EE-365 - W4
REVIEW
MAGNETIC CIRCUITS

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INDUCTOR AND CAPACITOR RESPONSE

As already mentioned before:

- the current lags the voltage by 90° in an inductor
- currents leads the voltage by 90° in a capacitor

Relation between voltages and currents can be summarized as:

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{j\omega L} = \frac{\mathbf{V}_L}{\omega L} e^{-j\pi/2}$$

$$\mathbf{I}_C = j\omega C \mathbf{V}_C = \omega C \mathbf{V}_C e^{j\pi/2}$$

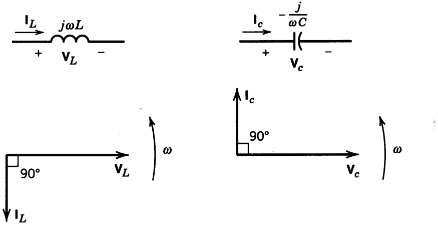

Figure 1 Phasor representation.

Inductor current i_L and capacitor voltage v_C are commonly used state variable:

$$i_L(t) = i_L(t_1) + \frac{1}{L}\int_{t_1}^{t} v_L\, dt, \quad t > t_1$$

$$v_C(t) = v_C(t_1) + \frac{1}{C}\int_{t_1}^{t} i_C\, dt, \quad t > t_1$$

- inductor current is the response to applied inductor voltage
- capacitor voltage is the response to change in capacitor current

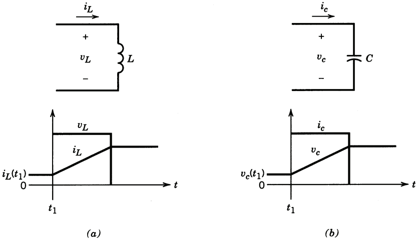

Figure 2 Inductor and Capacitor response.

Steady state condition imply repetition of the waveforms

- $v(t+T) = v(t)$
- $i(t+T) = i(t)$

In case of an inductor this leads to:

$$\frac{1}{T}\int_{t_1}^{t_1+T} v_L dt = 0$$

In the steady state average inductor voltage is zero

In case of a capacitor we have:

$$\frac{1}{T}\int_{t_1}^{t_1+T} i_C dt = 0$$

In the steady state average capacitor current is zero

These concepts will be frequently used for circuit analysis in the course

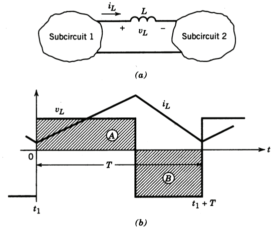

Figure 3 Inductor response in steady state.

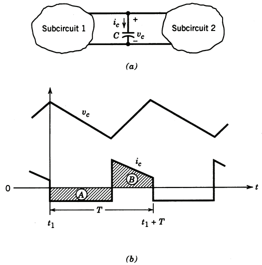

Figure 4 Capacitor response in steady state.

Magnetic components are widely used in power electronics

Inductors and Transformers are typical components found in many converters

Ampere's Law

- a current-carrying conductor produces a magnetic field of intensity $H, [A/m]$
- the line integral of the magnetic field intensity H, equals the total enclosed current

$$\oint H dl = \sum i$$

For practical circuits, this normally can be written as:

$$\sum_k H_k l_k = \sum_m N_m i_m$$

For circuit in Fig, 5 this becomes:

$$H_1 l_1 + H_g l_g = N_1 i_1$$

Right-hand rule can be used to determine the direction of H-field

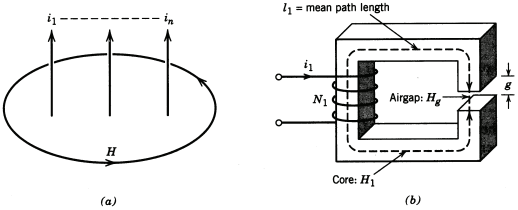

Figure 5 Ampere's Law: a) General formulation, b) Example.

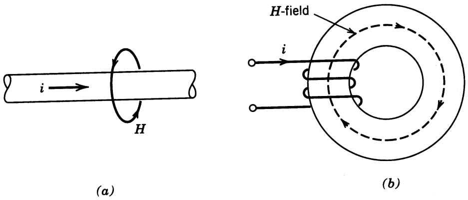

Figure 6 Right-hand rule: a) Generic case, b) Example.

Flux density B is related to field intensity H through the medium where field exist:

$$B = \mu H \quad [Wb/m^2] = [T]$$

The permeability μ of a medium is defined as:

$$\mu = \mu_0 \mu_r$$

where:

- μ_0 is the permeability of the air, $\mu_0 = 4\pi \cdot 10^{-7} H/m$
- μ_r is the relative permeability that may go up to several thousands

In case of saturation, incremental permeability $\mu_\Delta = \Delta B/\Delta H$ is much smaller

The magnetic flux Φ through an area can be found as surface integral of the B-field

$$\Phi = \int_A B dA$$

As the magnetic flux lines form closed loops, in-out net sum is zero

Continuity of flux imply:

$$\Phi = \int_{A_{closed\ loop}} B dA = 0 \quad \Rightarrow \quad \sum_k \Phi_k = 0$$

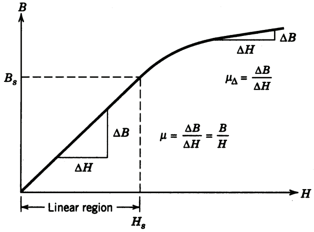

Figure 7 B-H relation.

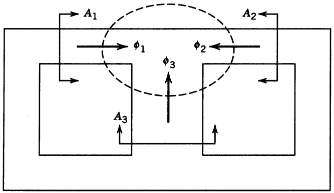

Figure 8 Continuity of flux (KCL for magnetics)

Ampere's law and continuity of flux can be used to define the Reluctance

$$\sum_k H_k l_k = \sum_k H_k \mu_k A_k \frac{l_k}{\mu_k A_k} = \sum_k B_k A_k \frac{l_k}{\mu_k A_k} = \sum_k \Phi_k \frac{l_k}{\mu_k A_k} = \Phi \sum_k \frac{l_k}{\mu_k A_k}$$

This further yields:

$$\Phi \sum_k \frac{l_k}{\mu_k A_k} = \sum_m N_m i_m$$

The magnetic Reluctance $\mathfrak{R}_k$ in the path of the magnetic lines is defined as:

$$\mathfrak{R}_k = \frac{l_k}{\mu_k A_k}$$

We have:

$$\Phi \sum_k \mathfrak{R}_k = \sum_m N_m i_m$$

For simple circuit as on figure we have: $\Phi \mathfrak{R} = Ni$

Knowing $\mathfrak{R}_k$ and i_m from circuit, flux can be calculated as:

$$\Phi = \frac{\sum_m N_m i_m}{\sum_k \mathfrak{R}_k}$$

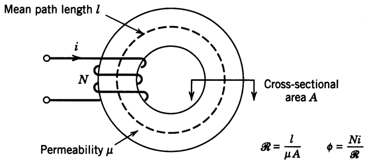

Figure 9 Magnetic Reluctance.

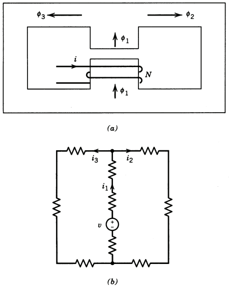

Figure 10 Magnetic-Electric Analogy.

FARADAY'S LAW, LENZ'S LAW, SELF-INDUCTANCE

For a stationary coil, convention indicates positive voltage where positive current enters, while for a chosen current direction, right-hand rule indicates positive flux direction

Faraday's Law relates a time varying flux linkage of the coil $N\Phi$ to induced voltage:

$$e = +\frac{d(N\Phi)}{dt} = N\frac{d\Phi}{dt}$$

Lenz's Law states that the current induced in a circuit due to a change or a motion in a magnetic field is so directed as to oppose the change in flux and to exert a mechanical force opposing the motion. Self-inductance of the coil is defined as:

$$L = \frac{N\Phi}{i} \Rightarrow N\Phi = Li$$

From Faraday's law we have for stationary coil:

$$e = L\frac{di}{dt} + i\frac{dL}{dt} = L\frac{di}{dt}$$

Considering $\Phi\mathfrak{R} = Ni$ we have:

$$L = \frac{N}{i}\frac{Ni}{\mathfrak{R}} = \frac{N^2}{\mathfrak{R}}$$

Coil inductance is a property of magnetic circuit and independent of i

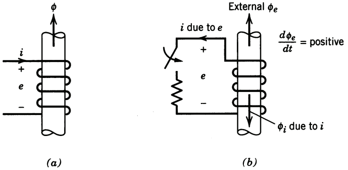

Figure 11 a) Flux direction and voltage polarity; b) Lenz's Law.

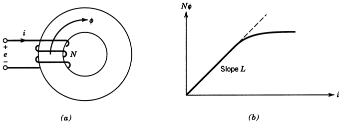

Figure 12 Self-inductance L

TRANSFORMER WITH LOSSLESS CORE (I)

A transformer consist of two or more coils that are magnetically coupled.

Total flux in coils 1 and 2 is given, respectively:

$$\Phi_1 = \Phi + \Phi_{l1}, \qquad \Phi_2 = -\Phi + \Phi_{l2}$$

where Φ_{l1}, Φ_{l2} are the leakage fluxes. The flux Φ links the two coils and is given by:

$$\Phi = \frac{N_1 i_1 - N_2 i_2}{\mathfrak{R}_c} = \frac{N_1 i_m}{\mathfrak{R}_c}$$

with $\mathfrak{R}_c$ being reluctance of the core, and i_m magnetizing current:

$$i_m = i_1 - \frac{N_2}{N_1} i_2$$

The leakage fluxes are given with:

$$\Phi_{l1} = \frac{N_1 i_1}{\mathfrak{R}_{l1}}, \qquad \Phi_{l2} = \frac{N_2 i_2}{\mathfrak{R}_{l2}}$$

with $\mathfrak{R}_{l1}, \mathfrak{R}_{l2}$ being Reluctances of the leakage paths

No matter how good is transformer design, certain portion of flux will leak outside the core

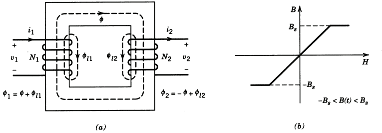

Figure 13 a) Cross Section; b) B-H characteristic.

TRANSFORMER WITH LOSSLESS CORE (II)

The voltages v_1, v_2 at the transformer terminals are given by:

$$v_1 = R_1 i_1 + N_1 \frac{d\Phi_1}{dt}, \qquad v_2 = -R_2 i_2 - N_2 \frac{d\Phi_2}{dt}$$

where R_1, R_2 account for the ohmic losses in the windings. Further development yields:

$$v_1 = R_1 i_1 + \frac{N_1^2}{\Re_{l1}} \frac{di_1}{dt} + \frac{N_1^2}{\Re_c} \frac{di_m}{dt}, \qquad v_2 = -R_2 i_2 - \frac{N_2^2}{\Re_{l2}} \frac{di_2}{dt} + \frac{N_1 N_2}{\Re_c} \frac{di_m}{dt}$$

We can simplify this by considering:

$$e_1 = \frac{N_1^2}{\Re_c} \frac{di_m}{dt} = L_m \frac{di_m}{dt}, \quad L_m = \frac{N_1^2}{\Re_c}, \quad L_{l1} = \frac{N_1^2}{\Re_{l1}}, \quad L_{l2} = \frac{N_2^2}{\Re_{l2}}$$

Resulting in:

$$v_1 = R_1 i_1 + L_{l1} \frac{di_1}{dt} + L_m \frac{di_m}{dt} = R_1 i_1 + L_{l1} \frac{di_1}{dt} + e_1$$

$$v_2 = -R_2 i_2 - L_{l2} \frac{di_2}{dt} + \frac{N_2}{N_1} e_1 = -R_2 i_2 - L_{l2} \frac{di_2}{dt} + e_2$$

This is the base for the transformer equivalent circuit

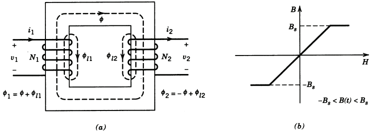

Figure 14 a) Cross Section; b) B-H characteristic.

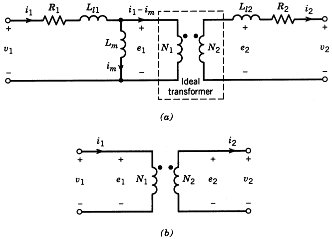

Figure 15 Equivalent circuits of a transformer

Ideal transformer circuit is result of several simplifications

- $R_1 = R_2 = 0$ - perfect conductors used for windings
- $\mathfrak{R}_c = 0 \Rightarrow \mu = \infty \Rightarrow L_m = \infty$
- $\mathfrak{R}_{l1} = \mathfrak{R}_{l2} = 0 \Rightarrow L_{l1} = L_{l2} = 0 \Rightarrow \Phi_{l1} = \Phi_{l2} = 0$

With these simplifications, transformer equivalent circuit reduces to ideal transformer:

$$v_1 = e_1, \quad v_2 = e_2 = \frac{N_2}{N_1} e_1 = \frac{N_2}{N_1} v_1 \Rightarrow \frac{v_1}{N_1} = \frac{v_2}{N_2}$$

In case of a real core with hysteresis characteristics, core losses are present

Core losses are modelled by equivalent resistance R_m in series or parallel to L_m

The total leakage inductance seen from one side, can be written as:

$$L_{l,total} = L_{l1} + L_{l2}^{'} \Rightarrow L_{l2}^{'} = (\frac{N_1}{N_2})^2 L_{l2}$$

Similarly, winding resistance can be reflected from the secondary side:

$$R_2^{'} = (\frac{N_1}{N_2})^2 R_2$$

The total leakage inductance is often specified in p.u. of the transformer apparent power

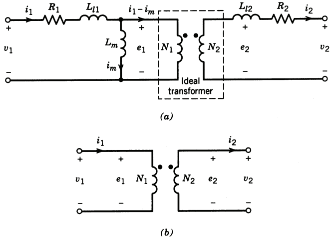

Figure 16 Equivalent circuits of a transformer

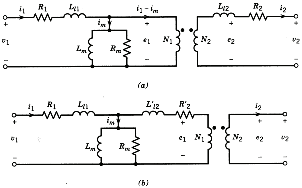

Figure 17 a) Core loss inc.; b) Reflected to prim.