Telecommunications Systems Exercises 6: Solutions

May 2025

6.1 Free-Space Path Loss and Antenna Gain

Given:

- Frequency: $f = 2.4$ GHz $= 2.4 \times 10^9$ Hz
- Speed of light: $c = 3 \times 10^8$ m/s
- Transmit power: $P_t = 0$ dBm $= 1$ mW $= 10^{-3}$ W

First, calculate the wavelength:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.4 \times 10^9} = 0.125 \text{ m}$$

The free-space received power formula is:

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2}$$

1. **Received power at 50 meters (Isotropic Antennas)**
   Given $G_t = G_r = 1$ and $R = 50$ m:

   $$P_r = \frac{(10^{-3}) \times 1 \times 1 \times (0.125)^2}{(4\pi \times 50)^2}$$

   Simplifying:

   $$(0.125)^2 = 0.015625, \quad (4\pi \times 50)^2 \approx 394784$$

   Thus:

   $$P_r = \frac{1.5625 \times 10^{-5}}{394784} \approx 3.957 \times 10^{-11} \text{ W}$$

   Converting to dBm:

   $$P_r \text{ (dBm)} = 10 \log_{10}(3.957 \times 10^{-8}) \approx -74 \text{ dBm}$$

   Answer: Received power is approximately -74 dBm.

2. **Received power with 6 dBi transmit antenna gain**
   Given $G_t = 6$ dBi $= 10^{6/10} = 3.981$, $G_r = 1$:

$$P_r = \frac{(10^{-3}) \times 3.981 \times 1 \times (0.125)^2}{(4\pi \times 50)^2}$$

$$P_r = 3.981 \times 3.957 \times 10^{-11} \approx 1.574 \times 10^{-10} \text{ W}$$

   Converting to dBm:

$$P_r \text{ (dBm)} = 10 \log_{10}(1.574 \times 10^{-7}) \approx -68 \text{ dBm}$$

   Answer: Received power is approximately -68 dBm.

3. **Maximum distance with receiver sensitivity of -90 dBm (Isotropic Antennas)**
   Required $P_r = 10^{-9}$ mW $= 10^{-12}$ W.

   Rearranging for distance:

$$R = \frac{\lambda}{4\pi}\sqrt{\frac{P_t G_t G_r}{P_r}}$$

   Substituting values:

$$R = \frac{0.125}{4\pi}\sqrt{\frac{10^{-3}}{10^{-12}}} = \frac{0.125}{4\pi} \times 31622.8$$

$$R = \frac{3952.85}{12.566} \approx 314.5 \text{ m}$$

   Answer: Maximum distance is approximately 314.5 meters.

4. **Maximum distance with 6 dBi (Tx) and 3 dBi (Rx) gains**
   Antenna gains:

$$G_t = 10^{6/10} = 3.981, \quad G_r = 10^{3/10} = 1.995$$

   Thus:

$$R = \frac{0.125}{4\pi}\sqrt{\frac{10^{-3} \times 3.981 \times 1.995}{10^{-12}}}$$

$$R = \frac{0.125}{4\pi} \times 89105.9$$

$$R = \frac{11138.24}{12.566} \approx 886 \text{ m}$$

   Answer: Maximum distance is approximately 886 meters.

6.2 Cellular System Design & User Capacity

6.2.1 Cell Size Based on Sensitivity Requirement

Given:

- Transmit Power: $P_t = +30\,\text{dBm}$
- Sensitivity: $P_{\text{rx,min}} = -104\,\text{dBm}$
- Margin: $M = 23\,\text{dB}$

$$L_{\max} = P_t - P_{\text{rx,min}} - M = 30 - (-104) - 23 = 111\,\text{dB}$$

We use the simplified path loss model:

$$L = 10\beta \log_{10}(R)$$

(a) For $\beta = 2$:

$$111 = 20\log_{10}(R_2) \Rightarrow \log_{10}(R_2) = \frac{111}{20} = 5.55$$

$$R_2 = 10^{5.55} \approx 354813.4\,\text{m} = 354.8\,\text{km}$$

(b) For $\beta = 3$:

$$111 = 30\log_{10}(R_3) \Rightarrow \log_{10}(R_3) = \frac{111}{30} = 3.7$$

$$R_3 = 10^{3.7} \approx 5011.9\,\text{m} = 5.01\,\text{km}$$

6.2.2 Architecture and Reuse

We use the approximation:

$$\text{SIR} \approx (\sqrt{3N})^{\beta}$$

$$\text{Target SIR} = 11.5\,\text{dB} \Rightarrow \text{SIR}_{\text{lin}} > 10^{11.5/10} \approx 14.13$$

(a) For $\beta = 2$:

- $N = 3$: SIR $= 9$
- $N = 4$: SIR $= 12$
- $N = 7$: SIR $= 21$

$\Rightarrow$ N = 7

(b) For $\beta = 3$:

- $N = 3$: SIR $= 27$

$\Rightarrow$ N = 3

6.2.3 Supported User Density for Largest Possible Cell

Given:

- Total bandwidth (uplink or downlink): 12.5 MHz
- Channel bandwidth: 200 kHz
- Number of channels: $\frac{12.5}{0.2} = 62.5 \Rightarrow 62$
- Users per channel: 8
- Total users per cluster: $62 \times 8 = 496$
- Area of a hexagonal cell: $A = \frac{3\sqrt{3}}{2}R^2$

(a) For $\beta = 2$, $R_2 = 354.8\,\text{km}$, $N = 7$:

$$\text{Users per cell} = \frac{496}{7} \approx 70.86$$

$$A = \frac{3\sqrt{3}}{2} \cdot (354.8)^2 \approx 326530\,\text{km}^2$$

$$\text{User density} = \frac{70.86}{326530} \approx 0.000217\,\text{users/km}^2$$

(b) For $\beta = 3$, $R_3 = 5.01\,\text{km}$, $N = 3$:

$$\text{Users per cell} = \frac{496}{3} \approx 165.33$$

$$A = \frac{3\sqrt{3}}{2} \cdot (5.01)^2 \approx 65.1\,\text{km}^2$$

$$\text{User density} = \frac{165.33}{65.1} \approx 2.54\,\text{users/km}^2$$

(c) **Observation:** Larger cells (with $\beta = 2$) cover more area but support significantly lower user density due to higher reuse requirements. Smaller cells (with $\beta = 3$) allow higher reuse and significantly higher user densities. Thus, increasing β improves spatial spectrum efficiency.

6.2.4 Supported User Density for Fixed Cell Size

Assume $R = R_3 = 5.01\,\text{km}$

(a) For $\beta = 2$:

$$L = 20\log_{10}(5.01) \approx 20 \cdot 0.7 = 14.02\,\text{dB} \ll 111\,\text{dB}$$

$\Rightarrow$ Yes, this cell size supports $\beta = 2$

(b) Reuse constraint for $\beta = 2$:

- $N = 7$ is required (SIR = 21)

$$\text{Users per cell} = \frac{496}{7} \approx 70.86$$
$$A = \frac{3\sqrt{3}}{2} \cdot (5.01)^2 \approx 65.1\,\text{km}^2$$
$$\text{User density} = \frac{70.86}{65.1} \approx 1.089\,\text{users/km}^2$$

6.3 Two-Ray Interference

6.3.1 Path Length Derivations

In the two-ray model, the received signal comprises:

- A direct Line-of-Sight (LOS) path
- A ground-reflected path

Given:

- Transmitter height: $h_t = 1$ m
- Receiver height: $h_r = 3$ m
- Horizontal distance: d

Using the Pythagorean theorem:

- Direct path length:
$$d_2 = \sqrt{(h_r - h_t)^2 + d^2}$$
- Ground-reflected path length:
$$d_1 = \sqrt{(h_r + h_t)^2 + d^2}$$

6.3.2 Phase Difference Computation

Given:

- Frequency: $f = 1.8$ GHz
- Speed of light: $c = 3 \times 10^8$ m/s

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.8 \times 10^9} = 0.1667 \text{ m}$$
$$\theta_1 = \frac{2\pi}{\lambda} \cdot d_1$$
$$\theta_2 = \frac{2\pi}{\lambda} \cdot d_2$$
$$\Delta\theta = \theta_1 - \theta_2$$

6.3.3 Electric Field Simulation

Assume a reference field $E_0 = 1$ at reference distance $d_0 = 1$ m.
Each electric field component attenuates with distance and accumulates a phase shift:

$$E_1 = \frac{E_0 d_0}{d_1} e^{j\theta_1}, \quad E_2 = \frac{E_0 d_0}{d_2} e^{j\theta_2}$$

The total field at the receiver:

$$E = E_1 + E_2$$

Magnitude of the received field:

$$|E| = \left| \frac{E_0 d_0}{d_1} e^{j\theta_1} + \frac{E_0 d_0}{d_2} e^{j\theta_2} \right|$$

6.3.4 MATLAB Simulation and Plots

Example MATLAB code:

```
hr = 3;
ht = 1;
d = logspace(0, 10, 1000);
f = 1.8e9;
c = 3e8;

d1 = sqrt((hr + ht)^2 + d.^2);
d2 = sqrt((ht - hr)^2 + d.^2);
delta_d = d1 - d2;

E0 = 1;
d0 = 1;
theta1 = d1 ./ (c / f) * 2 * pi;
theta2 = d2 ./ (c / f) * 2 * pi;
E1 = E0 * d0 ./ d1 .* exp(1i .* theta1);
E2 = E0 * d0 ./ d2 .* exp(1i .* theta2);
E = E1 + E2;
figure();
loglog(d, abs(E));
```

6.3.5 Questions

1. **How does the interference pattern evolve with increasing distance?**
   As distance increases, the path length difference grows, causing more rapid oscillations in phase and alternating regions of constructive and destructive interference.

2. **Where do deep nulls (signal drops) appear? What causes them?**
   Deep nulls appear when the two waves arrive nearly equal in amplitude but with a phase difference of π (180 degrees), leading to destructive interference. This occurs periodically based on distance.

3. **How does changing the transmitter or receiver height affect the pattern?**
   Increasing antenna heights increases the vertical path length difference, shifting the interference pattern. This can reduce the density of nulls in the near-field region.

4. **What happens to the interference if the frequency increases (e.g., to 2.4 GHz)?**
   A higher frequency leads to a shorter wavelength, increasing the rate of phase change with distance. This results in more frequent constructive/destructive regions and generally greater signal fading.

6.4 Noise Figure

Assume that the input signal power is P_s, input noise is N_0, the gain is G, internal noise is N_a, we can write the noise figure as:

$$F = \frac{\text{SNR}_{in}}{\text{SNR}_{out}} = \frac{\frac{P_s}{N_0}}{\frac{GP_s}{GN_0+N_a}} = 1 + \frac{N_a}{GN_0}$$

Thus, higher VGA gain leads to a lower noise figure if the internal noise is constant.

6.5 Doppler Shift

1. *What is the Doppler shift of the received signal from the satellite ?*

   The Doppler shift from satellite to plane is :

   $$\Delta f_{sp} = f_c \frac{v_s + v_p}{c} \cos(\theta) \approx 315.993 \text{ kHz}$$

2. *Is the Doppler shift an issue for the receiver ? Explain why.*

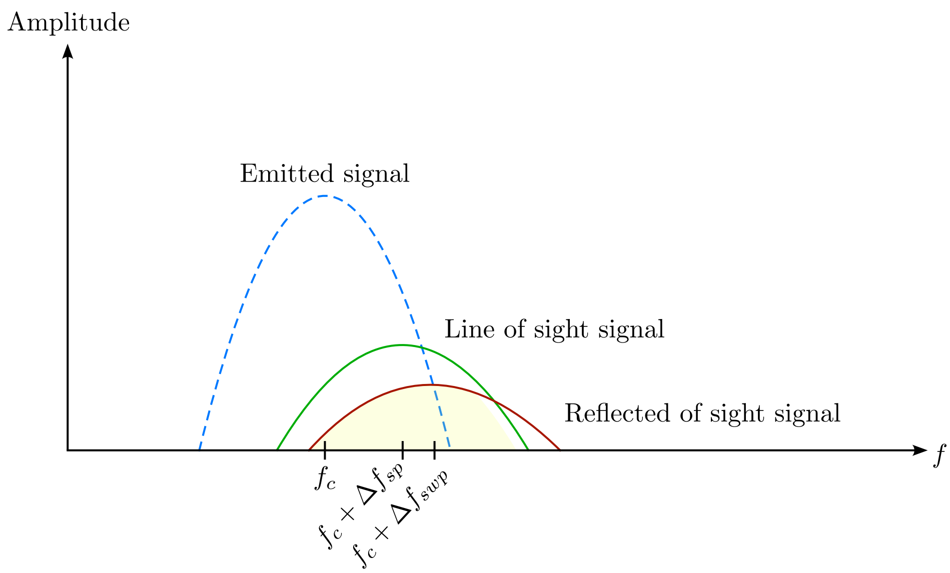
   The signal is shifted of more than 300 kHz, this is a problem because the receiver downconverts the signal from an expected carrier frequency. With such an offset, it will not find the signal where expected.

3. *What is the Doppler shift of the received signal from the reflection ?*

   The signal is first reflected with the Doppler shift between the satellite and the surface, and then received as this shifted signal which is shifted again by the plane's velocity. We estimate the Doppler shift of the reflected signal at the plane, Δf_{swp} in two steps : 1) calculate the Doppler shift of the satellite signal at the reflection spot on the ocean Δf_{sw} and 2) calculate the Doppler shift of the reflected signal at the plane Δf_{wp}.

$$\begin{aligned}
\Delta f_{swp} &= \Delta f_{sw} + \Delta f_{wp} \\
&= f_c \frac{v_s}{c} \cos(\pi/2 - \alpha) + (f_c + \Delta f_{sw}) \frac{v_p}{c} \cos(\pi/2 - \alpha) \\
&= f_c \frac{\sin(\alpha)}{c} \left(v_s + v_p + \frac{v_s v_p}{c} \sin(\alpha) \right) \\
&\approx 357.236 \text{ kHz}
\end{aligned}$$

4. *Consider that the transmitted signal has bandwidth B, sketch the emitted signal spectrum, and the received signals spectra and explain what will the reflected signal cause.*



As illustrated above, the received signal is shifted of Δf_{sp} from f_c, the reflected signal is shifted of Δf_{swp}. With a bandwidth of $B = 1$ MHz, the two received signals mostly overlap, causing interferences.

5. *Can you imagine a simple solution on the receiver side to avoid this second path ?*

The signal of a satellite always comes from above. Signals reflected from the ground / ocean always com from below. We can just put an antenna with one lobe pointing above and we will only receive signals from the satellite.

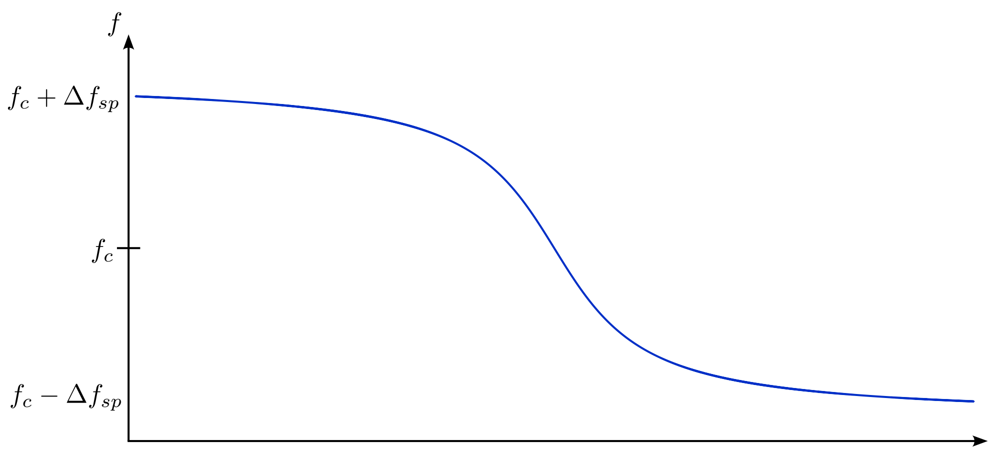
6. *Only focusing on the signal received from the satellite, assume that after a few minutes, the satellite flew above the plane and further and we now are in a symmetric situation as depicted above but with the satellite behind the plane. What is the Doppler shift now ?*

With a symmetric situation, all remains the same except θ that now is $\frac{\pi}{2} + \theta$, thus the Doppler shift simply changes of sign and we have:

$$\Delta f_{sp} \approx -315.993 \text{ kHz}$$

7. *Sketch how the Doppler shift of the received signal from the satellite evolves over time and discuss the consequences for the receiver.*

The Doppler shift starts high as the satellite moves towards the plane, crosses a point where there if no shift because relative velocity if orthogonal tor elative position, and then the Doppler shift becomes negative as the satellite moves away from the plane. The graph below is a Doppler curve and these are well known from engineers working on space telecommunication systems.



Indeed, not only is the Doppler shift large, it changes over time. The consequence is that the band on which the receiver must be able to detect the signal is larger, and the latter must track and compensate the Doppler shift to demodulate the transmitted information.