Table 13–6 Octant Location versus Arctangent Expressions

Octant	Arctan approximation
1st, or 8th	$\theta' = \dfrac{IQ}{I^2 + 0.28125Q^2}$
2nd, or 3rd	$\theta' = \pi/2 - \dfrac{IQ}{Q^2 + 0.28125I^2}$
4th, or 5th	$\theta' = \pi + \dfrac{IQ}{I^2 + 0.28125Q^2}$
6th, or 7th	$\theta' = -\pi/2 - \dfrac{IQ}{Q^2 + 0.28125I^2}$

$$\tan^{-1}(Q/I) = \pi/2 - \tan^{-1}(I/Q). \tag{13–109'}$$

Those properties allow us to create Table 13–6.

So we have to check the signs of Q and I, and see if $|Q| > |I|$, to determine the octant location, and then use the appropriate approximation in Table 13–6. The maximum angle approximation error is 0.26° for all octants.

When θ is in the 5th octant, the above algorithm will yield a θ' that's more positive than $+\pi$ radians. If we need to keep the θ' estimate in the range of $-\pi$ -to- $+\pi$, we can rotate any θ residing in the 5th quadrant $+\pi/4$ radians (45°), by multiplying $(I + jQ)$ by $(1 + j)$, placing it in the 6th octant. That multiplication yields new real and imaginary parts defined as

$$I' = (I - Q), \text{ and } Q' = j(I + Q). \tag{13–110}$$

Then the 5th octant θ' is estimated using I' and Q' with

$$\theta'_{5\text{th oct.}} = -3\pi/4 - \frac{I'Q'}{Q'^2 + 0.28125I'^2}. \tag{13–110'}$$

13.22 FREQUENCY DEMODULATION ALGORITHMS

In Section 9.2 we discussed the notion of measuring the instantaneous frequency of a complex sinusoidal signal by computing the derivative of the signal's instantaneous $\theta(n)$ phase as shown in Figure 13–60.

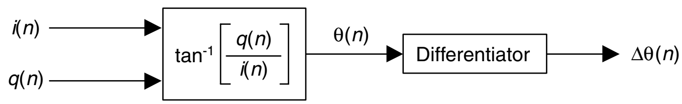

Figure 13–60 Frequency demodulator using an arctangent function.

This is the traditional discrete signal FM demodulation method, and it works fine. The demodulator's instantaneous output frequency is

$$f(n) = \frac{f_s[\Delta\theta_{rad}(n)]}{2\pi} \text{ Hz}, \tag{13–111}$$

where f_s is the sample rate in Hz.

Computing instantaneous phase $\theta(n)$ requires an arctangent operation, which is difficult to implement accurately without considerable computational resources. Here's a scheme for computing $\Delta\theta(n)$ for use in Eq. (13–111) without the intermediate $\theta(n)$ phase computation (and its pesky arctangent)[53,54]. We derive the $\Delta\theta(n)$ computation algorithm as follows, initially using continuous-time variables based on the following definitions:

$$\begin{aligned} i(t) &= \text{in-phase signal,} \\ q(t) &= \text{quadrature phase signal,} \\ \theta(t) &= \tan^{-1}[q(t)/i(t)] = \text{instantaneous phase,} \\ \Delta\theta(t) &= \text{time derivative of } \theta(t). \end{aligned} \tag{13–112}$$

First, we let $r(t) = q(t)/i(t)$ be the signal for which we're trying to compute the derivative of its arctangent. The time derivative of $\tan^{-1}[r(t)]$, a calculus identity, is

$$\Delta\theta(t) = \frac{d\{\tan^{-1}[r(t)]\}}{dt} = \frac{1}{1+r^2(t)} \frac{d[r(t)]}{dt}. \tag{13–113}$$

Because $d[r(t)]/dt = d[q(t)/i(t)]/dt$, we use the calculus identity for the derivative of a ratio to write

$$\frac{d[r(t)]}{dt} = \frac{d[q(t)/i(t)]}{dt} = \frac{i(t)\frac{d[q(t)]}{dt} - q(t)\frac{d[i(t)]}{dt}}{i^2(t)}. \tag{13–114}$$

Plugging Eq. (13–114)'s result into Eq. (13–113), we have

$$\Delta\theta(t) = \frac{1}{1+r^2(t)} \frac{i(t)\frac{d[q(t)]}{dt} - q(t)\frac{d[i(t)]}{dt}}{i^2(t)}. \tag{13–115}$$

Replacing $r(t)$ in Eq. (13–115) with $q(t)/i(t)$ yields

$$\Delta\theta(t) = \frac{1}{1+[q(t)/i(t)]^2} \cdot \frac{i(t)\dfrac{d[q(t)]}{dt} - q(t)\dfrac{d[i(t)]}{dt}}{i^2(t)}. \qquad (13\text{–}116)$$

We're getting there. Next we multiply the numerator and denominator of the first ratio in Eq. (13–116) by $i^2(t)$, and replace t with our discrete time variable index n to arrive at our final result of

$$\Delta\theta(n) = \frac{i(n)\dfrac{d[q(n)]}{dn} - q(n)\dfrac{d[i(n)]}{dn}}{i^2(n) + q^2(n)}. \qquad (13\text{–}117)$$

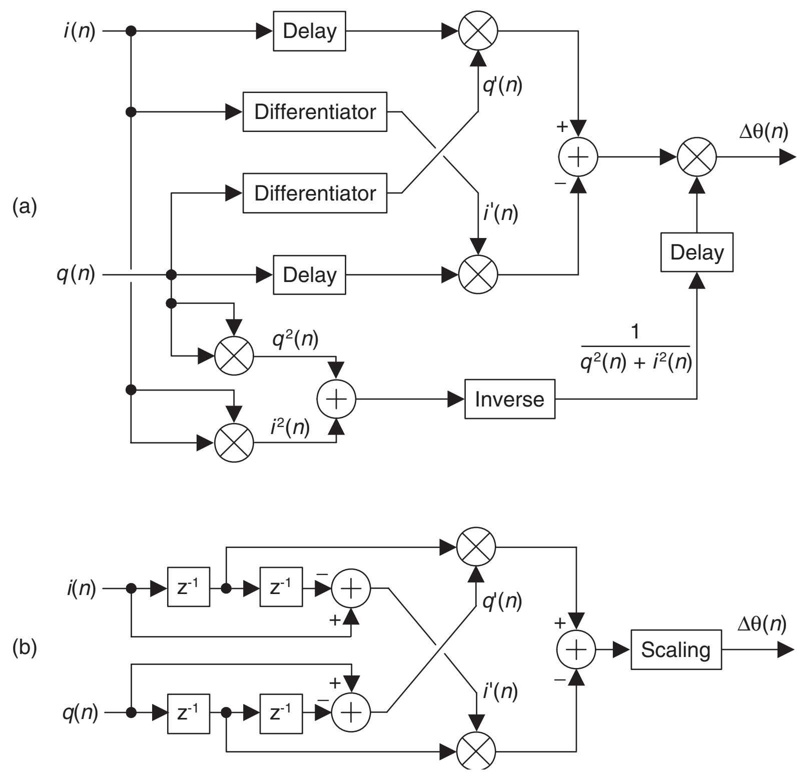

Figure 13–61 Frequency demodulator without arctangent: (a) standard process; (b) simplified process.

The implementation of this algorithm, where the derivatives of $i(n)$ and $q(n)$ are $i'(n)$ and $q'(n)$ respectively, is shown in Figure 13–61(a). The $\Delta\phi(n)$ output sequence is used in Eq. (13–111) to compute instantaneous frequency.

The Differentiator is an tapped-delay line FIR differentiating filter with an odd number of taps. Reference [54] reports acceptable results when the differentiator is a FIR filter having 1,0,–1 as coefficients. The Delay elements in Figure 13–61 are used to time-align $i(n)$ and $q(n)$ with the outputs of the differentiators such that the delay is $(K–1)/2$ samples when a K-tap differentiator is used. In practice, the Delay can be obtained by tapping off the center tap of the differentiating filter.

If the $i(n)+jq(n)$ signal is purely FM and *hard limited* such that $i^2(n)+q^2(n)$ = Constant, the denominator computations in Eq. (13–117) need not be performed. In this case, using the 1,0,–1 coefficient differentiators, the FM demodulator is simplified to that shown in Figure 13–61(b) where the Scaling operation is multiplication by the reciprocal of Constant.

13.23 DC REMOVAL

When we digitize analog signals using an analog-to-digital (A/D) converter, the converter's output typically contains some small DC bias: that is, the average of the digitized time samples is not zero. That DC bias may have come from the original analog signal or from imperfections within the A/D converter. Another source of DC bias contamination in DSP is when we truncate a discrete sequence from a B-bit representation to word widths less than B bits. Whatever the source, unwanted DC bias on a signal can cause problems. When we're performing spectrum analysis, any DC bias on the signal shows up in the frequency domain as energy at zero Hz, the $X(0)$ spectral sample. For an N-point FFT the $X(0)$ spectral value is proportional to N and becomes inconveniently large for large-sized FFTs. When we plot our spectral magnitudes, the plotting software will accommodate any large $X(0)$ value and squash down the remainder of the spectrum in which we are more interested.

A non-zero DC bias level in audio signals is particularly troublesome because concatenating two audio signals, or switching between two audio signals, results in unpleasant audible clicks. In modern digital communications systems, a DC bias on quadrature signals degrades system performance and increases bit error rates. With that said, it's clear that methods for DC removal are of interest to many DSP practitioners.

13.23.1 Block-Data DC Removal

If you're processing in non-real-time, and the signal data is acquired in blocks (fixed-length sequences) of block length N, DC removal is straightforward. We merely compute the average of our N time samples, and subtract that av-