# EE-365 - W1
# DC-DC CONVERTERS
# BUCK-BOOST, FLYBACK

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# DC-DC CONVERTERS

*Numerous topologies are available*

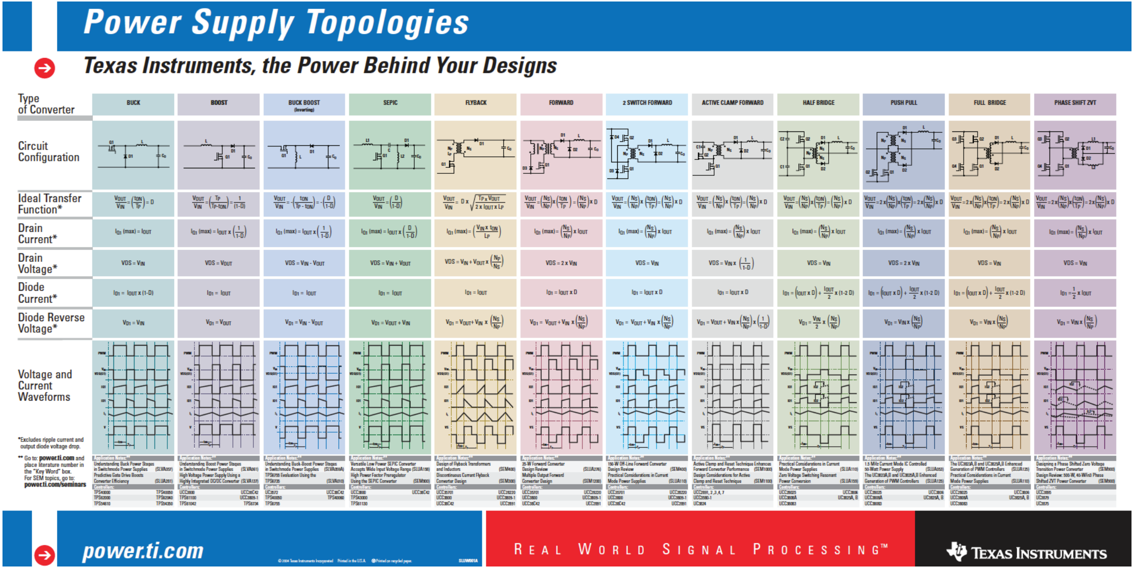


**Figure 1** Texas Instruments SMPS poster.

# BUCK-BOOST CONVERTER

*Non-isolated step-up-down DC-DC converter*

Third basic non-isolated converter is Buck-Boost:

- as the name indicates, output voltage can be lower or higher than input voltage
- polarity of output voltage is reversed
- it is used for many battery powered devices
- Buck-Boost converter could be obtained by cascade connection of Buck and Boost
- conversion ratio is then product of two converter ratios in cascade

This allows for output voltage to be lower or higher than input voltage

$$\frac{V_o}{V_d} = D\frac{1}{1-D}$$

During the $t_{on}$

- Switch is ON and Diode is OFF (reverse biased)
- input voltage source $V_d$ provides the energy to the inductor
- output capacitor is supplying the load

During the $t_{off}$

- Switch is OFF and Diode is ON
- energy stored in inductor is released to the load
- no energy is supplied by the input $V_d$

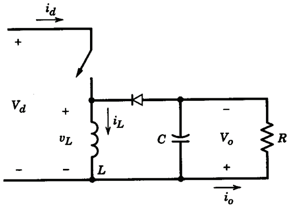


**Figure 2** Buck-Boost Converter.

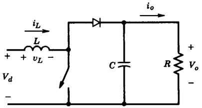


**Figure 3** Boost Converter.

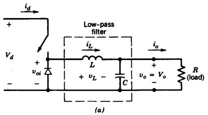


**Figure 4** Buck Converter.

During the CCM:

- inductor current $i_L$ is flowing continuously (it never reaches zero value)
- the average inductor current is equal to $I_L = I_d + I_o$

For the Buck-Boost converter we have:

$$v_L = L\frac{di_L}{dt}, \quad i_C = C\frac{dV_o}{dt}, \quad i_L(t_{on}) = i_d \quad i_L(t_{off}) = i_C + i_R = C\frac{dV_o}{dt} + \frac{V_o}{R}$$

During the $t_{on}$ - Switch is ON and Diode is OFF

$$v_L = V_d, \quad i_L = i_d, \quad i_C = C\frac{dV_o}{dt} = -i_o = -\frac{V_o}{R}$$

During the $t_{off}$ - Switch is OFF and Diode is ON

$$v_L = -V_o, \quad i_L = i_C + i_o = C\frac{dV_o}{dt} + \frac{V_o}{R}$$

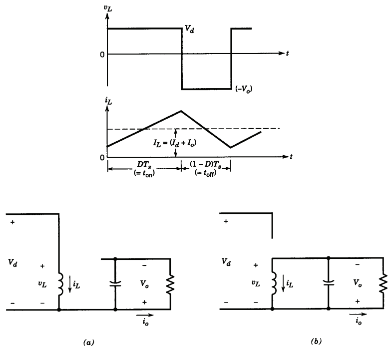


**Figure 5** Buck-Boost Converter during CCM.

Considering the steady state, the average voltage across the inductor is zero

$$\int_0^{T_s} v_L dt = 0 \quad \Rightarrow \quad \int_0^{t_{on}} v_L dt + \int_{t_{on}}^{T_s} v_L dt = 0 \quad \Rightarrow \quad V_d t_{on} - V_o t_{off} = 0$$

Finally, input-output voltage relation (transfer function) of a Boost converter is:

$$\frac{V_o}{V_d} = \frac{t_{on}}{t_{off}} = \frac{D}{1-D}$$

Neglecting the power losses:

$$P_d = P_o \Rightarrow V_d I_d = V_o I_o$$

$$\frac{I_o}{I_d} = \frac{V_d}{V_o} = \frac{1-D}{D}$$

At the boundary condition, inductor current goes to zero at the of a period (note that $I_o = I_L - I_d = (1-D)I_L$):

$$I_{LB} = \frac{1}{2} i_{L,peak} = \frac{V_d}{2L} t_{on} = \frac{T_s V_d}{2L} D = \frac{T_s V_o}{2L}(1-D) \quad \Rightarrow \quad I_{oB} = \frac{T_s V_o}{2L}(1-D)^2$$

With constant $V_o$, $I_{oB}$ can be plotted as function of duty cycle $D$, and we can calculate::

$$I_{LB,max}(D=0) = \frac{T_s V_o}{2L} \Rightarrow I_{LB} = (1-D) I_{LB,max}, \qquad I_{oB,max}(D=0) = \frac{T_s V_o}{2L} \Rightarrow I_{oB} = (1-D)^2 I_{oB,max}$$

For a given $D$ with constant $V_o$: if $I_o$ drops below $I_{oB}$ (or $I_L$ drops below $I_{LB}$) Buck-Boost converter changes its mode from CCM to DCM

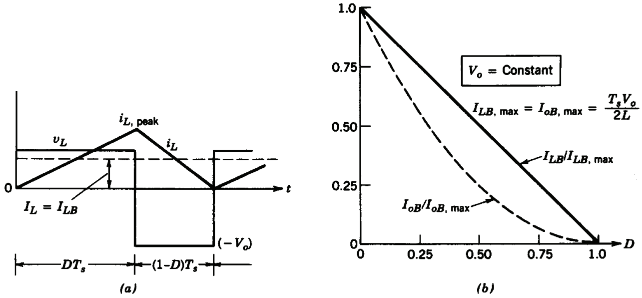


Figure 6 Current waveform at the boundary of CCM and DCM.

We assume that Buck-Boost converter:

- operates with constant $V_d$ and $D$ when a load decreases
- while $D$ would be modified by control, we assume that is not changed
- decrease of load ($P_o = P_d$) reduces inductor average current
- $t_{off}$ interval is split into two sub-intervals
- during $\Delta_1 T_s$ the inductor current is reducing to zero
- during $\Delta_2 T_s$ the inductor current/voltage is zero and load is supplied by capacitor

The average voltage across the inductor is zero

$$\int_0^{T_s} v_L dt = 0 \quad \Rightarrow \quad V_d D T_s + -V_o \Delta_1 T_s = 0 \quad \Rightarrow \quad \frac{V_o}{V_d} = \frac{D}{\Delta_1} = \frac{I_d}{I_o}$$

Inductor peak current and average currents are:

$$i_{L,peak} = \frac{V_d}{L} D T_s \quad \Rightarrow \quad I_L = i_{L,peak} \frac{D + \Delta_1}{2} = \frac{V_d}{2L} D T_s (D + \Delta_1)$$

Finally $D$ as function of load current for various $V_o/V_d$ is:

$$D = \frac{V_o}{V_d} \sqrt{\frac{I_o}{I_{oB,max}}}$$

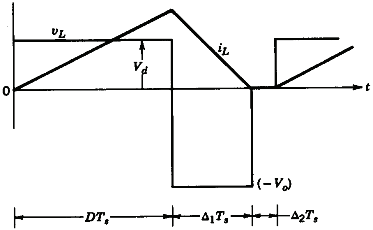


**Figure 7** Buck-Boost Converter during DCM.

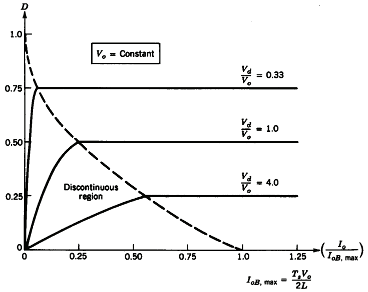


**Figure 8** Buck-Boost characteristic with constant $V_o$.

Considering the CCM operation, we can derive sizing rules for the $L$ and $C$

During $t_{on} = DT_s$ interval, current rise for $\Delta I_L$ (peak-to-peak ripple)

$$\Delta I_L = \frac{V_d}{L} DT_s$$

Inductor average current is:

$$I_L = I_d + I_o = \frac{D}{1-D} I_o + I_o = \frac{1}{1-D} I_o = \frac{V_o}{R} \frac{1}{1-D} = \frac{V_d}{R} \frac{D}{(1-D)^2}$$

$L$ can be selected as:

$$L = \frac{V_d}{\Delta I_L} DT_s = \frac{V_o}{\Delta I_L}(1-D)T_s$$

Output capacitor voltage ripple:

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o DT_s}{C} = \frac{V_o}{R} \frac{DT_s}{C}$$

From here we have:

$$\frac{\Delta V_o}{V_o} = \frac{DT_s}{RC} = \frac{DT_s}{\tau} \Rightarrow C = \frac{DT_s}{R} \frac{V_o}{\Delta V_o}$$

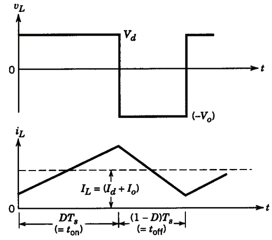


**Figure 9** Buck-Boost converter inductor current ripple.

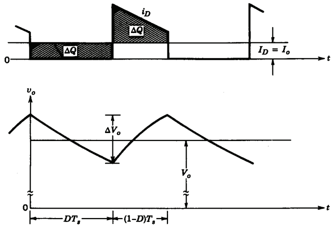


**Figure 10** Buck-Boost converter output voltage ripple.

# FLYBACK CONVERTER

*Using coupled inductors and air-gap for energy exchange...*

# FLYBACK CONVERTER

Flyback converter is derived from the Buck-Boost converter

- second winding is added to the inductor to achieve galvanic isolation
- this is not truly a transformer, but a two-winding inductor (observe the dots and secondary diode)
- primary current and secondary current do not flow at the same time - as in regular transformer
- energy is stored in the air gap and released to the output
- two-winding inductor thus achieves energy storage and galvanic isolation, at the same time
- we can still analyse circuit using the transformer notion
- magnetizing inductance design will not aim for the highest possible value
- Flyback converter is widely used SMPS topology for low power applications

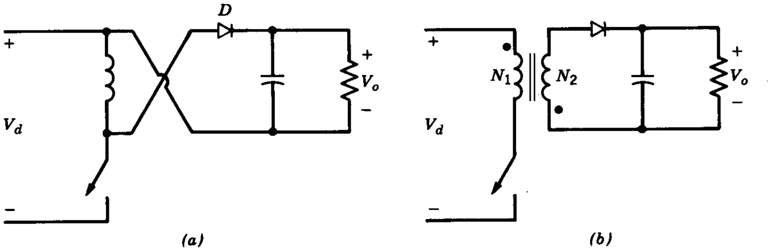


**Figure 11** Flyback converter.

Using transformer representation for two winding inductor:

During the $t_{on}$

- Switch is ON
- Diode is OFF (reverse biased due to winding polarities)
- output capacitor is supplying the load
- CCM mode implies incomplete demagnetization of the core
- inductor core flux increases linearly from the initial value

$$\Phi(t) = \Phi(0) + \frac{V_d}{N_1}t \quad \Rightarrow \quad \hat{\Phi} = \Phi(t_{on}) = \Phi_0 + \frac{V_d}{N_1}t_{on}$$

During the $t_{off}$

- Switch is OFF and Diode is ON
- voltage accross secondary winding is $v_2 = -V_o$
- energy is released to the secondary side
- inductor core flux decreases from the initial value

$$\Phi(t) = \hat{\Phi} - \frac{V_o}{N_2}(t - t_{on}) \quad \Rightarrow \quad \Phi(T_s) = \Phi_0 + \frac{V_d}{N_1}t_{on} - \frac{V_o}{N_2}(T_s - t_{on})$$

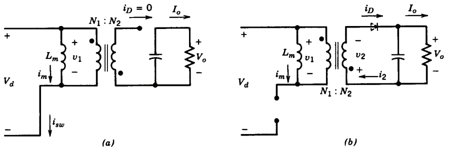


**Figure 12** Flyback converter: a) switch ON; b) switch OFF.

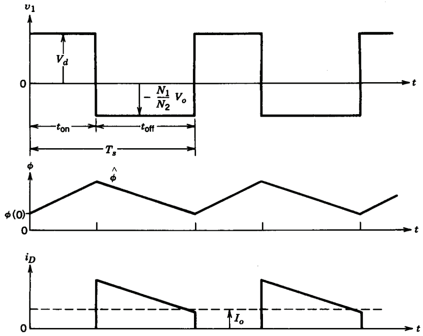


**Figure 13** Flyback converter waveforms.

Considering that net change of flux through the core iz zero in steady state:

$$\Phi(T_s) = \Phi(0) \quad \Rightarrow \quad \frac{V_o}{V_d} = \frac{N_2}{N_1}\frac{D}{1-D}$$

Voltage transfer ratio of Flyback converter:

- depends on duty cycle $D$ as for Buck-Boost converter
- transformer turns ratio allow for additional voltage adaptation

During the $t_{on}$ we have $v_1 = V_d$ and rise of inductor current:

$$i_m(t) = i_{sw}(t) = I_m(0) + \frac{V_d}{L_m}t \quad \Rightarrow \quad \hat{I}_m = \hat{I}_{sw} = I_m(0) + \frac{V_d}{L_m}t_{on}$$

During the $t_{off}$ we have $v_1 = -(N_1/N_2)V_o$ and $i_{sw} = 0$

Inductor current $i_m$ and diode current $i_D$ can be expressed as:

$$i_m(t) = \hat{I}_m - \frac{V_o(N_1/N_2)}{L_m}(t - t_{on})$$

$$i_D(t) = \frac{N_1}{N_2}i_m(t) = \frac{N_1}{N_2}\left[\hat{I}_m - \frac{V_o(N_1/N_2)}{L_m}(t - t_{on})\right]$$

With average diode current being $I_o$ we have switch stresses as:

$$\hat{I}_m = \hat{I}_{sw} = \frac{N_2}{N_1}\frac{1}{1-D}I_o + \frac{N_1}{N_2}\frac{(1-D)T_s}{2L_m}V_o, \quad v_{sw} = V_d + \frac{N_1}{N_2}V_o = \frac{V_d}{1-D}$$

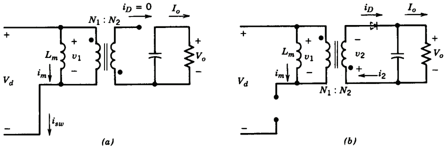


Figure 14 Flyback converter: a) switch ON; b) switch OFF.

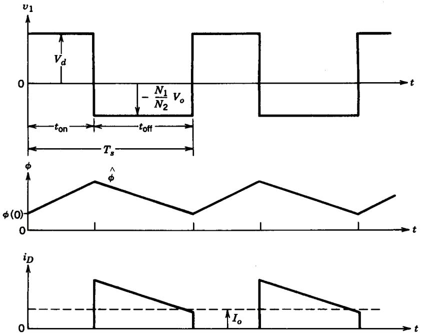

Figure 15 Flyback converter waveforms.

# VOLTAGE TRANSFER FUNCTION, DUTY CYCLE RANGE

- Input-output voltage transfer function $\frac{U_{out}}{U_{in}} = f\left(D\right)$ of a Flyback converter is already derived
- it is easy to solve it for the duty cycle $D = f\left(U_{in}, U_{out}\right)$
- IC PWM controller has limited maximum duty cycle
- Input voltage has minimum, rated and maximum value

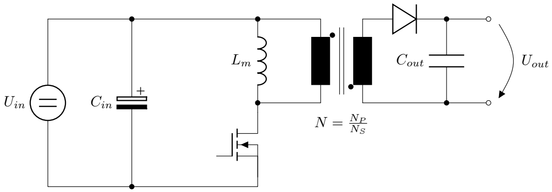


**Figure 16** The power path of a Flyback Converter (leakage inductance and auxiliary supply windings are now shown).

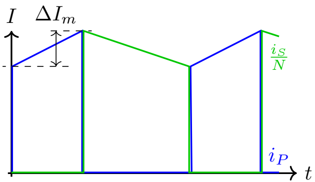


**Figure 17** Current of the Flyback Converter.

- Blue - Primary MOSFET current waveform
- Green - Secondary DIODE current waveform
- Magnetizing current - triangular current waveform with ripple $\Delta I_m$
- While the structured looks like a transformer, these are coupled inductors

# MAGNETIZING INDUCTANCE, CCM, DCM

- Flyback should operate in CCM until $P_{out,min,CCM}$
- DCM operation below that power is OK
- Recall equations for primary side current (at CCM-DCM border it starts from zero)
- Recall simple geometry of a triangle - relation between average and peak value of current

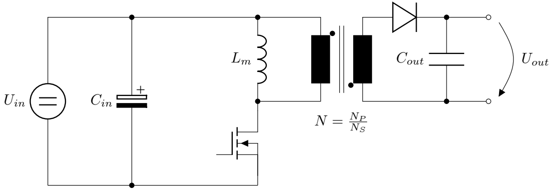


**Figure 18** The power path of a Flyback Converter (leakage inductance and auxiliary supply windings are now shown).

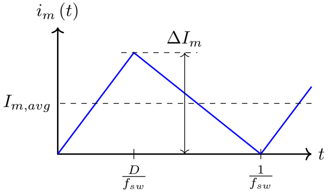


**Figure 19** Inductor curent $i_m(t)$ at the border to discontinuous conduction mode.

Magnetizing current peak and Average value are:

$$\Delta I_m = \frac{U_{in}}{L_m} \cdot t_{on} = \frac{U_{in}}{L_m} \cdot \frac{D}{f_{sw}} \qquad I_{m,avg} = \frac{P_{out}}{U_{in}} \cdot \frac{1}{D}$$

$L_m$ can be calculated considering current shape at the CCM-DCM border and:

$$\Delta I_m = 2 \cdot I_{m,avg}$$

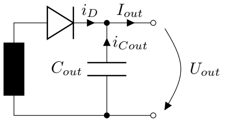


**Figure 20** The secondary side with the relevant currents.

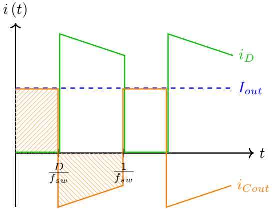


**Figure 21** Kirchhoff's current low for the output capacitor.

- Output capacitance $C_{out}$ should fulfill requested ripple requirements
- Worst operating conditions should be considered
- In the steady state, the capacitor voltage $u_{Cout} = \frac{1}{C_{out}} \cdot \int \{i_{Cout}\}dt$ has periodic ripple - net voltage gain should be zero
- voltage also has average value - output voltage
- Two shaded areas, representing charge, have to be identical - choose one

$$Q_{Cout,1} = I_{out} \cdot \frac{D}{f_{sw}} = \frac{P_{out}}{U_{out}} \cdot \frac{U_{out}/U_{in}}{U_{out}/U_{in} + 1/N} \cdot \frac{1}{f_{sw}}$$

$$C_{out} = \frac{Q_{Cout,1}}{\Delta U} = \frac{P_{out}}{U_{out}} \cdot \frac{U_{out}/U_{in}}{U_{out}/U_{in} + 1/N} \cdot \frac{1}{f_{sw} \cdot \Delta U_{out}}$$

- the charge $Q$ – and thus the ripple – increase with $P_{out}$ - Choose correctly power for design calculations!
- Input voltage $U_{in}$ also has an effect one the $C_{out}$ - Choose correctly input voltage for design calculations!
- You should fulfill requirements under all operating conditions

# INPUT CAPACITANCE, HOLD-UP TIME, SAFETY DISCHARGE

DC-DC converters are often part of a larger power supply system connected to the AC supply through a rectifier

- input rectifier is connected to AC supply line ($50Hz$) and provides DC voltage as input to DC-DC stage
- rectifier normally has Capacitor at the output, smoothing DC voltage at the input od DC-DC converter
- DC-DC stage regulates the output DC voltage, and operates correctly with defined input voltage range ($V_{in-min}, V_{in-max}$)
- AC network may have problems, and there may be complete loss of AC power supply for some time
- often, a loss of a half-cycle ($10ms$) of AC supply, should not be visible at the output of DC-DC converter
- **HOLD-UP** time is defined as the time required for the output voltage to remain within regulation after the AC input voltage is removed
- capacitor between AC-DC and DC-DC stage must be sized to buffer loss of AC supply during half-cycle ($t_{UP} = 10ms$)
- Your task 1: **Calculate C** for scenario "adapted" for your problem (simplified problem, since efficiency is considered as 100%)
- Your task 2: **Calculate R** to discharge C, in defined time, once we turn off everything ($\tau = RC$, it takes roughly $5\tau$ to discharge)

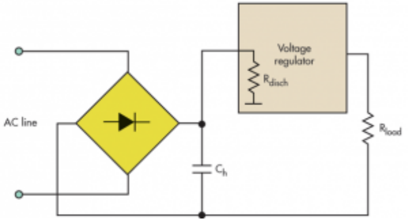


**Figure 22** Simplified illustration of DC-DC converter fed from a AC-DC converter

Energy stored in capacitor $C$ at voltage $V$:

$$E = \frac{1}{2}CV^2$$

Energy difference in capacitor $C$ due to voltage drop:

$$\Delta E = \frac{1}{2}C(V_{high}^2 - V_{low}^2)$$

Energy needed to supply output rated power:

$$\Delta E = P_{out} \cdot t_{UP}$$

**Adequate MOSFET and DIODE should be selected considering:**

- Maximum voltage stress of the MOSFET in the circuit
- Maximum current of the MOSFET in the circuit
- Maximum voltage stress of the DIODE in the circuit
- Maximum current stress of the DIODE in the circuit

- Calculate theoretically from the circuit
- Use PLECS to verify (if in doubt)
- Determined values are minimum requirements
- List of devices is provided to you
- Choose final devices accordingly

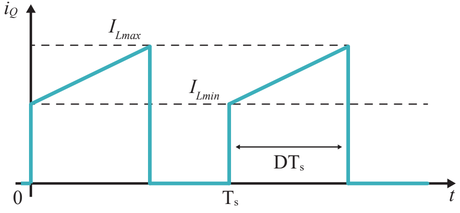


**Figure 23** Current waveform of the MOSFET in the steady-state

**Expressions for RMS and AVERAGE current can be easily derived**

- recall basic definition for RMS calculation

$$I_{rms} = \sqrt{\frac{1}{T_s}\int_0^{T_s} i^2(t)dt}$$

- recall basic definition for AVG calculation

$$I_{avg} = \frac{1}{T_s}\int_0^{T_s} i(t)dt$$

- from the typical current waveforms define $i(t)$ for MOSFET
- from the typical current waveforms define $i(t)$ for DIODE
- these calculations are available in many textbooks

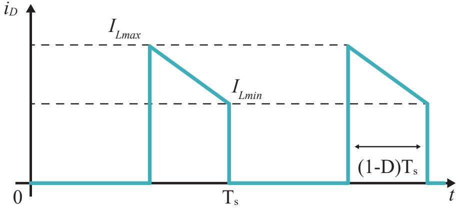


**Figure 24** Current waveforms of the diode in the steady-state

# OTHER FLYBACK CONVERTER TOPOLOGIES

Two-Switch Flyback Converter

- $T_1$ and $T_2$ are turned ON and OFF simultaneously
- voltage ratings of switches is one half of the the single-switch version
- diodes provide freewheeling path to dissipate energy stored in leakage inductance

Paralleling Flyback Converters

- added redundancy in the system
- increased effective switching frequency seen at the output
- modularity of design

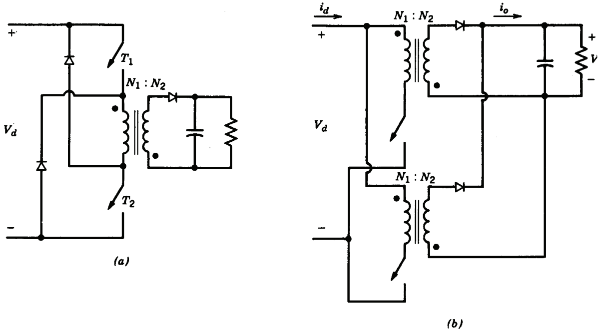


**Figure 25** a) Two-switch Flyback converter; b) paralleled Flyback converters.

# TWO-SWITCH FLYBACK CONVERTER

*Minimizing the impact of leakage inductance*

# 2-SWITCH FLYBACK CONVERTER

Single-switch Flyback suffers from:

- high turn-off voltage stress of primary switch
- ringing due to leakage inductance and switch output capacitance
- energy stored in the leakage inductance
- increased switching loss
- need for RCD clamp, further increasing complexity and losses

Two-switch Flyback:

- increased complexity: 2 MOSFETs, 2 Diodes
- diodes effectively clamp and reduce switching overvoltages
- diodes provide return path for the leakage inductance discharge
- MOSFET turn off loss is reduced

Material is adopted from the paper:
D. Murthy-Bellur, M.K. Kazimierczuk, "Two-switch flyback PWM DC-DC converter in continuous-conduction mode", Int. J. Circ. Theor. Appl. 2011; 39: 1145-1160

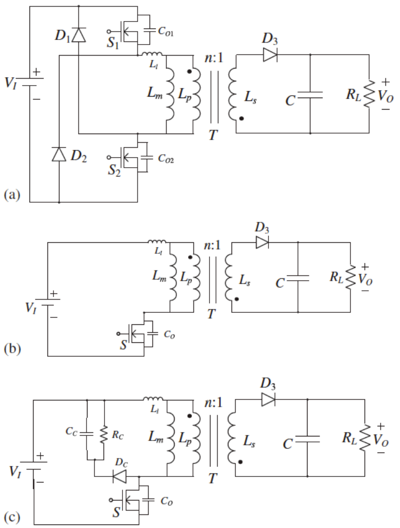


**Figure 26** (a) two-switch flyback; (b) single-switch flyback;(c) single-switch flyback PWM DC-DC converter with RCD clamp.

# 2-SWITCH FLYBACK CONVERTER - CCM

## Relevant waveforms in CCM in steady state

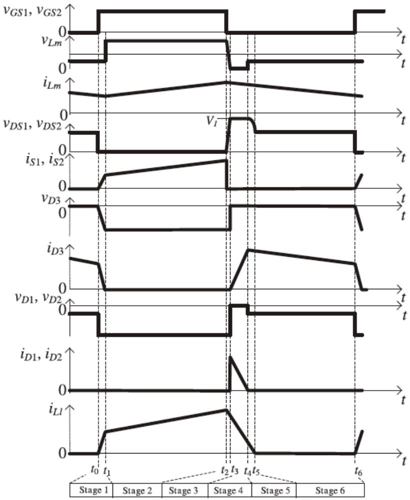


**Figure 27** Voltage and current waveforms of the two-switch flyback in CCM

## 6 stages (intervals) during the switching period

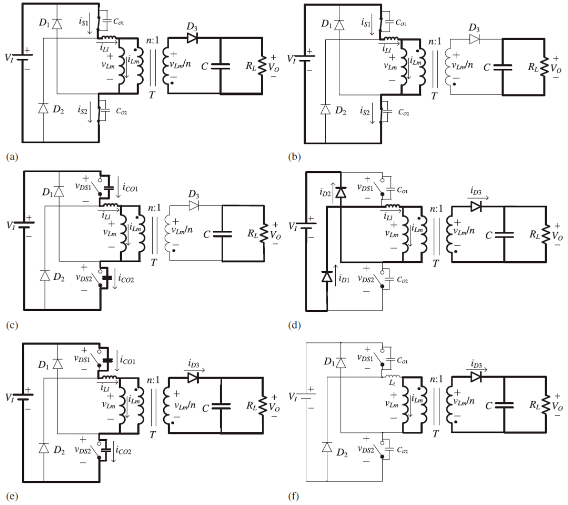


**Figure 28** Equivalent circuits of the two-switch flyback

## Nomenclature

- Active switches: $S_1$ and $S_2$
- Diodes: $D_1, D_2$ and $D_3$
- Magnetizing inductance: $L_m$
- Total leakage inductance: $L_l$
- Output capacitor: $C$
- Output capacitance of switches: $C_{O1}$ and $C_{O2}$
- Transformer turns ratio: $n$
- Input DC voltage: $V_I$
- Output DC voltage: $V_O$
- Load resistance: $R_L$
- Switching period: $T_s$

## 6 stages (intervals) during the switching period

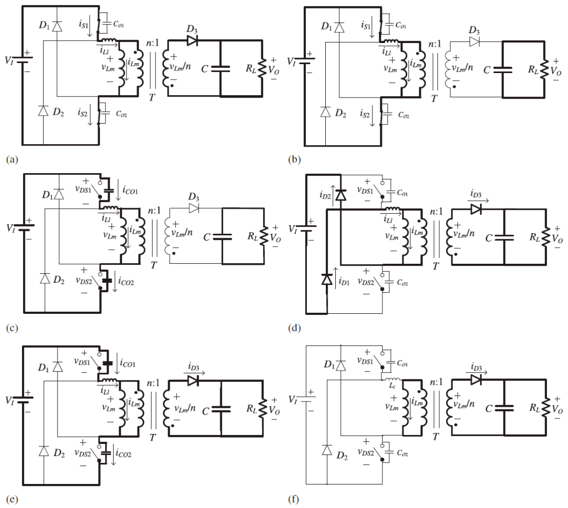

Figure 29 Equivalent circuits of the two-switch flyback

During the time interval $[t_0 < t \leq t_1]$

- $D_1$ and $D_2$ are OFF, $D_3$ is ON (conducting) and $S_1$ and $S_2$ are **turned ON**
- due to leakage inductance $L_l$, magnetizing current $i_{L_m}$ cannot be instantaneously transferred from the secondary to the primary
- voltage across clamping diodes $D_1$ and $D_2$ is: $v_{D1} = v_{D2} = -V_I$
- $D_1$ and $D_2$ are reversed biased (blocking), hence: $i_{D1}(t) = i_{D2}(t) = 0$
- the voltage across magnetizing inductance $L_m$ is: $v_{L_m} = -nV_O$. Hence:

$$i_{L_m}(t) = \frac{nV_O}{L_m}(t - t_0) + i_{L_m}(t_0), \quad i_{L_m}(t_0) = init.v.(CCM)$$

- the voltage across leakage inductance $L_l$ is $V_{L_l} = V_I + nV_O$
- the leakage inductance $L_l$ limits the rise of current through $S_1$ and $S_2$

$$i_{S_1}(t) = i_{S_2}(t) = i_{L_l}(t) = \underbrace{\frac{V_I + nV_O}{L_l}}_{\text{slope}}(t - t_0) + i_{L_l}(t_0), \quad i_{L_l}(t_0) = 0$$

- the current through $D_3$ is:

$$i_{D_3}(t) = -n(i_{L_l}(t) - i_{L_m}(t)) = -n\frac{V_I L_m + nV_O(L_m + L_l)}{L_m L_l}(t - t_0) + n i_{L_m}(t_0)$$

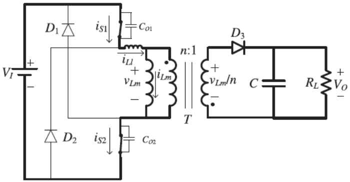


**Figure 30** Equivalent circuit during the Stage 1

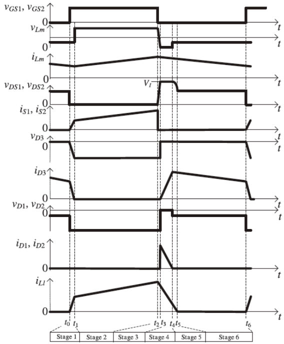


**Figure 31** Voltage and current waveforms...

# 2-SWITCH FLYBACK CONVERTER - STAGE 1 (CONT.)

During the time interval $[t_0 < t \leq t_1]$

- the current through $D_3$ is:

$$i_{D_3}(t) = -n(i_{L_l}(t) - i_{L_m}(t)) = -n\frac{V_I L_m + nV_O(L_m + L_l)}{L_m L_l}(t - t_0) + n i_{L_m}(t_0)$$

- in a good design $L_m \gg L_l$ and simplification yields:

$$i_{D_3}(t) = \underbrace{-n\frac{V_I + nV_O}{L_l}}_{\text{slope}}(t - t_0) + n i_{L_m}(t_0)$$

- the voltages across switches are: $v_{S_1} = v_{S_2} = 0$
- the stage 1 ends at the time $t = t_1$ when the $i_{L_l} = i_{L_m}$ and $i_{D_3}(t) = 0$
- substituting $i_{D_3}(t_1) = 0$ into simplified expression yields duration of the stage as:

$$\Delta t_1 = t_1 - t_0 = \frac{L_l}{V_I + nV_O} i_{L_m}(t_0)$$

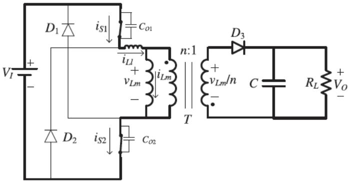


**Figure 32** Equivalent circuit during the Stage 1

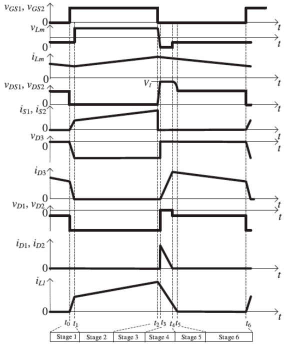


**Figure 33** Voltage and current waveforms...

# 2-SWITCH FLYBACK CONVERTER - STAGE 2

During the time interval $[t_1 < t \leq t_2]$

- $S_1$ and $S_2$ are ON; all diodes $D_1$, $D_2$ and $D_3$ are OFF
- the output load $R_L$ is supplied by the output capacitor $C$
- the voltages across switches are: $v_{S_1} = v_{S_2} = 0$
- the primary side current is:

$$i_{S_1}(t) = i_{S_2}(t) = i_{L_l}(t) = i_{L_m}(t) = \underbrace{\frac{V_I}{L_m + L_l}}_{\text{slope}}(t - t_1) + i_{L_m}(t_1)$$

- the peak current of the magnetizing inductance $L_m$ is:

$$i_{L_m}(t_2) = \frac{V_I D T_s}{L_m + L_l} + i_{L_m}(t_1) = \frac{V_I D}{f_s(L_m + L_l)} + i_{L_m}(t_1)$$

- the peak-to-peak value of the ripple current through magnetizing inductance is

$$\Delta i_{L_m} = i_{L_m}(t_2) - i_{L_m}(t_1) = \frac{V_I D}{f_s(L_m + L_l)}$$

- the stage 2 ends at the time $t = t_2$ when both switches $S_1$ and $S_2$ are **turned OFF**

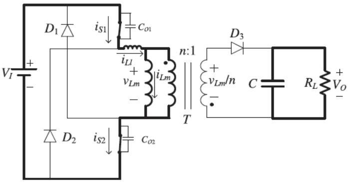


**Figure 34** Equivalent circuit during the Stage 2

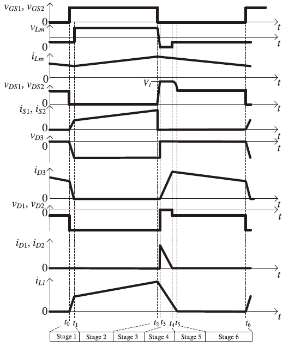


**Figure 35** Voltage and current waveforms...

# 2-SWITCH FLYBACK CONVERTER - STAGE 3

During the time interval $[t_2 < t \leq t_3]$

- $S_1, S_2, D_1, D_2$ and $D_3$ are all OFF; the $R_L$ is supplied by the output capacitor $C$
- the magnetizing current (equal to leakage current) is charging output capacitances $C_{O1}$ and $C_{O2}$ of primary switches $S_1$ and $S_2$
- an LC resonant circuit is formed due to opening of switches $S_1$ and $S_2$
- the primary side current is:

$$i_{C_{O1}}(t) = i_{C_{O2}}(t) = i_{L_l}(t) = i_{L_m}(t) = \frac{V_I}{Z_1}\sin\omega_1(t-t_2) + i_{L_m}(t_2)\cos\omega_1(t-t_2)$$

- where:

$$Z_1 = \sqrt{\frac{(L_m + L_l)(C_{O1}+C_{O2})}{C_{O1}C_{O2}}} \qquad \omega_1 = \sqrt{\frac{C_{O1}+C_{O2}}{(L_m+L_l)C_{O1}C_{O2}}}$$

- assuming identical output capacitances $C_{O1} = C_{O2} = C_O$, the switch voltages are:

$$v_{DS1}(t) = v_{DS2}(t) = v_{C_{O1}}(t) = v_{C_{O2}}(t) = \frac{i_{L_m}(t_2)Z_1}{2}\sin\omega_1(t-t_2) + \frac{V_I}{2}(t_2)\cos\omega_1(t-t_2)$$

- the stage 3 ends at the time $t = t_3$ when the voltage across each switch $S_1$ and $S_2$ equals $V_I$, resulting in clamping diodes $D_1, D_2$ (and $D_3$) being **turned ON**

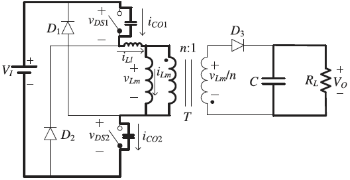

Figure 36 Equivalent circuit during the Stage 3

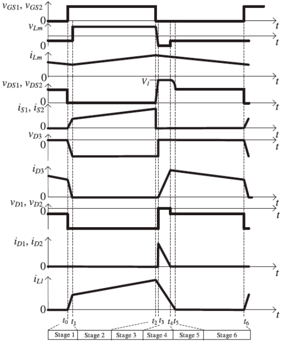


Figure 37 Voltage and current waveforms...

# 2-SWITCH FLYBACK CONVERTER - STAGE 4

During the time interval $[t_3 < t \leq t_4]$

- $S_1, S_2$ are OFF, all diodes $D_1, D_2$ and $D_3$ are ON
- the voltage across each switch is clamped to $V_I + V_F$, where $V_F$ is the forward voltage drop across the clamping diode
- the current through the leakage inductance is charging the input voltage suource $V_I$ through the diodes $D_1$ and $D_2$

$$i_{D1}(t) = i_{D2}(t) = i_{L_l}(t) = \underbrace{-\frac{V_I - nV_O}{L_l}}_{\text{slope}}(t - t_3) + i_{L_l}(t_3)$$

- where $i_{L_l}(t_3)$ is the initial leakage inductance current
- this is regenerative clamping mode
- the stage 4 ends at the time $t = t_4$ when the rectifier diode current $i_{D3}(t)$ equals the reflected magnetizing current $nI_{L_m}$, which results in diodes $D_1$ and $D_2$ being **turned OFF**

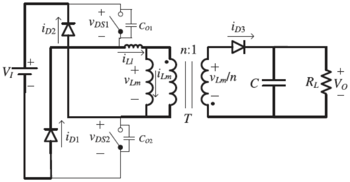

Figure 38 Equivalent circuit during the Stage 4

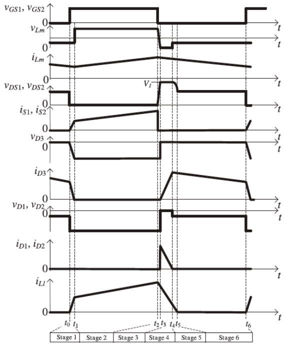

Figure 39 Voltage and current waveforms...

# 2-SWITCH FLYBACK CONVERTER - STAGE 5

During the time interval $[t_4 < t \leq t_5]$

- $S_1, S_2, D_1$, and $D_2$ are all OFF; $D_3$ is ON
- the magnetizing inductance $L_m$ is clamped due to conduction of $D_3$
- the resonant current on the the primary is:

$$i_{L_l}(t) = i_{C_{O1}}(t) = i_{C_{O2}}(t) = i_{L_l}(t_4)\cos\omega_2(t - t_4)$$

- where $i_{L_l}(t_4)$ is the initial leakage inductance current
- the voltage across the leakage inductance $L_l$ is

$$v_{L_l}(t) = -i_{L_l}(t_4) Z_2 \sin\omega_2(t - t_4)$$

$$Z_2 = \sqrt{\frac{L_l(C_{O1} + C_{O2})}{C_{O1}C_{O2}}} \qquad \omega_2 = \sqrt{\frac{C_{O1} + C_{O2}}{L_l C_{O1} C_{O2}}}$$

- assuming identical output capacitances $C_{O1} = C_{O2} = C_O$, the switch voltages are:

$$v_{DS1}(t) = v_{DS2}(t) = v_{C_{O1}}(t) = v_{C_{O2}}(t) = V_I - i_{L_l}(t_4)\sin\omega_2(t - t_4)$$

- the stage 5 ends at the time $t = t_5$ when the leakage currents $i_{L_l}(t)$ drops to zero

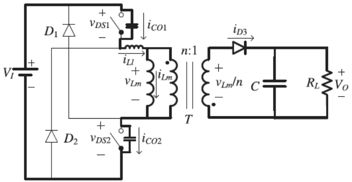


**Figure 40** Equivalent circuit during the Stage 5

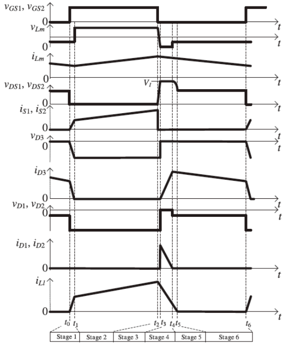


**Figure 41** Voltage and current waveforms...

# 2-SWITCH FLYBACK CONVERTER - STAGE 6

During the time interval $[t_5 < t \leq t_6]$

- $S_1, S_2, D_1$ and $D_2$ are OFF, $D_3$ is ON
- resonance between $L_l, C_{O1}$ and $C_{O2}$ has stopped
- the voltage across magnetizing inductance $L_m$ is: $v_{L_m} = -nV_O$
- Magnetizing current and rectifier diode $D_3$ current are:

$$i_{L_m}(t) = \underbrace{-\frac{nV_O}{L_m}}_{\text{slope}}(t - t_5) + i_{L_m}(t_5), \qquad i_{D_3}(t) = \underbrace{-\frac{n^2 V_O}{L_m}}_{\text{slope}}(t - t_5) + n i_{L_m}(t_5)$$

- the voltages across the switches $S_1$ and $S_2$, and clamping diodes $D_1$ and $D_2$ are:

$$v_{DS1}(t) = v_{DS2}(t) = \frac{V_I + nV_O}{2} \qquad v_{D1}(t) = v_{D2}(t) = \frac{nV_O - V_I}{2}$$

- there are no currents on the primary side
- the stage 6 ends at the time $t = t_6$ when the switches $S_1$ and $S_2$ are **turned ON**

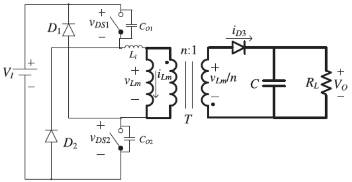


Figure 42 Equivalent circuit during the Stage 6

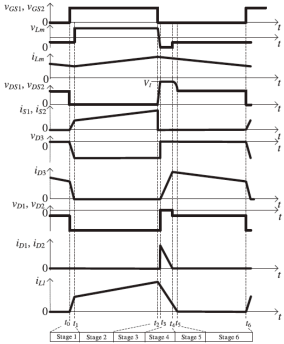


Figure 43 Voltage and current waveforms...

# DC VOLTAGE TRANSFER FUNCTION

**Relevant waveforms in CCM in steady state**

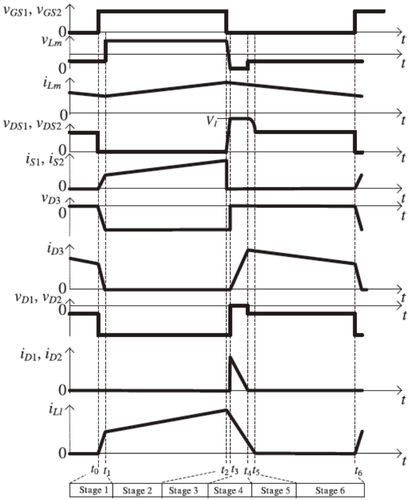


**Figure 44** Voltage and current waveforms of the two-switch flyback in CCM

Input-output voltage relation:

- Applying the volt-second balance to the magnetizing inductance voltage

$$0 = V_I(\Delta t_2 - \Delta t_3 - \Delta t_4) - nV_O(\Delta t_1 + \Delta t_5 + \Delta t_6)$$

$$\frac{V_O}{V_I} = \frac{\Delta t_2 - \Delta t_3 - \Delta t_4}{n(\Delta t_1 + \Delta t_5 + \Delta t_6)}$$

- neglecting duration of stages 1, 3, 4, and 5, as it is typically much smaller in comparison to duration of stages 2 and 6, one has:

$$\frac{V_O}{V_I} = \frac{\Delta t_2}{n\Delta t_6} = \frac{D}{n(1 - D)}$$

- the same voltage transfer function as for the single switch flyback

# DEVICE STRESSES

**Relevant waveforms in CCM in steady state**

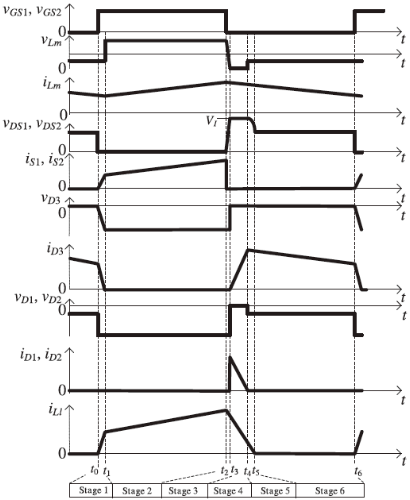


**Figure 45** Voltage and current waveforms of the two-switch flyback in CCM

Primary side switches $S_1$ and $S_2$

- maximum voltage appears during the stage 4
- maximum current appears during the stage 2

$$V_{S1,2(max)} = V_{I(max)}, \qquad I_{S1,2(max)} = \frac{I_{Omax}}{n(1-D_{max})} + \frac{\Delta I_{L_m}}{2}$$

Clamping diodes $D_1$ and $D_2$

- maximum voltage appears during the stage 1 and 2
- maximum current appears during the stage 4

$$V_{D1,2(max)} = V_{I(max)}, \qquad I_{D1,2(max)} = \frac{I_{Omax}}{n(1-D_{max})} + \frac{\Delta I_{L_m}}{2}$$

Rectifier diode $D_3$

- maximum voltage appears during the stage 2
- maximum current appears during the stage 4

$$V_{D3(max)} = \frac{V_I}{n} + V_O, \qquad I_{D3(max)} = \frac{I_{Omax}}{1-D_{max}} + \frac{n\Delta I_{L_m}}{2}$$