14

Inductor Design

This chapter treats the design of magnetic elements such as filter inductors, using the K_g method. With this method, the maximum flux density B_{max} is specified in advance, and the element is designed to attain a given copper loss.

The design of a basic filter inductor is discussed in Sections 14.1 and 14.1.5. In the filter inductor application, it is necessary to obtain the required inductance, avoid saturation, and obtain an acceptable low dc winding resistance and copper loss. The geometrical constant K_g is a measure of the effective magnetic size of a core, when dc copper loss and winding resistance are the dominant constraints [1,2]. Design of a filter inductor involves selection of a core having a K_g sufficiently large for the application, then computing the required air gap, turns, and wire size. A simple step-by-step filter inductor design procedure is given. Values of K_g for common ferrite core shapes are tabulated in Appendix D.

Extension of the K_g method to multiple-winding elements is covered in Section 14.3. In applications requiring multiple windings, it is necessary to optimize the wire sizes of the windings so that the overall copper loss is minimized. It is also necessary to write an equation that relates the peak flux density to the applied waveforms or to the desired winding inductance. Again, a simple step-by-step transformer design approach is given.

The goal of the K_g approach of this chapter is the design of a magnetic device having a given copper loss. Core loss is not specifically addressed in the K_g approach, and B_{max} is a given fixed value. In the next chapter, the flux density is treated as a design variable to be optimized. This allows the overall loss (i.e., core loss plus copper loss) to be minimized.

14.1 FILTER INDUCTOR DESIGN CONSTRAINTS

A filter inductor employed in a CCM buck converter is illustrated in Fig. 14.1(a). In this application, the value of inductance L is usually chosen such that the inductor current ripple peak magnitude Δi is a small fraction of the full-load inductor current dc component I, as illustrated in Fig. 14.1(b). As illustrated in Fig. 14.2, an air gap is employed that is sufficiently large to prevent saturation of the core by the peak

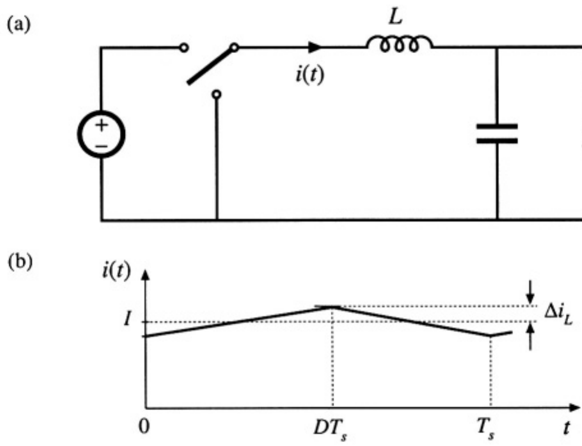

Fig. 14.1 Filter inductor employed in a CCM buck converter: (a) circuit schematic, (b) inductor current waveform.

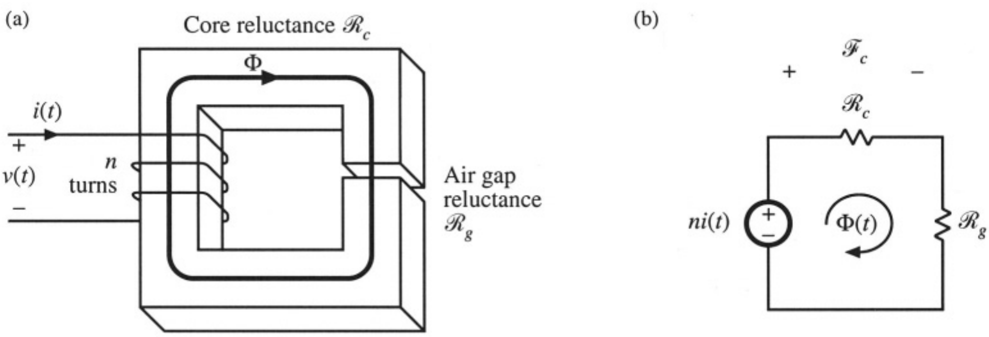

Fig. 14.2 Filter inductor: (a) structure, (b) magnetic circuit model.

current $I + \Delta i$.

Let us consider the design of the filter inductor illustrated in Figs. 14.1 and 14.2. It is assumed that the core and proximity losses are negligible, so that the inductor losses are dominated by the low-frequency copper losses. The inductor can therefore be modeled by the equivalent circuit of Fig. 14.3, in which R represents the dc resistance of the winding. It is desired to obtain a given inductance L and given winding resistance R. The inductor should not saturate when a given worst-case peak current I_{max} is applied. Note that specification of R is equivalent to specification of the copper loss P_{cu}, since

$$P_{cu} = I_{rms}^2 R \tag{14.1}$$

The influence of inductor winding resistance on converter efficiency and output voltage is modeled in Chapter 3.

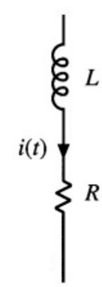

Fig. 14.3 Filter inductor equivalent circuit.

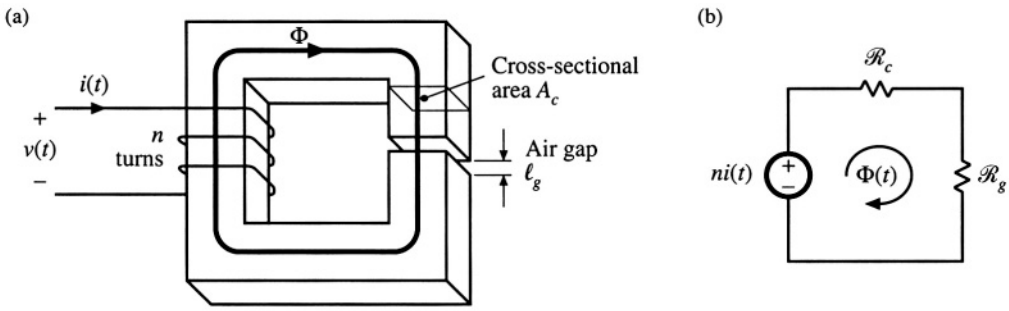

Fig. 14.4 Filter inductor: (a) assumed geometry, (b) magnetic circuit.

It is assumed that the inductor geometry is topologically equivalent to Fig. 14.4(a). An equivalent magnetic circuit is illustrated in Fig. 14.4(b). The core reluctance $\mathcal{R}_c$ and air gap reluctance $\mathcal{R}_g$ are

$$\mathcal{R}_c = \frac{\ell_c}{\mu_c A_c}$$
$$\mathcal{R}_g = \frac{\ell_g}{\mu_0 A_c} \tag{14.2}$$

where ℓ_c is the core magnetic path length, A_c is the core cross-sectional area, μ_c is the core permeability, and ℓ_g is the air gap length. It is assumed that the core and air gap have the same cross-sectional areas. Solution of Fig. 14.4(b) yields

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right) \tag{14.3}$$

Usually, $\mathcal{R}_c \ll \mathcal{R}_g$, and hence Eq. (14.3) can be approximated as

$$ni \approx \Phi\mathcal{R}_g \tag{14.4}$$

The air gap dominates the inductor properties. Four design constraints now can be identified.

14.1.1 Maximum Flux Density

Given a peak winding current I_{max}, it is desired to operate the core flux density at a maximum value B_{max}. The value of B_{max} is chosen to be less than the worst-case saturation flux density B_{sat} of the core material.

Substitution of $\Phi = BA_c$ into Eq. (14.4) leads to

$$ni = BA_c\mathcal{R}_g \tag{14.5}$$

Upon letting $I = I_{max}$ and $B = B_{max}$, we obtain

$$nI_{max} = B_{max}A_c\mathcal{R}_g = B_{max}\frac{\ell_g}{\mu_0} \tag{14.6}$$

This is the first design constraint. The turns ratio n and the air gap length ℓ_g are unknowns.

14.1.2 Inductance

The given inductance value L must be obtained. The inductance is equal to

$$L = \frac{n^2}{\mathcal{R}_g} = \frac{\mu_0 A_c n^2}{\ell_g} \tag{14.7}$$

This is the second design constraint. The turns ratio n, core area A_c, and gap length ℓ_g are unknown.

14.1.3 Winding Area

As illustrated in Fig. 14.5, the winding must fit through the window, i.e., the hole in the center of the core. The cross-sectional area of the conductor, or bare area, is A_W. If the winding has n turns, then the area of copper conductor in the window is

$$nA_W \tag{14.8}$$

If the core has window area W_A, then we can express the area available for the winding conductors as

$$K_u W_A \tag{14.9}$$

where K_u is the *window utilization factor,* or *fill factor*. Hence, the third design constraint can be expressed as

$$K_u W_A \geq nA_W \tag{14.10}$$

The fill factor K_u is the fraction of the core window area that is filled with copper. K_u must lie between zero and one. As discussed in [1], there are several mechanism that cause K_u to be less than unity. Round wire does not pack perfectly; this reduces K_u by a factor of 0.7 to 0.55, depending on the winding technique. The wire has insulation; the ratio of wire conductor area to total wire area varies from approximately 0.95 to 0.65, depending on the wire size and type of insulation. The bobbin uses some of the window area. Insulation may be required between windings and/or winding layers. Typical values of K_u for cores with winding bobbins are: 0.5 for a simple low-voltage inductor, 0.25 to 0.3 for an off-line transformer, 0.05 to 0.2 for a high-voltage transformer supplying several kV, and 0.65 for a low-voltage foil transformer or inductor.

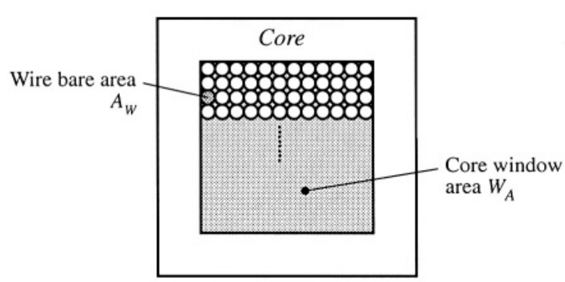

Fig. 14.5 The winding must fit in the core window area.

14.1.4 Winding Resistance

The resistance of the winding is

$$R = \rho \frac{\ell_b}{A_W} \tag{14.11}$$

where ρ is the resistivity of the conductor material, ℓ_b is the length of the wire, and A_W is the wire bare area. The resistivity of copper at room temperature is $1.724 \cdot 10^{-6}\ \Omega\text{–cm}$. The length of the wire comprising an n-turn winding can be expressed as

$$\ell_b = n(MLT) \tag{14.12}$$

where (MLT) is the mean-length-per-turn of the winding. The mean-length-per-turn is a function of the core geometry. Substitution of Eq. (14.12) into (14.11) leads to

$$R = \rho \frac{n(MLT)}{A_W} \tag{14.13}$$

This is the fourth constraint.

14.1.5 The Core Geometrical Constant K_g

The four constraints, Eqs. (14.6), (14.7), (14.10), and (14.13), involve the quantities A_c, W_A, and MLT, which are functions of the core geometry, the quantities I_{max}, B_{max}, μ_0, L, K_u, R, and ρ, which are given specifications or other known quantities, and n, ℓ_g, and A_W, which are unknowns. Elimination of the unknowns n, ℓ_g, and A_W leads to the following equation:

$$\frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u} \tag{14.14}$$

The quantities on the right side of this equation are specifications or other known quantities. The left side of the equation is a function of the core geometry alone. It is necessary to choose a core whose geometry satisfies Eq. (14.14).

The quantity

$$K_g = \frac{A_c^2 W_A}{(MLT)} \tag{14.15}$$

is called the core geometrical constant. It is a figure-of-merit that describes the effective electrical size of magnetic cores, in applications where copper loss and maximum flux density are specified. Tables are included in Appendix D that list the values of K_g for several standard families of ferrite cores. K_g has dimensions of length to the fifth power.

Equation (14.14) reveals how the specifications affect the core size. Increasing the inductance or peak current requires an increase in core size. Increasing the peak flux density allows a decrease in core size, and hence it is advantageous to use a core material that exhibits a high saturation flux density. Allowing a larger winding resistance R, and hence larger copper loss, leads to a smaller core. Of course, the increased copper loss and smaller core size will lead to a higher temperature rise, which may be unacceptable. The fill factor K_u also influences the core size.

Equation (14.15) reveals how core geometry affects the core capabilities. An inductor capable of meeting increased electrical requirements can be obtained by increasing either the core area A_c, or the window area W_A. Increase of the core area requires additional iron core material. Increase of the window area implies that additional copper winding material is employed. We can trade iron for copper, or vice versa, by changing the core geometry in a way that maintains the K_g of Eq. (14.15).

14.2 A STEP-BY-STEP PROCEDURE

The procedure developed in Section 14.1 is summarized below. This simple filter inductor design procedure should be regarded as a first-pass approach. Numerous issues have been neglected, including detailed insulation requirements, conductor eddy current losses, temperature rise, roundoff of number of turns, etc.

The following quantities are specified, using the units noted:

Wire resistivity	ρ	(Ω–cm)
Peak winding current	I_{max}	(A)
Inductance	L	(H)
Winding resistance	R	(Ω)
Winding fill factor	K_u	
Maximum operating flux density	B_{max}	(T)

The core dimensions are expressed in cm:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.

1. *Determine core size*

$$K_g \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u} 10^8 \quad (\mathrm{cm}^5) \tag{14.16}$$

Choose a core which is large enough to satisfy this inequality. Note the values of A_c, W_A, and MLT for this core. The resistivity ρ of copper wire is $1.724 \cdot 10^{-6}$ Ω–cm at room temperature, and $2.3 \cdot 10^{-6}$ Ω–cm at 100°C.

2. *Determine air gap length*

$$\ell_g = \frac{\mu_0 L I_{max}^2}{B_{max}^2 A_c} 10^4 \quad (\mathrm{m}) \tag{14.17}$$

with A_c expressed in cm^2. $\mu_0 = 4\pi \cdot 10^{-7}$ H/m. The air gap length is given in meters. The value expressed in Eq. (14.17) is approximate, and neglects fringing flux and other nonidealities.

Core manufacturers sell gapped cores. Rather than specifying the air gap length, the equivalent quantity A_L is used. A_L is equal to the inductance, in mH, obtained with a winding of 1000 turns. When A_L is specified, it is the core manufacturer's responsibility to obtain the correct gap length. Equation (14.17) can be modified to yield the required A_L, as follows:

$$A_L = \frac{10 B_{max}^2 A_c^2}{L I_{max}^2} \quad \text{(mH/1000 turns)} \tag{14.18}$$

where A_c is given in cm^2, L is given in Henries, and B_{max} is given in Tesla.

3. *Determine number of turns*

$$n = \frac{L I_{max}}{B_{max} A_c} 10^4 \tag{14.19}$$

4. *Evaluate wire size*

$$A_W \le \frac{K_u W_A}{n} \quad (\text{cm}^2) \tag{14.20}$$

Select wire with bare copper area less than or equal to this value. An American Wire Gauge table is included in Appendix D.

As a check, the winding resistance can be computed:

$$R = \frac{\rho n (MLT)}{A_w} \quad (\Omega) \tag{14.21}$$

14.3 MULTIPLE-WINDING MAGNETICS DESIGN VIA THE K_g METHOD

The K_g method can be extended to the case of multiple-winding magnetics, such as the transformers and coupled inductors described in Sections 13.5.3 to 13.5.5. The desired turns ratios, as well as the desired winding voltage and current waveforms, are specified. In the case of a coupled inductor or flyback transformer, the magnetizing inductance is also specified. It is desired to select a core size, number of turns for each winding, and wire sizes. It is also assumed that the maximum flux density B_{max} is given.

With the K_g method, a desired copper loss is attained. In the multiple-winding case, each winding contributes some copper loss, and it is necessary to allocate the available window area among the various windings. In Section 14.3.1 below, it is found that total copper loss is minimized if the window area is divided between the windings according to their apparent powers. This result is employed in the following sections, in which an optimized K_g method for coupled inductor design is developed.

14.3.1 Window Area Allocation

The first issue to settle in design of a multiple-winding magnetic device is the allocation of the window area A_W among the various windings. It is desired to design a device having k windings with turns ratios $n_1 : n_2 : \ldots : n_k$. These windings must conduct rms currents $I_1, I_2, \ldots, I_k$ respectively. It should be noted that the windings are effectively in parallel: the winding voltages are ideally related by the turns ratios

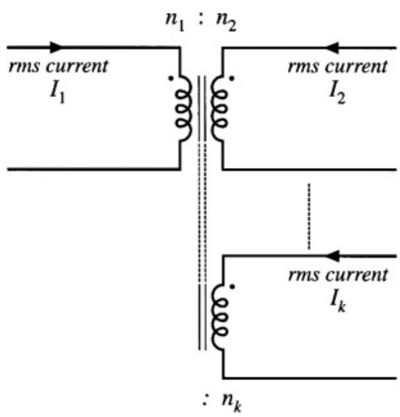

Fig. 14.6 It is desired to optimally allocate the window area of a k–winding magnetic element to minimize low-frequency copper losses, with given rms winding currents and turns ratios.

$$\frac{v_1(t)}{n_1} = \frac{v_2(t)}{n_2} = \cdots = \frac{v_k(t)}{n_k} \tag{14.22}$$

However, the winding rms currents are determined by the loads, and in general are not related to the turns ratios. The device is represented schematically in Fig. 14.6.

The relevant geometrical parameters are summarized in Fig. 14.7(a). It is necessary to allocate a portion of the total window area W_A to each winding, as illustrated in Fig. 14.7(b). Let α_j be the fraction of the window area allocated to winding j, where

$$\begin{aligned} &0 < \alpha_j < 1 \\ &\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1 \end{aligned} \tag{14.23}$$

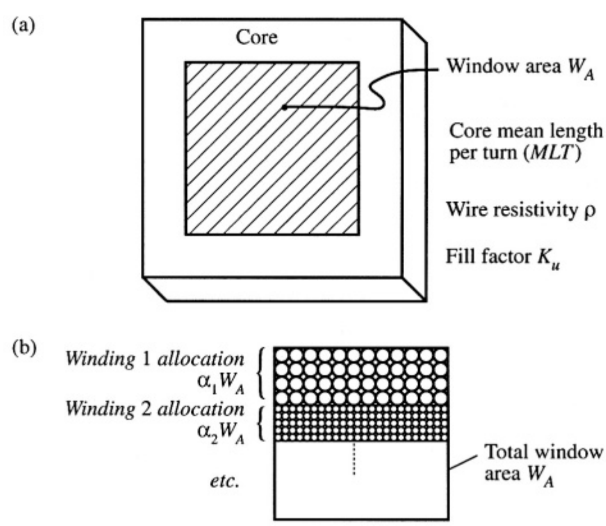

Fig. 14.7 Basic core topology, including window area W_A enclosed by core (a). The window is allocated to the various windings to minimize low-frequency copper loss (b).

The low-frequency copper loss $P_{cu,j}$ in winding j depends on the dc resistance R_j of winding j, as follows:

$$P_{cu,j} = I_j^2 R_j \tag{14.24}$$

The resistance of winding j is

$$R_j = \rho \frac{\ell_j}{A_{W,j}} \tag{14.25}$$

where ρ is the wire resistivity, ℓ_j is the length of the wire used for winding j, and $A_{W,j}$ is the cross-sectional area of the wire used for winding j. These quantities can be expressed as

$$\ell_j = n_j (MLT) \tag{14.26}$$

$$A_{W,j} = \frac{W_A K_u \alpha_j}{n_j} \tag{14.27}$$

where (MLT) is the winding mean-length-per-turn, and K_u is the winding fill factor. Substitution of these expressions into Eq. (14.25) leads to

$$R_j = \rho \frac{n_j^2 (MLT)}{W_A K_u \alpha_j} \tag{14.28}$$

The copper loss of winding j is therefore

$$P_{cu,j} = \frac{n_j^2 i_j^2 \rho (MLT)}{W_A K_u \alpha_j} \tag{14.29}$$

The total copper loss of the k windings is

$$P_{cu,tot} = P_{cu,1} + P_{cu,2} + \cdots + P_{cu,k} = \frac{\rho (MLT)}{W_A K_u} \sum_{j=1}^{k} \left(\frac{n_j^2 I_j^2}{\alpha_j} \right) \tag{14.30}$$

It is desired to choose the α_js such that the total copper loss $P_{cu,tot}$ is minimized. Let us consider what happens when we vary one of the αs, say α_1, between 0 and 1.

When $\alpha_1 = 0$, then we allocate zero area to winding 1. In consequence, the resistance of winding 1 tends to infinity. The copper loss of winding 1 also tends to infinity. On the other hand, the other windings are given maximum area, and hence their copper losses can be reduced. Nonetheless, the total copper loss tends to infinity.

When $\alpha_1 = 1$, then we allocate all of the window area to winding 1, and none to the other windings. Hence, the resistance of winding 1, as well as its low-frequency copper loss, are minimized. But the copper losses of the remaining windings tend to infinity.

As illustrated in Fig. 14.8, there must be an optimum value of α_1 that lies between these two extremes, where the total copper loss is minimized. Let us compute the optimum values of $\alpha_1, \alpha_2, \ldots, \alpha_k$ using the method of Lagrange multipliers. It is desired to minimize Eq. (14.30), subject to the constraint of Eq. (14.23). Hence, we define the function

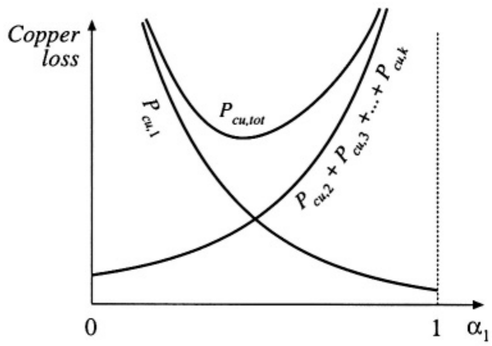

Fig. 14.8 Variation of copper losses with α_1.

$$f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi) = P_{cu,tot}(\alpha_1, \alpha_2, \cdots, \alpha_k) + \xi\, g(\alpha_1, \alpha_2, \cdots, \alpha_k) \tag{14.31}$$

where

$$g(\alpha_1, \alpha_2, \cdots, \alpha_k) = 1 - \sum_{j=1}^{k} \alpha_j \tag{14.32}$$

is the constraint that must equal zero, and ξ is the Lagrange multiplier. The optimum point is the solution of the system of equations

$$\begin{aligned}
\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_1} &= 0 \\
\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_2} &= 0 \\
&\vdots \\
\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_k} &= 0 \\
\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \xi} &= 0
\end{aligned} \tag{14.33}$$

The solution is

$$\xi = \frac{\rho(MLT)}{W_A K_u} \left(\sum_{j=1}^{k} n_j I_j \right)^2 = P_{cu,tot} \tag{14.34}$$

$$\alpha_m = \frac{n_m I_m}{\sum_{n=1}^{\infty} n_j I_j} \tag{14.35}$$

This is the optimal choice for the αs, and the resulting minimum value of $P_{cu,tot}$.

According to Eq. (14.22), the winding voltages are proportional to the turns ratios. Hence, we can express the α_js in the alternate form

$$\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j} \tag{14.36}$$

by multiplying and dividing Eq. (14.35) by the quantity V_m/n_m. It is irrelevant whether rms or peak voltages are used. Equation (14.36) is the desired result. It states that the window area should be allocated to the various windings in proportion to their apparent powers. The numerator of Eq. (14.36) is the apparent power of winding m, equal to the product of the rms current and the voltage. The denominator is the sum of the apparent powers of all windings.

As an example, consider the PWM full-bridge transformer having a center-tapped secondary, as illustrated in Fig. 14.9. This can be viewed as a three-winding transformer, having a single primary-side winding of n_1 turns, and two secondary-side windings, each of n_2 turns. The winding current waveforms $i_1(t)$, $i_2(t)$, and $i_3(t)$ are illustrated in Fig. 14.10. Their rms values are

$$I_1 = \sqrt{\frac{1}{2T_s}\int_0^{2T_s} i_1^2(t)dt} = \frac{n_2}{n_1} I \sqrt{D} \tag{14.37}$$

$$I_2 = I_3 = \sqrt{\frac{1}{2T_s}\int_0^{2T_s} i_2^2(t)dt} = \frac{1}{2} I \sqrt{1+D} \tag{14.38}$$

Substitution of these expressions into Eq. (14.35) yields

$$\alpha_1 = \frac{1}{\left(1 + \sqrt{\frac{1+D}{D}}\right)} \tag{14.39}$$

$$\alpha_2 = \alpha_3 = \frac{1}{2}\frac{1}{\left(1 + \sqrt{\frac{D}{1+D}}\right)} \tag{14.40}$$

If the design is to be optimized at the operating point $D = 0.75$, then one obtains

$$\begin{aligned} \alpha_1 &= 0.396 \\ \alpha_2 &= 0.302 \\ \alpha_3 &= 0.302 \end{aligned} \tag{14.41}$$

So approximately 40% of the window area should be allocated to the primary winding, and 30% should

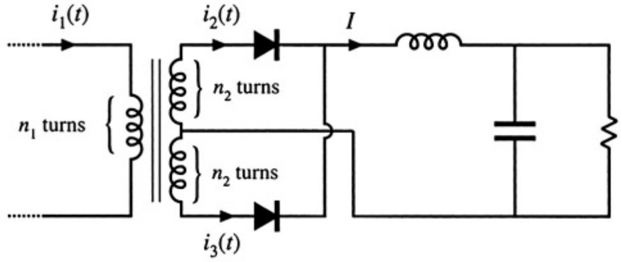

Fig. 14.9 PWM full-bridge transformer example.

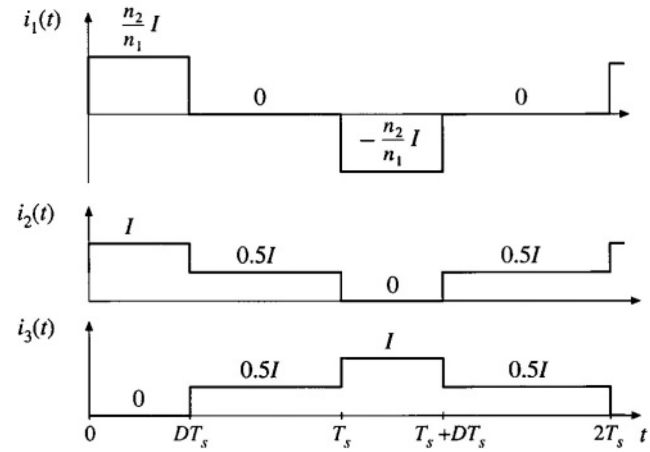

Fig. 14.10 Transformer waveforms, PWM full-bridge converter example.

be allocated to each half of the center-tapped secondary. The total copper loss at this optimal design point is found from evaluation of Eq. (14.34):

$$\begin{aligned} P_{cu,tot} &= \frac{\rho(MLT)}{W_A K_u}\left(\sum_{j=1}^{3} n_j I_j\right)^2 \\ &= \frac{\rho(MLT) n_2^2 I^2}{W_A K_u}\left(1 + 2D + 2\sqrt{D(1+D)}\right) \end{aligned} \tag{14.42}$$

14.3.2 Coupled Inductor Design Constraints

Let us now consider how to design a k-winding coupled inductor, as discussed in Section 13.5.4 and illustrated in Fig. 14.11. It is desired that the magnetizing inductance be a specified value L_M, referred to winding 1. It is also desired that the numbers of turns for the other windings be chosen according to desired turns ratios. When the magnetizing current $i_M(t)$ reaches its maximum value $I_{M,max}$, the coupled inductor should operate with a given maximum flux density B_{max}. With rms currents $I_1, I_2, \ldots, I_k$ applied to the respective windings, the total copper loss should be a desired value P_{cu} given by Eq. (14.34). Hence, the design procedure involves selecting the core size and number of primary turns so that the desired magnetizing inductance, the desired flux density, and the desired total copper loss are achieved. Other quantities, such as air gap length, secondary turns, and wire sizes, can then be selected. The derivation follows the derivation for the single winding case (Section 14.1), and incorporates the window area optimization of Section 14.3.1.

The magnetizing current $i_M(t)$ can be expressed in terms of the winding currents $i_1(t), i_2(t), \ldots, i_k(t)$ by solution of Fig. 14.11 (a) (or by use of Ampere's Law), as follows:

$$i_M(t) = i_1(t) + \frac{n_2}{n_1} i_2(t) + \cdots + \frac{n_k}{n_1} i_k(t) \tag{14.43}$$

By solution of the magnetic circuit model of Fig. 14.1 1(b), we can write

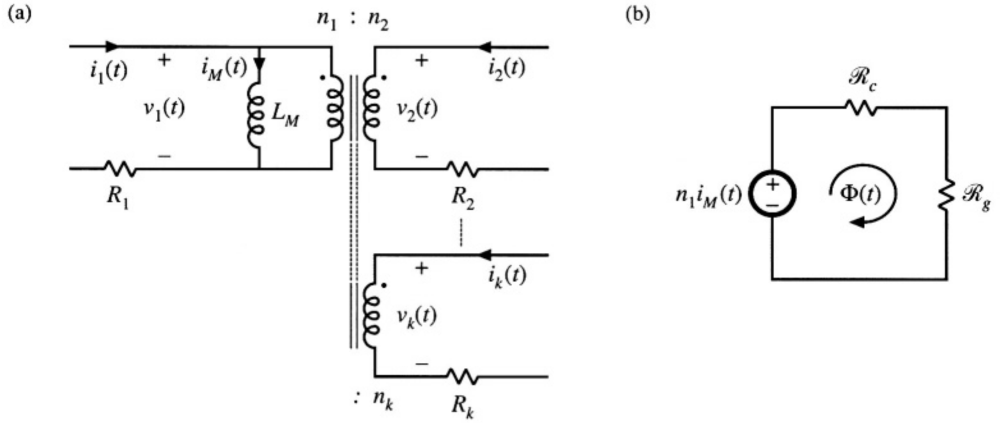

Fig. 14.11 A k-winding magnetic device, with specified turns ratios and waveforms: (a) electrical circuit model, (b) a magnetic circuit model.

$$n_1 i_M(t) = B(t) A_c \mathcal{R}_g \tag{14.44}$$

This equation is analogous to Eq. (14.4), and assumes that the reluctance $\mathcal{R}_g$ of the air gap is much larger than the reluctance $\mathcal{R}_c$ of the core. As usual, the total flux $\Phi(t)$ is equal to $B(t)A_c$. Leakage inductances are ignored.

To avoid saturation of the core, the instantaneous flux density $B(t)$ must be less than the saturation flux density of the core material, B_{sat}. Let us define $I_{M,max}$ as the maximum value of the magnetizing current $i_M(t)$. According to Eq. (14.44), this will lead to a maximum flux density B_{max} given by

$$n_1 I_{M,max} = B_{max} A_c \mathcal{R}_g = B_{max} \frac{\ell_g}{\mu_0} \tag{14.45}$$

For a value of $I_{M,max}$ given by the circuit application, we should use Eq. (14.45) to choose the turns n_1 and gap length ℓ_g such that the maximum flux density B_{max} is less than the saturation density B_{sat}. Equation (14.45) is similar to Eq. (14.6), but accounts for the magnetizations produced by multiple winding currents.

The magnetizing inductance L_M, referred to winding 1, is equal to

$$L_M = \frac{n_1^2}{\mathcal{R}_g} = n_1^2 \frac{\mu_0 A_c}{\ell_g} \tag{14.46}$$

This equation is analogous to Eq. (14.7).

As shown in Section 14.3.1, the total copper loss is minimized when the core window area W_A is allocated to the various windings according to Eq. (14.35) or (14.36). The total copper loss is then given by Eq. (14.34). Equation (14.34) can be expressed in the form

$$P_{cu} = \frac{\rho (MLT) n_1^2 I_{tot}^2}{W_A K_u} \tag{14.47}$$

where

$$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_1} I_j \tag{14.48}$$

is the sum of the rms winding currents, referred to winding 1.

We can now eliminate the unknown quantities ℓ_g and n_1 from Eqs. (14.45), (14.46), and (14.47). Equation (14.47) then becomes

$$P_{cu} = \frac{\rho (MLT) L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 A_c^2 W_A K_u} \tag{14.49}$$

We can now rearrange this equation, by grouping terms that involve the core geometry on the left-hand side, and specifications on the right-hand side:

$$\frac{A_c^2 W_A}{(MLT)} = \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 K_u P_{cu}} \tag{14.50}$$

The left-hand side of the equation can be recognized as the same K_g term defined in Eq. (14.15). Therefore, to design a coupled inductor that meets the requirements of operating with a given maximum flux density B_{max}, given primary magnetizing inductance L_M, and with a given total copper loss P_{cu}, we must select a core that satisfies

$$K_g \geq \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 K_u P_{cu}} \tag{14.51}$$

Once such a core is found, then the winding 1 turns and gap length can be selected to satisfy Eqs. (14.45) and (14.46). The turns of windings 2 through k are selected according to the desired turns ratios. The window area is allocated among the windings according to Eq. (14.35), and the wire gauges are chosen using Eq. (14.27).

The procedure above is applicable to design of coupled inductors. The results are applicable to design of flyback and SEPIC transformers as well, although it should be noted that the procedure does not account for the effects of core or proximity loss. It also can be extended to design of other devices, such as conventional transformers—doing so is left as a homework problem.

14.3.3 Design Procedure

The following quantities are specified, using the units noted:

Wire effective resistivity	ρ	(Ω–cm)
Total rms winding currents, referred to winding 1	$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_i} I_j$	(A)
Peak magnetizing current, referred to winding 1	$I_{M,max}$	(A)
Desired turns ratios	n_2/n_1, n_3/n_1, etc.	

Magnetizing inductance, referred to winding 1	L_M	(H)
Allowed total copper loss	P_{cu}	(W)
Winding fill factor	K_u	
Maximum operating flux density	B_{max}	(T)

The core dimensions are expressed in cm:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.

1. *Determine core size*

$$K_g \geq \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 P_{cu} K_u} 10^8 \qquad (\text{cm}^5) \tag{14.52}$$

Choose a core which is large enough to satisfy this inequality. Note the values of A_c, W_A, and *MLT* for this core. The resistivity ρ of copper wire is $1.724 \cdot 10^{-6}\ \Omega$–cm at room temperature, and $2.3 \cdot 10^{-6}\ \Omega$–cm at 100°C.

2. *Determine air gap length*

$$\ell_g = \frac{\mu_0 L_M I_{M,max}^2}{B_{max}^2 A_c} 10^4 \qquad (\text{m}) \tag{14.53}$$

Here, B_{max} is expressed in Tesla, A_c is expressed in cm^2, and ℓ_g is expressed in meters. The permeability of free space is $\mu_0 = 4\pi \cdot 10^{-7}$ H/m. This value is approximate, and neglects fringing flux and other nonidealities.

3. *Determine number of winding 1 turns*

$$n_1 = \frac{L_M I_{M,max}}{B_{max} A_c} 10^4 \tag{14.54}$$

Here, B_{max} is expressed in Tesla and A_c is expressed in cm^2.

3. *Determine number of secondary turns*

Use the desired turns ratios:

$$\begin{aligned} n_2 &= \left(\frac{n_2}{n_1}\right) n_1 \\ n_3 &= \left(\frac{n_3}{n_1}\right) n_1 \\ &\vdots \end{aligned} \tag{14.55}$$

4. *Evaluate fraction of window area allocated to each winding*

$$\begin{aligned} \alpha_1 &= \frac{n_1 I_1}{n_1 I_{tot}} \\ \alpha_2 &= \frac{n_2 I_2}{n_1 I_{tot}} \\ &\vdots \\ \alpha_k &= \frac{n_k I_k}{n_1 I_{tot}} \end{aligned} \tag{14.56}$$

5. *Evaluate wire sizes*

$$\begin{aligned} A_{w1} &\le \frac{\alpha_1 K_u W_A}{n_1} \\ A_{w2} &\le \frac{\alpha_2 K_u W_A}{n_2} \\ &\vdots \end{aligned} \tag{14.57}$$

Select wire with bare copper area less than or equal to these values. An American Wire Gauge table is included in Appendix D.

14.4 EXAMPLES

14.4.1 Coupled Inductor for a Two-Output Forward Converter

As a first example, let us consider the design of coupled inductors for the two-output forward converter illustrated in Fig. 14.12. This element can be viewed as two filter inductors that are wound on the same core. The turns ratio is chosen to be the same as the ratio of the output voltages. The magnetizing inductance performs the function of filtering the switching harmonics for both outputs, and the magnetizing current is equal to the sum of the reflected winding currents.

At the nominal full-load operating point, the converter operates in the continuous conduction mode with a duty cycle of D = 0.35. The switching frequency is 200 kHz. At this operating point, it is desired that the ripple in the magnetizing current have a peak magnitude equal to 20% of the dc component of magnetizing current.

The dc component of the magnetizing current I_M is

$$\begin{aligned} I_M &= I_1 + \frac{n_2}{n_1} I_2 \\ &= (4\text{ A}) + \frac{12}{28}(2\text{ A}) \\ &= 4.86\text{ A} \end{aligned} \tag{14.58}$$

The magnetizing current ripple Δi_M can be expressed as

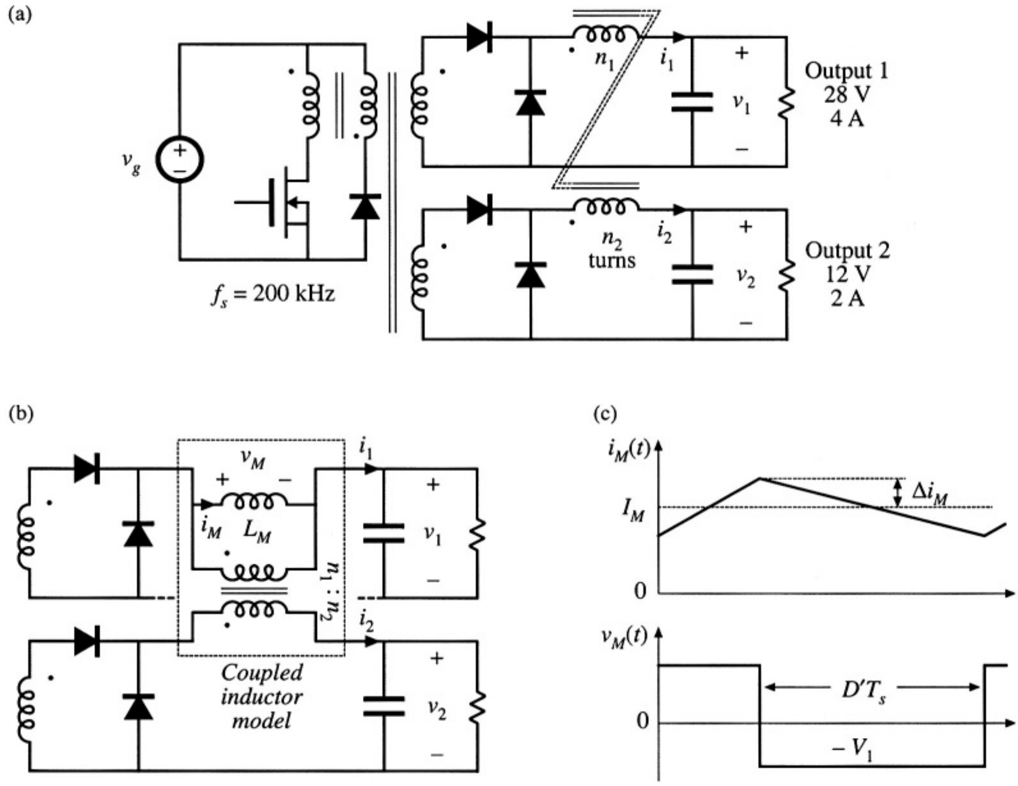

Fig. 14.12 Two-output forward converter example: (a) circuit schematic, (b) coupled inductor model inserted into converter secondary-side circuit, (c) magnetizing current and voltage waveforms of coupled inductor, referred to winding 1.

$$\Delta i_M = \frac{V_1 D' T_s}{2 L_M} \tag{14.59}$$

Since we want Δi_M to be equal to 20% of I_M, we should choose L_M as follows:

$$\begin{aligned} L_M &= \frac{V_1 D' T_s}{2 \Delta i_M} \\ &= \frac{(28\text{ V})(1-0.35)(5\ \mu\text{s})}{2(4.86\text{ A})(20\%)} \\ &= 47\ \mu\text{H} \end{aligned} \tag{14.60}$$

The peak magnetizing current, referred to winding 1, is therefore

$$I_{M,max} = I_M + \Delta i_M = 5.83\text{ A} \tag{14.61}$$

Since the current ripples of the winding currents are small compared to the respective dc components, the rms values of the winding currents are approximately equal to the dc components: $I_1 = 4$ A, $I_2 = 2$ A. Therefore, the sum of the rms winding currents, referred to winding 1, is

$$I_{tot} = I_1 + \frac{n_2}{n_1} I_2 = 4.86 \text{ A} \tag{14.62}$$

For this design, it is decided to allow 0.75 W of copper loss, and to operate the core at a maximum flux density of 0.25 Tesla. A fill factor of 0.4 is assumed. The required K_g is found by evaluation of Eq. (14.52), as follows:

$$\begin{aligned} K_g &\geq \frac{(1.724 \cdot 10^{-6}\ \Omega\text{-cm})(47\ \mu\text{H})^2(4.86\ \text{A})^2(5.83\ \text{A})^2}{(0.25\ \text{T})^2(0.75\ \text{W})(0.4)} 10^8 \\ &= 16 \cdot 10^{-3}\ \text{cm}^5 \end{aligned} \tag{14.63}$$

A ferrite PQ 20/16 core is selected, which has a K_g of $22.4 \cdot 10^{-3}$ cm^5. From Appendix D, the geometrical parameters for this core are: $A_c = 0.62$ cm^2, $W_A = 0.256$ cm^2, and $MLT = 4.4$ cm.

The air gap is found by evaluation of Eq. (14.53) as follows:

$$\begin{aligned} \ell_g &= \frac{\mu_0 L_M I_{M,max}^2}{B_{max}^2 A_c} 10^4 \\ &= \frac{(4\pi \cdot 10^{-7}\text{H/m})(47\ \mu\text{H})(5.83\ \text{A})^2}{(0.25\ \text{T})^2(0.62\ \text{cm}^2)} 10^4 \\ &= 0.52\ \text{mm} \end{aligned} \tag{14.64}$$

In practice, a slightly longer air gap would be necessary, to allow for the effects of fringing flux and other nonidealities. The winding 1 turns are found by evaluation of Eq. (14.54):

$$\begin{aligned} n_1 &= \frac{L_M I_{M,max}}{B_{max} A_c} 10^4 \\ &= \frac{(47\ \mu\text{H})(5.83\ \text{A})}{(0.25\ \text{T})(0.62\ \text{cm}^2)} 10^4 \\ &= 17.6\ \text{turns} \end{aligned} \tag{14.65}$$

The winding 2 turns are chosen according to the desired turns ratio:

$$\begin{aligned} n_2 &= \left(\frac{n_2}{n_1}\right) n_1 \\ &= \left(\frac{12}{28}\right)(17.6) \\ &= 7.54\ \text{turns} \end{aligned} \tag{14.66}$$

The numbers of turns are rounded off to $n_1 = 17$ turns, $n_2 = 7$ turns (18:8 would be another possible choice). The window area W_A is allocated to the windings according to the fractions from Eq. (14.56):

$$\begin{aligned} \alpha_1 &= \frac{n_1 I_1}{n_1 I_{tot}} = \frac{(17)(4\ \text{A})}{(17)(4.86\ \text{A})} = 0.8235 \\ \alpha_2 &= \frac{n_2 I_2}{n_1 I_{tot}} = \frac{(7)(2\ \text{A})}{(17)(4.86\ \text{A})} = 0.1695 \end{aligned} \tag{14.67}$$

The wire sizes can therefore be chosen as follows:

$$A_{w1} \le \frac{\alpha_1 K_u W_A}{n_1} = \frac{(0.8235)(0.4)(0.256\,\mathrm{cm}^2)}{(17)} = 4.96 \cdot 10^{-3}\,\mathrm{cm}^2 \quad \text{use AWG \#21}$$

$$A_{w2} \le \frac{\alpha_2 K_u W_A}{n_2} = \frac{(0.1695)(0.4)(0.256\,\mathrm{cm}^2)}{(7)} = 2.48 \cdot 10^{-3}\,\mathrm{cm}^2 \quad \text{use AWG \#24} \tag{14.68}$$

14.4.2 CCM Flyback Transformer

As a second example, let us design the flyback transformer for the converter illustrated in Fig. 14.13. This converter operates with an input voltage of 200 V, and produces an full-load output of 20 V at 5A. The switching frequency is 150 kHz. Under these operating conditions, it is desired that the converter operate in the continuous conduction mode, with a magnetizing current ripple equal to 20% of the dc component of magnetizing current. The duty cycle is chosen to be $D = 0.4$, and the turns ratio is $n_2/n_1 = 0.15$. A copper loss of 1.5 W is allowed, not including proximity effect losses. To allow room for isolation between the primary and secondary windings, a fill factor of $K_u = 0.3$ is assumed. A maximum flux density of $B_{max} = 0.25$ T is used; this value is less than the worst-case saturation flux density B_{sat} of the ferrite core material.

By solution of the converter using capacitor charge balance, the dc component of the magnetizing current can be found to be

$$I_M = \left(\frac{n_2}{n_1}\right)\frac{1}{D'}\frac{V}{R} = 1.25\ \mathrm{A} \tag{14.69}$$

Hence, the magnetizing current ripple should be

$$\Delta i_M = (20\%)I_M = 0.25\ \mathrm{A} \tag{14.70}$$

and the maximum value of the magnetizing current is

$$I_{M,max} = I_M + \Delta i_M = 1.5\ \mathrm{A} \tag{14.71}$$

To obtain this ripple, the magnetizing inductance should be

$$L_M = \frac{V_g D T_s}{2\Delta i_M} = 1.07\ \mathrm{mH} \tag{14.72}$$

The rms value of the primary winding current is found using Eq. (A.6) of Appendix A, as follows:

$$I_1 = I_M \sqrt{D}\sqrt{1 + \frac{1}{3}\left(\frac{\Delta i_M}{I_M}\right)^2} = 0.796\ \mathrm{A} \tag{14.73}$$

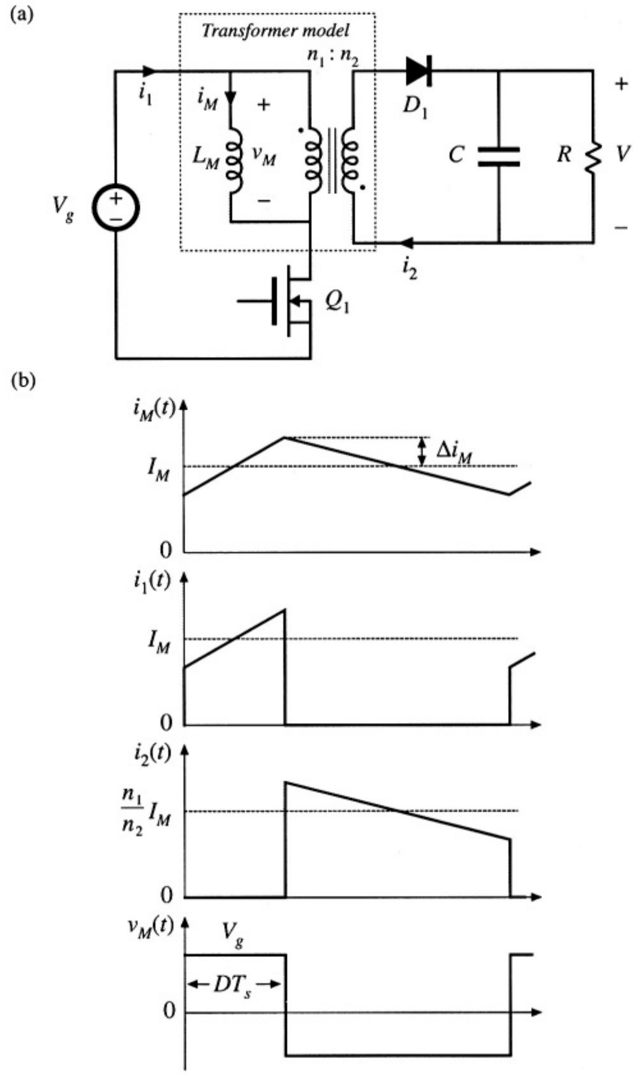

Fig. 14.13 Flyback transformer design example: (a) converter schematic, (b) typical waveforms.

The rms value of the secondary winding current is found in a similar manner:

$$I_2 = \frac{n_1}{n_2} I_M \sqrt{D'} \sqrt{1 + \frac{1}{3}\left(\frac{\Delta i_M}{I_M}\right)^2} = 6.50 \text{ A} \tag{14.74}$$

Note that I_2 is not simply equal to the turns ratio multiplied by I_1. The total rms winding current is equal to:

$$I_{tot} = I_1 + \frac{n_2}{n_1} I_2 = 1.77 \text{ A} \tag{14.75}$$

We can now determine the necessary core size. Evaluation of Eq. (14.52) yields

$$\begin{aligned} K_g &\geq \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 P_{cu} K_u} 10^8 \\ &= \frac{\left(1.724 \cdot 10^{-6}\Omega\text{-cm}\right)\left(1.07 \cdot 10^{-3}\text{ H}\right)^2\left(1.77\text{ A}\right)^2\left(1.5\text{ A}\right)^2}{\left(0.25\text{ T}\right)^2\left(1.5\text{ W}\right)\left(0.3\right)} 10^8 \\ &= 0.049\text{ cm}^5 \end{aligned} \quad (14.76)$$

The smallest EE core listed in Appendix D that satisfies this inequality is the EE30, which has $K_g = 0.0857\text{ cm}^5$. The dimensions of this core are

$$\begin{array}{ll} A_c & 1.09\text{ cm}^2 \\ W_A & 0.476\text{ cm}^2 \\ MLT & 6.6\text{ cm} \\ \ell_m & 5.77\text{ cm} \end{array} \quad (14.77)$$

The air gap length ℓ_g is chosen according to Eq. (14.53):

$$\begin{aligned} \ell_g &= \frac{\mu_0 L_M I_{M,max}^2}{B_{max}^2 A_c} 10^4 \\ &= \frac{\left(4\pi \cdot 10^{-7}\text{H/m}\right)\left(1.07 \cdot 10^{-3}\text{ H}\right)\left(1.5\text{ A}\right)^2}{\left(0.25\text{ T}\right)^2\left(1.09\text{ cm}^2\right)} 10^4 \\ &= 0.44\text{ mm} \end{aligned} \quad (14.78)$$

The number of winding 1 turns is chosen according to Eq. (14.54), as follows:

$$\begin{aligned} n_1 &= \frac{L_M I_{M,max}}{B_{max} A_c} 10^4 \\ &= \frac{\left(1.07 \cdot 10^{-3}\text{ H}\right)\left(1.5\text{ A}\right)}{\left(0.25\text{ T}\right)\left(1.09\text{ cm}^2\right)} 10^4 \\ &= 58.7\text{ turns} \end{aligned} \quad (14.79)$$

Since an integral number of turns is required, we round off this value to

$$n_1 = 59 \quad (14.80)$$

To obtain the desired turns ratio, n_2 should be chosen as follows:

$$\begin{aligned} n_2 &= \left(\frac{n_2}{n_1}\right) n_1 \\ &= \left(0.15\right) 59 \\ &= 8.81 \end{aligned} \quad (14.81)$$

We again round this value off, to

$$n_2 = 9 \tag{14.82}$$

The fractions of the window area allocated to windings 1 and 2 are selected in accordance with Eq. (14.56):

$$\begin{aligned} \alpha_1 &= \frac{I_1}{I_{tot}} = \frac{(0.796\ \text{A})}{(1.77\ \text{A})} = 0.45 \\ \alpha_2 &= \frac{n_2 I_2}{n_1 I_{tot}} = \frac{(9)(6.5\ \text{A})}{(59)(1.77\ \text{A})} = 0.55 \end{aligned} \tag{14.83}$$

The wire gauges should therefore be

$$\begin{aligned} A_{W1} &\le \frac{\alpha_1 K_u W_A}{n_1} = 1.09 \cdot 10^{-3}\ \text{cm}^2 \quad \text{— use \#28 AWG} \\ A_{W2} &\le \frac{\alpha_2 K_u W_A}{n_2} = 8.88 \cdot 10^{-3}\ \text{cm}^2 \quad \text{— use \#19 AWG} \end{aligned} \tag{14.84}$$

The above American Wire Gauges are selected using the wire gauge table given at the end of Appendix D.

The above design does not account for core loss or copper loss caused by the proximity effect. Let us compute the core loss for this design. Figure Fig. 14.14 contains a sketch of the *B–H* loop for this design. The flux density $B(t)$ can be expressed as a dc component (determined by the dc value of the magnetizing current I_M), plus an ac variation of peak amplitude ΔB that is determined by the current ripple Δi_M. The maximum value of $B(t)$ is labeled B_{max}; this value is determined by the sum of the dc component and the ac ripple component. The core material saturates when the applied $B(t)$ exceeds B_{sat}; hence, to avoid saturation, B_{max} should be less than B_{sat}. The core loss is determined by the amplitude of the ac variations in $B(t)$ i.e., by ΔB.

The ac component ΔB is determined using Faraday's law, as follows. Solution of Faraday's law for the derivative of $B(t)$ leads to

$$\frac{dB(t)}{dt} = \frac{v_M(t)}{n_1 A_c} \tag{14.85}$$

As illustrated in Fig. 14.15, the voltage applied during the first subinterval is $v_M(t) = V_g$. This causes the

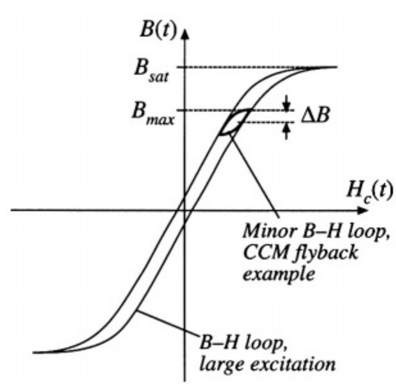

Fig. 14.14 *B–H* loop for the flyback transformer design example.

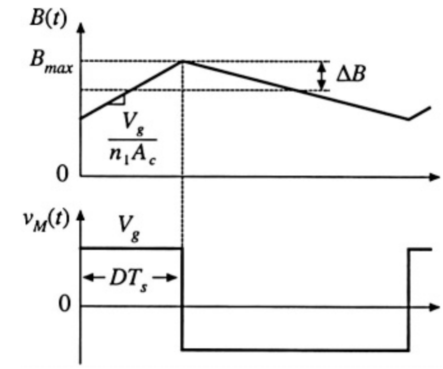

Fig. 14.15 Variation of flux density $B(t)$, flyback transformer example.

flux density to increase with slope

$$\frac{dB(t)}{dt} = \frac{V_g}{n_1 A_c} \tag{14.86}$$

Over the first subinterval $0 < t < DT_s$, the flux density $B(t)$ changes by the net amount $2\Delta B$. This net change is equal to the slope given by Eq. (14.86), multiplied by the interval length DT_s:

$$\Delta B = \left(\frac{V_g}{n_1 A_c}\right)\left(DT_s\right) \tag{14.87}$$

Upon solving for ΔB and expressing A_c in cm^2, we obtain

$$\Delta B = \frac{V_g DT_s}{2 n_1 A_c} 10^4 \tag{14.88}$$

For the flyback transformer example, the peak ac flux density is found to be

$$\begin{aligned}\Delta B &= \frac{(200\ \text{V})(0.4)(6.67\ \mu\text{s})}{2(59)(1.09\ \text{cm}^2)} 10^4 \\ &= 0.041\ \text{T}\end{aligned} \tag{14.89}$$

To determine the core loss, we next examine the data provided by the manufacturer for the given core material. A typical plot of core loss is illustrated in Fig. 14.16. For the values of ΔB and switching frequency of the flyback transformer design, this plot indicates that 0.04 W will be lost in every cm^3 of the core material. Of course, this value neglects the effects of harmonics on core loss. The total core loss P_{fe} will therefore be equal to this loss density, multiplied by the volume of the core:

$$\begin{aligned}P_{fe} &= (0.04\ \text{W/cm}^3)(A_c \ell_m) \\ &= (0.04\ \text{W/cm}^3)(1.09\ \text{cm}^2)(5.77\ \text{cm}) \\ &= 0.25\ \text{W}\end{aligned} \tag{14.90}$$

This core loss is less than the copper loss of 1.5 W, and neglecting the core loss is often warranted in designs that operate in the continuous conduction mode and that employ ferrite core materials.

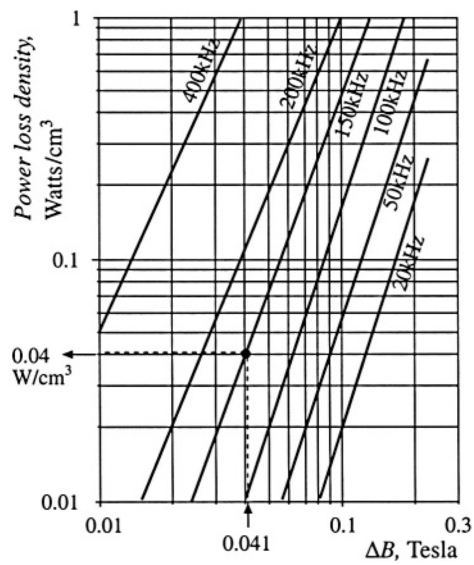

Fig. 14.16 Determination of core loss density for the flyback transformer design example.

14.5 SUMMARY OF KEY POINTS

1. A variety of magnetic devices are commonly used in switching converters. These devices differ in their core flux density variations, as well as in the magnitudes of the ac winding currents. When the flux density variations are small, core loss can be neglected. Alternatively, a low-frequency material can be used, having higher saturation flux density.

2. The core geometrical constant K_g is a measure of the magnetic size of a core, for applications in which copper loss is dominant. In the K_g design method, flux density and total copper loss are specified. Design procedures for single-winding filter inductors and for conventional multiple-winding transformers are derived.

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PROBLEMS

14.1 A simple buck converter operates with a 50 kHz switching frequency and a dc input voltage of $V_g = 40$ V. The output voltage is $V = 20$ V. The load resistance is $R \geq 4\ \Omega$.

(a) Determine the value of the output filter inductance L such that the peak-to-average inductor current ripple Δi is 10% of the dc component I.

(b) Determine the peak steady-state inductor current I_{max}

(c) Design an inductor which has the values of L and I_{max} from parts (a) and (b). Use a ferrite EE core, with $B_{max} = 0.25$ T. Choose a value of winding resistance such that the inductor copper loss is less than or equal to 1 W at room temperature. Assume $K_u = 0.5$. Specify: core size, gap length, wire size (AWG), and number of turns.

14.2 A boost converter operates at the following quiescent point: $V_g = 28$ V, $V = 48$ V, $P_{load} = 150$ W, $f_s = 100$ kHz. Design the inductor for this converter. Choose the inductance value such that the peak current ripple is 10% of the dc inductor current. Use a peak flux density of 0.225 T, and assume a fill factor of 0.5. Allow copper loss equal to 0.5% of the load power, at room temperature. Use a ferrite PQ core. Specify: core size, air gap length, wire gauge, and number of turns.

14.3 Extension of the K_g approach to design of two-winding transformers. It is desired to design a transformer having a turns ratio of 1:*n*. The transformer stores negligible energy, no air gap is required, and the ratio of the winding currents $i_2(t)/i_1(t)$ is essentially equal to the turns ratio n. The applied primary volt-seconds λ_1 are defined for a typical PWM voltage waveform $v_1(t)$ in Fig. 13.45(b); these volt-seconds should cause the maximum flux density to be equal to a specified value $B_{max} = \Delta B$. You may assume that the flux density $B(t)$ contains no dc bias, as in Fig. 13.46. You should allocate half of the cove window area to each winding. The total copper loss P_{cu} is also specified. You may neglect proximity losses.

(a) Derive a transformer design procedure, in which the following quantities are specified: total copper loss P_{cu}, maximum flux density B_{max}, fill factor K_u, wire resistivity ρ, rms primary current I_1, applied primary volt-seconds λ_1, and turns ratio 1:*n*. Your procedure should yield the following data: required core geometrical constant K_g, primary and secondary turns n_1 and n_2, and primary and secondary wire areas A_{w1} and A_{w2}.

(b) The voltage waveform applied to the transformer primary winding of the Ćuk converter [Fig. 6.41(c)] is equal to the converter input voltage V_g while the transistor conducts, and is equal to $-V_g D/(1-D)$ while the diode conducts. This converter operates with a switching frequency of 100 kHz, and a transistor duty cycle D equal to 0.4. The dc input voltage is $V_g = 120$ V, the dc output voltage is $V = 24$ V, and the load power is 200 W. You may assume a fill factor of $K_u = 0.3$. Use your procedure of part (a) to design a transformer for this application, in which $B_{max} = 0.15$ T, and $P_{cu} = 0.25$ W at 100°C. Use a ferrite PQ core. Specify: core size, primary and secondary turns, and wire gauges.

14.4 Coupled inductor design. The two-output forward converter of Fig. 13.47(a) employs secondary-side coupled inductors. An air gap is employed.

Design a coupled inductor for the following application: $V_1 = 5$ V, $V_2 = 15$ V, $I_1 = 20$ A, $I_2 = 4$ A, $D = 0.4$. The magnetizing inductance should be equal to 8 μH, referred to the 5 V winding. You may assume a fill factor K_u of 0.5. Allow a total of 1 W of copper loss at 100°C, and use a peak flux density of

$B_{max} = 0.2$ T. Use a ferrite EE core. Specify: core size, air gap length, number of turns and wire gauge for each winding.

14.5 Flyback transformer design. A flyback converter operates with a 160 Vdc input, and produces a 28 Vdc output. The maximum load current is 2 A. The transformer turns ratio is 8:1. The switching frequency is 100 kHz. The converter should be designed to operate in the discontinuous conduction mode at all load currents. The total copper loss should be less than 0.75 W.

(a) Choose the value of transformer magnetizing inductance L_M such that, at maximum load current, $D_3 = 0.1$ (the duty cycle of subinterval 3, in which all semiconductors are off). Please indicate whether your value of L_M is referred to the primary or secondary winding. What is the peak transistor current? The peak diode current?

(b) Design a flyback transformer for this application. Use a ferrite pot core with $B_{max} = 0.25$ Tesla, and with fill factor $K_u = 0.4$. Specify: core size, primary and secondary turns and wire sizes, and air gap length.

(c) For your design of part (b), compute the copper losses in the primary and secondary windings. You may neglect proximity loss.

(d) For your design of part (b), compute the core loss. Loss data for the core material is given by Fig. 13.20. Is the core loss less than the copper loss computed in Part (c)?