Power Systems Analysis,
Mock-up Exam,
Part II

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Questions:
3 points for this section

Remember to write down the calculations and eventual considerations that allowed you to derive the result. For part II, you can get a maximum of ⇒ +3 points and a minimum of ⇒ 0 point.
⟵ Insert your SCIPER number on the left and write your full name below. ↓

Surname and Name:

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Exercise

In the event of a massive blackout, the Transmission System Operator (TSO) is responsible for restoring (black-start) a power station to operation without relying on the external electric power transmission network. You are asked to study the black-start process of a small generator G_1 supplying auxiliary power to a larger power station (represented with the load U_1) located 100 km from the generator. The circuit is visible in Figure 1.

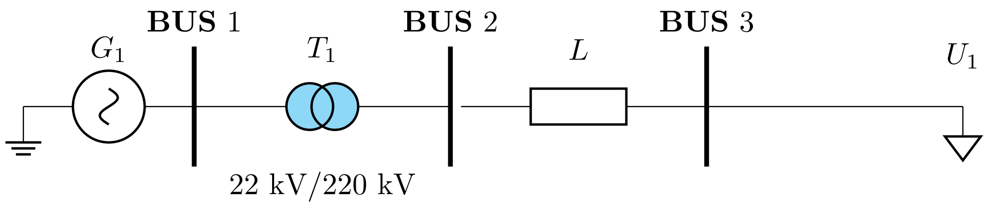

Figure 1: Single phase equivalent circuit of the power grid used during the black-start process.

Questions:

1. **Case 1:** The generator supplies the transformer T_1 and the line L, but the load is disconnected. The voltage at the end of the line (BUS 3) is 220 kV.
   (i) Compute the ABCD parameters of the line, with no approximation.
   (ii) Calculate voltage, active and reactive power at the start of the line (BUS 2).
   (iii) Determine the voltage at the generator terminal (BUS 1) and its internal EMF.
2. **Case 2:** Consider the load U_1 to be supplied, with a voltage of 220 kV. The load is consuming 150 MVA at a power factor of 0.95 lagging.
   (i) Calculate voltage, active and reactive power at the start of the line (BUS 2).
   (ii) Calculate the voltage at the generator terminal (BUS 1) and its internal EMF.
3. **Step 3:** For both the loading conditions of Case 1 and Case 2, draw the phasor diagram of the generator and elaborate on the requirements of the synchronous generator excitation system to withstand both no load and supply conditions.

Little help: The TSO is advising to perform all the calculations in p.u., with the bases:

$$V_{1b} = 22 \text{ kV}, \quad V_{2b} = V_{3b} = 220 \text{ kV}, \quad S_b = 300 \text{ MW}.$$

and reminding you that to obtain the per-unit power, knowing the per-unit voltage $\bar{v}$ and the per-unit current $\bar{i}$, you simply have to do: $\bar{s} = \bar{v}\bar{i}^*$.

Data:

The characteristics of L, T_1 and G_1 are contained in Table 1, 2 and 3, respectively.

Parameter	Value
Resistance (r)	$0.020 \cdot 10^{-3}$ [Ohm/m]
Reactance (x)	$0.268 \cdot 10^{-3}$ [Ohm/m]
Susceptance (b)	$4.300 \cdot 10^{-9}$ [S/m]
Conductance (g)	$0.007 \cdot 10^{-9}$ [S/m]
Length (l)	$100.0 \cdot 10^{3}$ [m]

Table 1: Nominal Parameters of Transmission Line

Parameter	Value
Nominal Voltage Primary (V_1)	22 kV
Nominal Voltage Secondary (V_2)	220 kV
Nominal Power (S_n)	300 MVA
Short Circuit Voltage (v_{cc})	10%
Short Circuit Resistance (r_{cc})	0

Table 2: Nominal Parameters of Transformer T_1

Parameter	Value
Nominal Power (S_n)	250 MVA
Nominal Voltage (V_n)	20 kV
Synchronous Reactance (x_s)	1.7241 pu

Table 3: Nominal Parameters of Generator G_1

Solution

Case 1:

The generator supplies the transformer T_1 and the line L, but the load is disconnected. The voltage at the end of the line (BUS 3) is 220 kV.

1. **Voltage at the end of the line (BUS 3):**

   With the following bases, chosen by the TSO:

$$V_{1b} = 22 \text{ kV}, \quad V_{2b} = V_{3b} = 220 \text{ kV}, \quad S_b = 300 \text{ MW}.$$

   the consequent base currents for the two sides of Transformer 1 are:

$$I_{1b} = \frac{S_b}{\sqrt{3}V_{1b}} = 7.87\,\text{kA}$$
$$I_{2b} = \frac{S_b}{\sqrt{3}V_{2b}} = 0.787\,\text{kA}$$

   while for the impedance:

$$Z_{1b} = \frac{V_{1b}^2}{S_b} = 1.61\,\Omega$$
$$Z_{2b} = \frac{V_{2b}^2}{S_b} = 161\ \ \Omega$$

   For line L, the impedance per-unit of length of the line is:

$$\bar{z} = r + jx = [0.020 + j0.268] \cdot 10^{-3} \quad [\text{Ohm/m}]$$

   and given a line of 100 km:

$$\bar{Z} = (r + jx) \cdot 100 \cdot 10^3 = 2.00 + j26.80 \quad [\text{Ohm}]$$

   Similarly, for the admittance:

$$\bar{y} = g + jb = [0.007 + j4.30] \cdot 10^{-9} \quad [\text{S/m}]$$

   and given a line of 100 km:

$$\bar{Y} = (g + jb) \cdot 100 \cdot 10^3 = [0.7 + j430] \cdot 10^{-6} \quad [\text{S}]$$

   The propagation constant $\bar{\gamma}$ is:

$$\bar{\gamma} = \sqrt{\bar{z} \cdot \bar{y}} = [0.0409 + j1.0742] \cdot 10^{-6} \quad [m^{-1}]$$
$$\bar{\gamma}L = \sqrt{\bar{Z} \cdot \bar{Y}} = 0.0041 + j0.1074 \qquad\qquad [\quad]$$

   and the characteristic impedance Z_0:

$$\bar{Z}_0 = \sqrt{\frac{\bar{z}}{\bar{y}}} = [249.83 - j9.11] \quad [\text{Ohm}]$$

so that:

$$\begin{aligned}\alpha = \Re(\bar{\gamma}) &= 0.0409 \cdot 10^{-6} \quad [m^{-1}]\\ \beta = \Im(\bar{\gamma}) &= 0.0615 \cdot 10^{-3} \quad [^\circ \cdot m^{-1}]\\ \text{abs}(Z_0) &= 249.99 \quad [\text{Ohm}]\\ \text{angle}(Z_0) &= -2.08 \quad [^\circ]\end{aligned}$$

Knowing the $\bar{\gamma}$ and $\bar{Z}_0$ we can compute the 4 constants of the line, without any approximation:

$$\begin{cases}\bar{A} = \cosh \bar{\gamma} L & = 0.9942 + j0.0004\\ \bar{B} = \bar{Z}_0 \sinh \bar{\gamma} L & = 1.9922 + j26.7489\\ \bar{C} = \frac{1}{\bar{Z}_0} \sinh \bar{\gamma} L & = 6.3575 \cdot 10^{-7} + j4.2917 \cdot 10^{-4}\end{cases} \tag{1}$$

and in per unit:

$$\begin{cases}\bar{a} = \bar{A} & = 0.9942 + j0.0004\\ \bar{b} = \frac{\bar{B}}{Z_{b2}} & = 0.0123 + j0.1658\\ \bar{c} = \bar{C} Z_{b2} & = 0.0001 + j0.0692\end{cases} \tag{2}$$

2. **Calculate voltage, active and reactive power at the start of the line (BUS 2).** Using the ABCD parameters of the transmission line, the relationship between the sending end (BUS 2) and the receiving end (BUS 3) is given by:

$$\begin{bmatrix}\bar{e}_2\\ \bar{i}_2\end{bmatrix} = \begin{bmatrix}\bar{a} & \bar{b}\\ \bar{c} & \bar{d}\end{bmatrix}\begin{bmatrix}\bar{e}_3\\ \bar{i}_3\end{bmatrix}$$

Given $\bar{E}_3 = 220\text{kV} = 1\text{p.u.}$ and $\bar{I}_3 = 0 + j0$ (no load connected),

$$\bar{e}_2 = \bar{a}\bar{e}_3 + \bar{b}\bar{i}_3 = 0.9942 + j0.0004\,\text{p.u.}$$

$$\bar{i}_2 = \bar{c}\bar{e}_3 + \bar{d}\bar{i}_3 = 0.0001 + j0.0692\,\text{p.u.}$$

Please note that the voltage magnitude is higher than at the start cause the line is mainly capacitive. Using the formula for complex power:

$$\begin{aligned}\bar{s} &= \bar{v}_2 \bar{i}_2^* = 0.0001 - j0.0688\,\text{p.u.}\\ p &= \Re(\bar{s}) = 0.0001\,\text{p.u.}\\ q &= \Im(\bar{s}) = -j0.0688\,\text{p.u.}\end{aligned}$$

3. **Voltage at the terminal of the generator (BUS 1) and the internal EMF of the generator:**

Considering the transformer's impedance,

$$\bar{e}_1 = \bar{e}_2 + \bar{z}_T \bar{i}_2 = 0.9873 + j0.0004\,\text{p.u.}$$

The internal EMF of the generator $\bar{e}_i$ can be found by adding the generator reactance $jX_d\bar{i}_1$ to the terminal voltage e_1:

$$\bar{e}_i = \bar{e}_1 + jX_d\bar{i}_1 = 0.8689 + j0.0006\,\text{p.u.}$$

Case 2

Consider the load U_1 to be supplied, with a voltage of 220 kV. The load is consuming 150 MVA at a power factor of 0.95 lagging.

1. **Calculate voltage, active and reactive power at the start of the line (BUS 2).** The apparent power s in per unit is computed as:

$$\bar{s}_3 = \frac{\bar{S}_3}{S_b} = \frac{150}{300} \exp(j \arccos 0.85) = 0.4750 + j0.1561$$

While the voltage is still fixed at 1 pu, the current can be computed knowing the apparent power $\bar{S}$:

$$\bar{e}_3 = 1\,\text{p.u.}$$
$$\bar{i}_3 = \frac{\bar{s}_3^*}{\bar{e}_3} = 0.4750 - j0.1561\,\text{p.u.}$$

and as a consequence:

$$\bar{e}_2 = \bar{a}\bar{e}_3 + \bar{b}\bar{i}_3 = 1.0260 + j0.0773\,\text{p.u.}$$

$$\bar{i}_2 = \bar{c}\bar{e}_3 + \bar{d}\bar{i}_3 = 0.4724 - j0.0858\,\text{p.u.}$$

Using the formula for complex power:

$$\bar{s} = \bar{v}_2\bar{i}_2^* = 0.4781 + j0.1245\,\text{p.u.}$$
$$p = \Re(\bar{s}) = 0.4781\,\text{p.u.}$$
$$q = \Im(\bar{s}) = j0.1245\,\text{p.u.}$$

2. **Calculate the voltage at generator terminal (BUS 1) and its internal EMF.** Considering the transformer's impedance,

$$\bar{e}_1 = \bar{e}_2 + \bar{z}_T\bar{i}_2 = 1.0346 + j0.1245\,\text{p.u.}$$

The internal EMF of the generator $\bar{e}_i$ can be found by adding the generator reactance $jX_d\bar{i}_1$ to the terminal voltage v_1:

$$\bar{e}_i = \bar{e}_1 + jX_d\bar{i}_1 = 1.1812 + j0.9323\,\text{p.u.}$$

3. **Step 3: For both the loading conditions of Step 1 and Step 2, draw the phasor diagram of the generator and elaborate on the requirements of the synchronous generator excitation system to withstand both no load and supply conditions.**

   Here below are two phasor diagrams for a generator supplying a line in no load condition (Figure 2), therefore capacitive, and under load (Figure 3), therefore inductive.

   No load: Under no load conditions, the generator is connected to the line without delivering any real power (P). In this scenario, the line exhibits capacitive behavior, causing the voltage to rise and the generator to be under-excited. This means the excitation voltage E_i projection over the terminal voltage E_1 is smaller than the $|E_1|$ and the generator draws reactive power from the system, acting like an inductor. This condition is represented as $|\bar{E}_i| \cos \delta < |\bar{E}_i|$.

   Load: Under load conditions, the generator is supplying both real power (P) and reactive power (Q) to the system. The system exhibits inductive behavior, causing the voltage to drop. In this scenario, the generator is over-excited, his condition is represented as $|\bar{E}_i| \cos \delta > |\bar{E}_i|$.

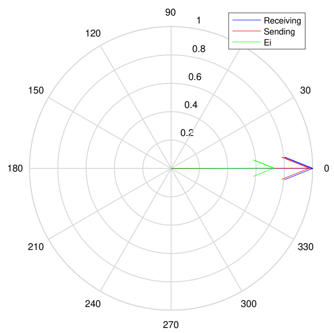

Figure 2: Phasor Diagram under no load

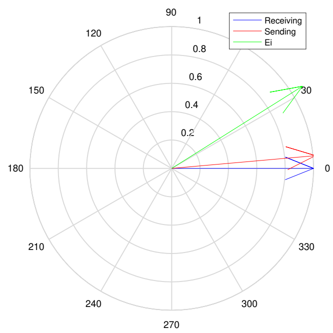

Figure 3: Phasor Diagram under load