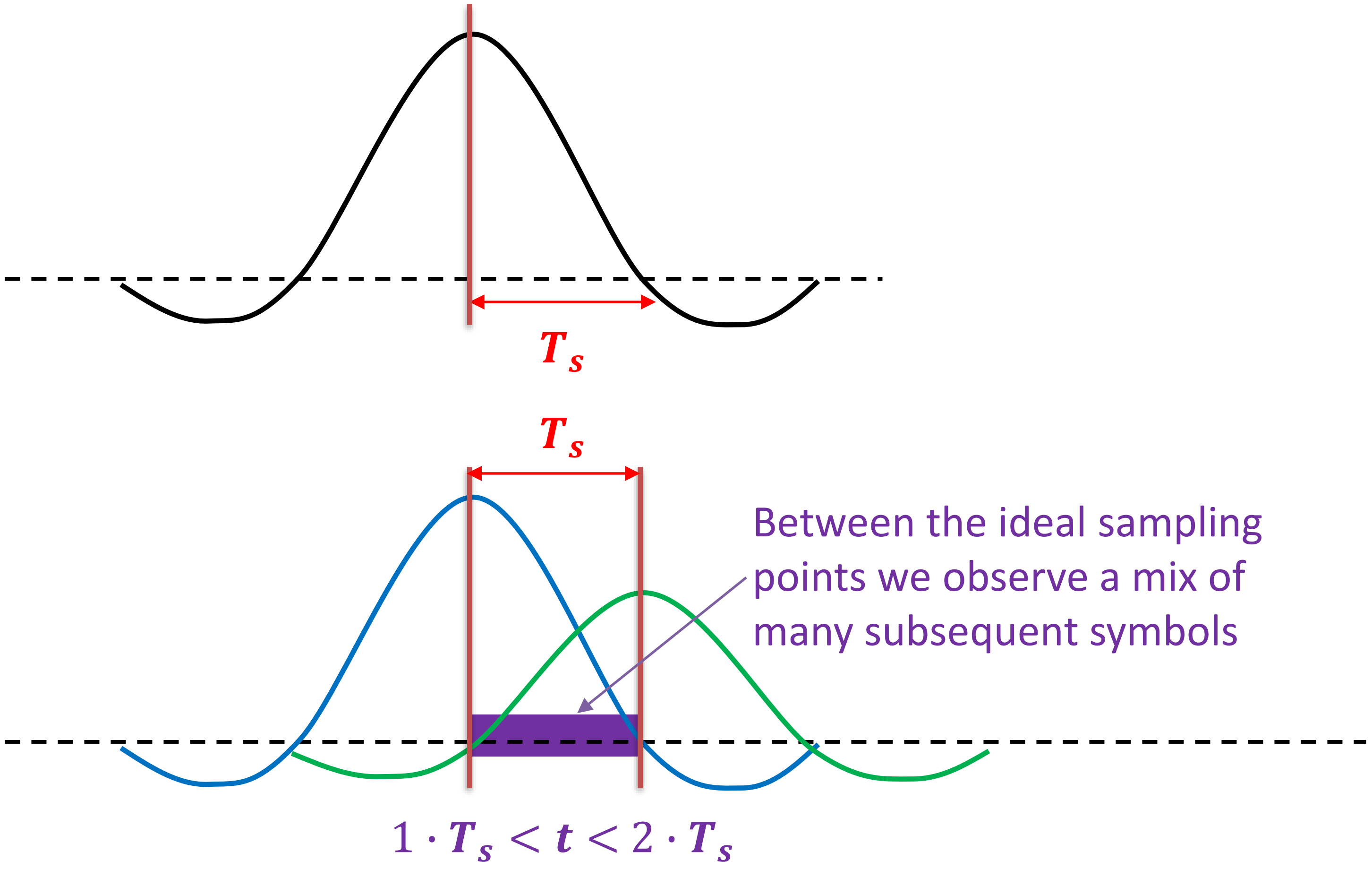
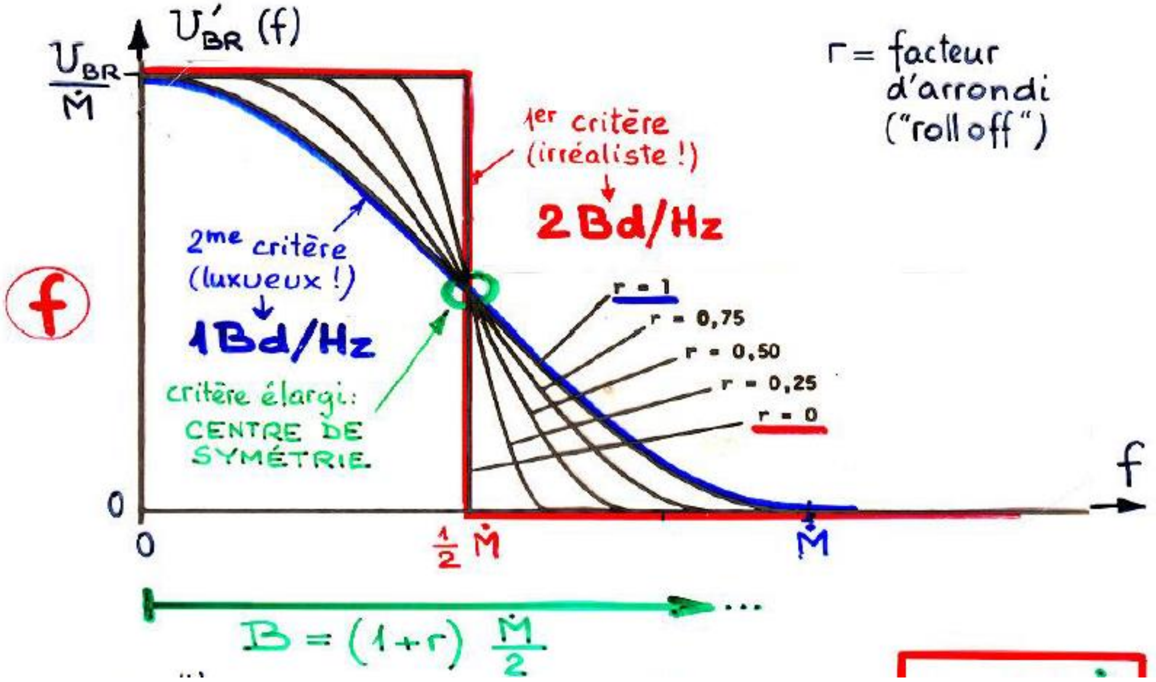
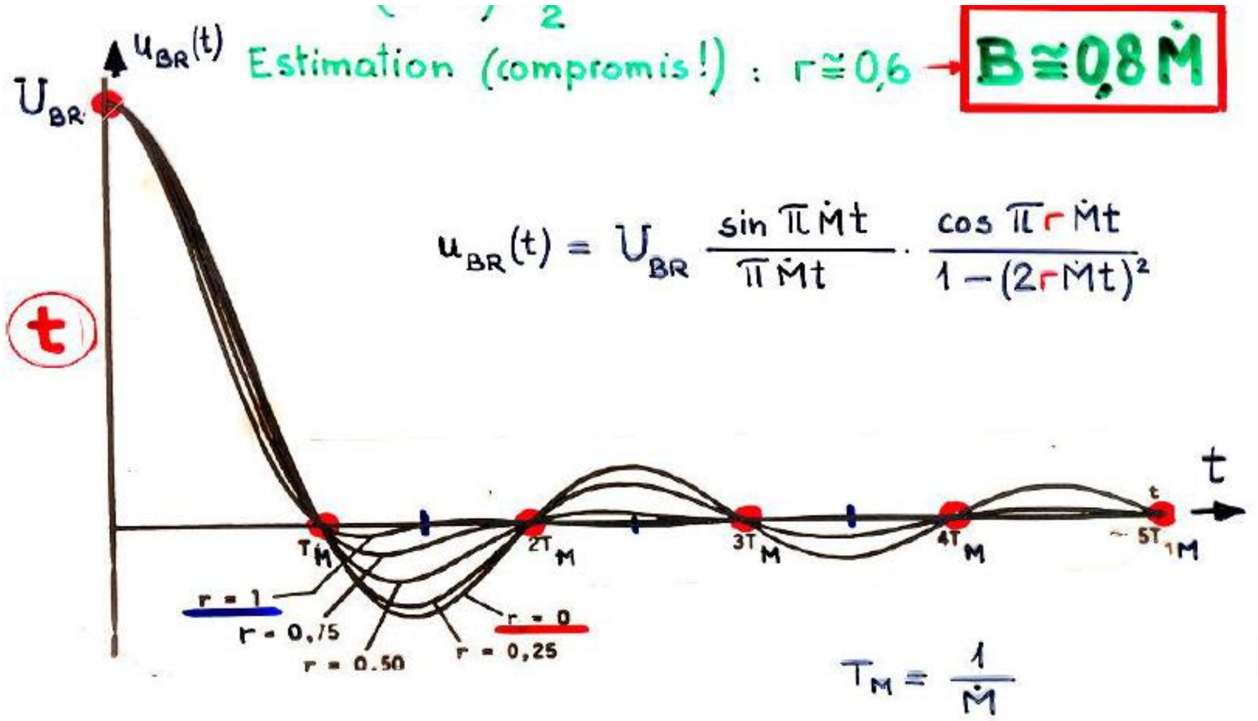


Pulse Shape Choice

One family of filters respecting the Nyquist criterion is called Raised cosine

- These filter have a parameter called rolloff factor, which offers a tradeoff between the bandwidth used and the importance of the sampling time precision.





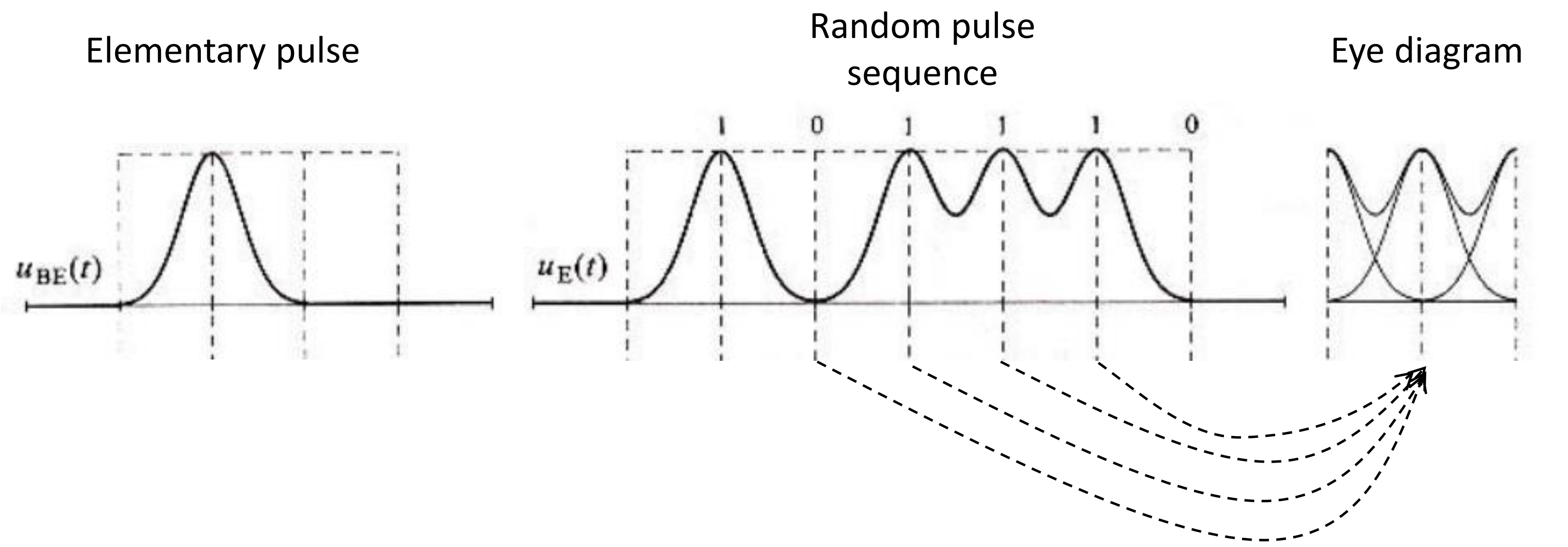
The Eye Diagram

Observing a signal exactly at $\mathrm{k} \cdot \boldsymbol{T_s}$ is often very hard.

We are interested in how the signal looks around $\mathrm{k} \cdot \boldsymbol{T_s}$

- How big is the time window in which we see almost no interference?
- How big is the interference with a small time offset?

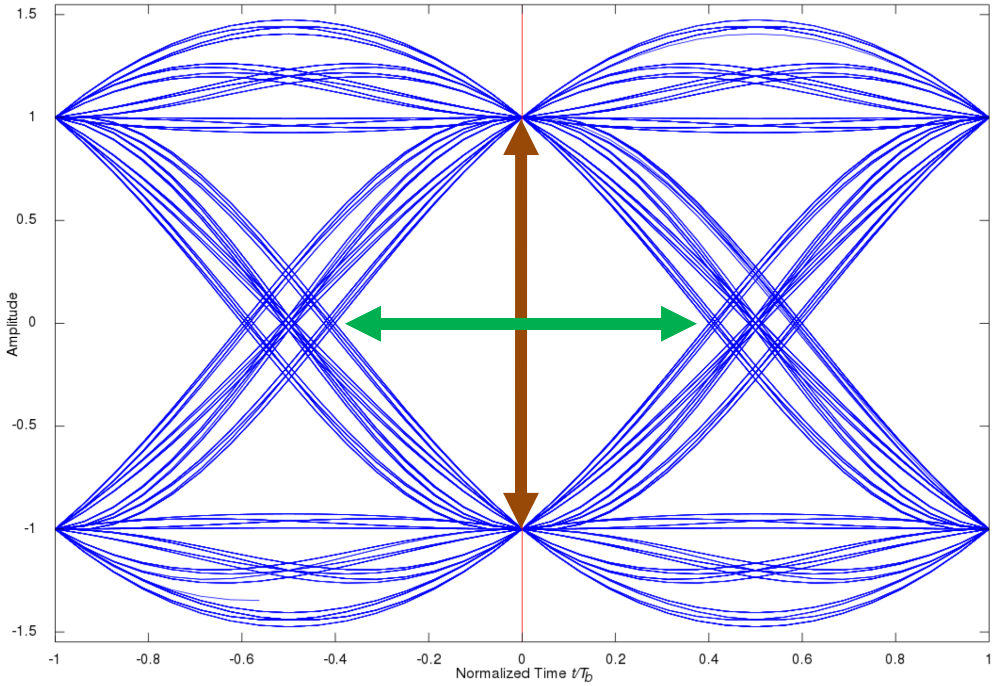
The eye diagram looks at a long period of randomly chosen symbols and plots them on top of each other



Eye Diagram Quality Metrics

The opening of the eye indicates the robustness of the signal

- **Vertical opening** of the eye: indicates the tolerance to offsets in the value (noise) at a given time offset
- **Horizontal opening** of the eye: indicates the tolerance to a time offset at a given value offset/threshold



Complex-Valued “Eye Diagrams”

With QAM signals, we can also plot a 2D eye diagram by plotting in-phase and quadrature components of the signal on the real and imaginary axes of the complex plane