EE-432 Systeme de Telecommunication

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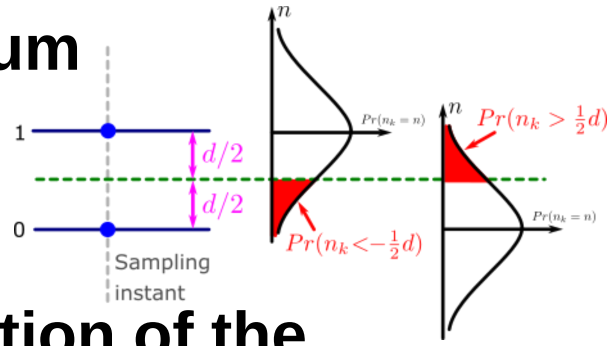
Digital RF (Carrier) Communication
Linear Modulation: From PAM to QAM

Recap from Week-8

- **Transmitted signals are distorted by additive white Gaussian noise**
- **Errors are caused by noise that is larger than ½ the minimum distance between constellation points (symbol levels)**



- **The error rate has the form $\varepsilon = \alpha \cdot Q\left(\frac{1}{2}\frac{d}{\sigma}\right)$, where α is a function of the employed modulation**
- **We are generally interested in the error rate as a function of the signal power and the noise (or the noise PSD)**
  - Express the minimum distance d as a function of the power P (and the noise as a function of N_0 and the bandwidth)
  - For example for M-PAM we obtain $\varepsilon_M = 2 \cdot \frac{M-1}{M} \cdot Q\left(\sqrt{\frac{3}{M^2-1} SNR_M}\right) = 2 \cdot \frac{M-1}{M} \cdot Q\left(\sqrt{\frac{3}{M^2-1} \frac{P}{B \cdot N_0}}\right)$

Error Rate from an Energy Perspective (Binary PCM)

- **Longer symbols correspond to expending more power for each symbol**
  - Is there a “fair” way to account for the increase of power when extending symbol duration?

- **Idea: start from an energy-perspective**
  - Using the “energy per symbol” $E_s = P_s \cdot T_s$ allows to express error rate performance independently from the symbol rate, but still relating to the thermal noise constant PSD
  - Assume minimum bandwidth as $B_s = \frac{1}{2 \cdot T_s}$

$$SNR = \frac{P_s \cdot T_s}{T_s \cdot B_s \cdot N_0} = 2 \cdot \frac{E_s}{N_0}$$

$$\varepsilon_2 = Q\left(\sqrt{2\frac{E_s}{N_0}}\right)$$

Error Rate from an Energy Perspective (M-PAM)

- **Increasing constellation order M allows to reduce the bandwidth (impact on noise as $\sigma^2 = N_0 \cdot B$**
- **Consider a M-PAM signal with a given bit rate f_{bit}, a pulse shaping filter with minimum spectrum occupation, and power $\bar{P}_s$**
- **Calculate the error rate ε_M as a function of the one-sided noise PSD N_0 and the bit rate f_{bit}**
  - Some useful expressions/hints

$$\varepsilon_M = 2 \cdot \frac{M-1}{M} \cdot Q\left(\sqrt{\frac{3}{M^2-1} SNR_M}\right)$$

Bandwidth: $\frac{f_{bit}}{2\log_2 M}$

Symbol-duration: $T_s = \frac{\log_2 M}{f_{bit}}$ Bit-duration: $T_{bit} = \frac{1}{f_{bit}}$

Bipolar M-PAM Error Probability with Fixed Throughput

- **First we obtain the Error Probability as a function of the symbol energy for the minimum required bandwidth**

$$\varepsilon_M = 2 \cdot \frac{M-1}{M} \cdot Q\left(\sqrt{\frac{3 \cdot 2}{M^2-1}\frac{E_s}{N_0}}\right)$$

- **Fixing the throughput f_{bit} implies that** $T_s = \frac{\log_2 M}{f_{bit}}$ and with $E_s = \bar{P}_s \cdot \frac{\log_2 M}{f_{bit}}$

$$\varepsilon_M = 2 \cdot \frac{M-1}{M} \cdot Q\left(\sqrt{\frac{3 \cdot 2}{M^2-1}\frac{\bar{P}_s}{N_0}\frac{\log_2 M}{f_{bit}}}\right)$$

  - Increasing the constellation implies a significant penalty in error rate performance
  - the increase in symbol energy when packing more bits with a fixed E_b provides only a slight compensation for the penalty

Energy per Bit

- **Similar to the "energy per symbol, we can define the "energy per bit"**

$$E_{\boldsymbol{b}} = \frac{\bar{P}_S \cdot \log_2 M}{f_{bit}} = E_s / \log_2 M$$

- E_b **provides another level of normalization that simplifies the comparison as it removes the dependency on the number of bits per symbol**

$$\varepsilon_M = 2 \cdot \frac{M-1}{M} \cdot Q\left(\sqrt{\frac{3 \cdot 2}{M^2 - 1} \frac{E_b}{N_0} \cdot \log_2 M}\right)$$

  - Constant "energy per bit" is equivalent to saying we pack more bits into a symbol and extend the symbol duration accordingly.

Week 8: Table of Contents

- **Digital RF (Carrier) Communication**

  Linear Modulation: From PAM to QAM

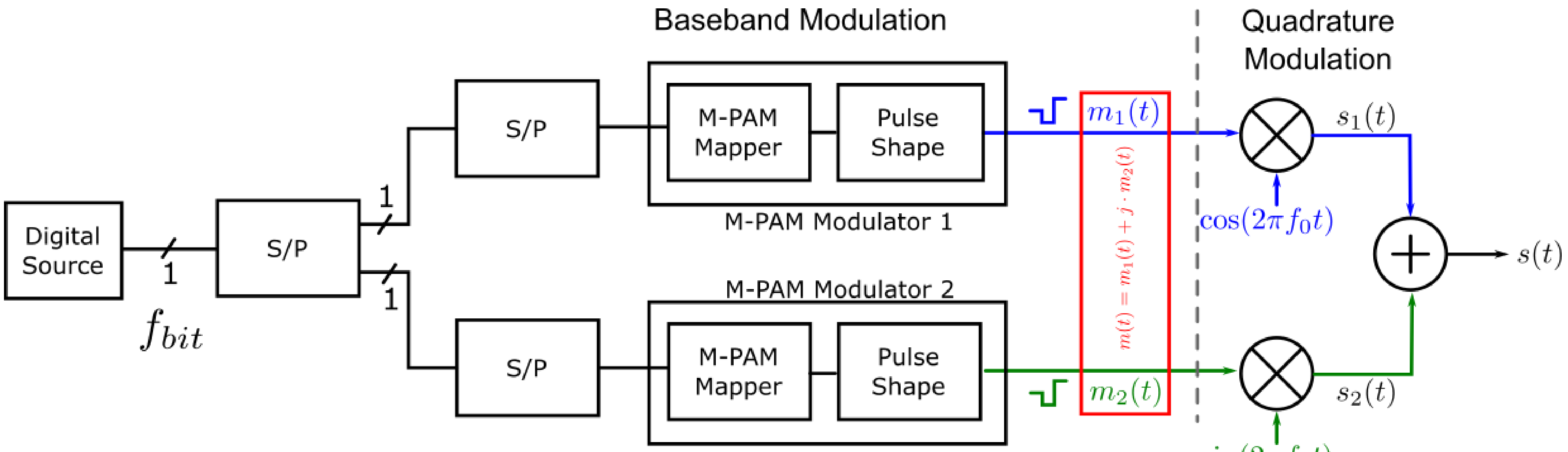
- **Channel Capacity**

From RF to Baseband (Carrier Modulation)

- **Can we use the same linear modulation concept to generate modulated Radio-Frequency (RF) signals?**

Converting two modulated baseband signals to RF

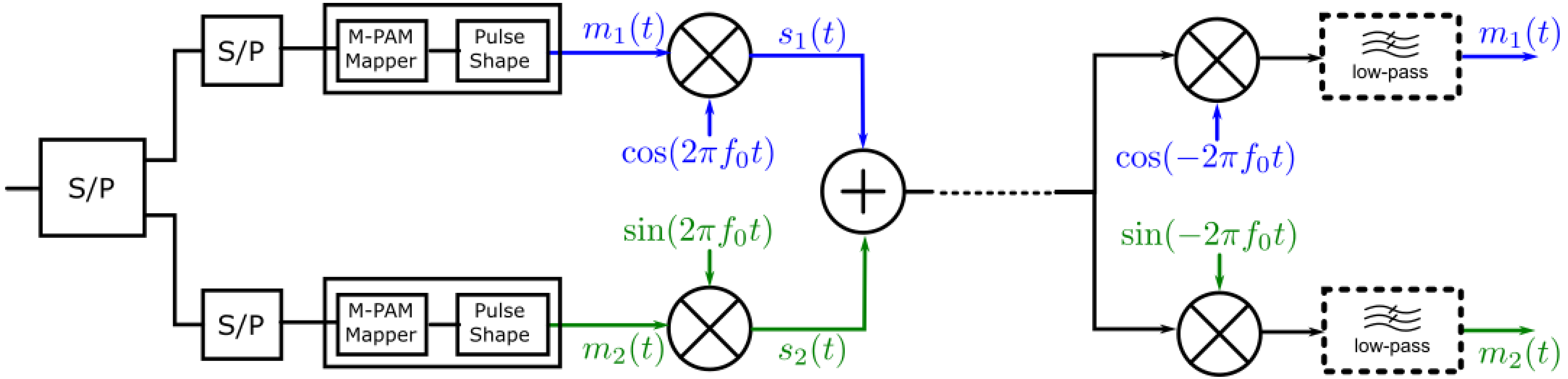
- "Carrier Modulation": conversion to RF based on Quadrature Modulator (see AM)
- The RF signal can accommodate two real-valued baseband signals in two orthogonal carriers



$$m(t) = m_1(t) + j \cdot m_2(t)$$

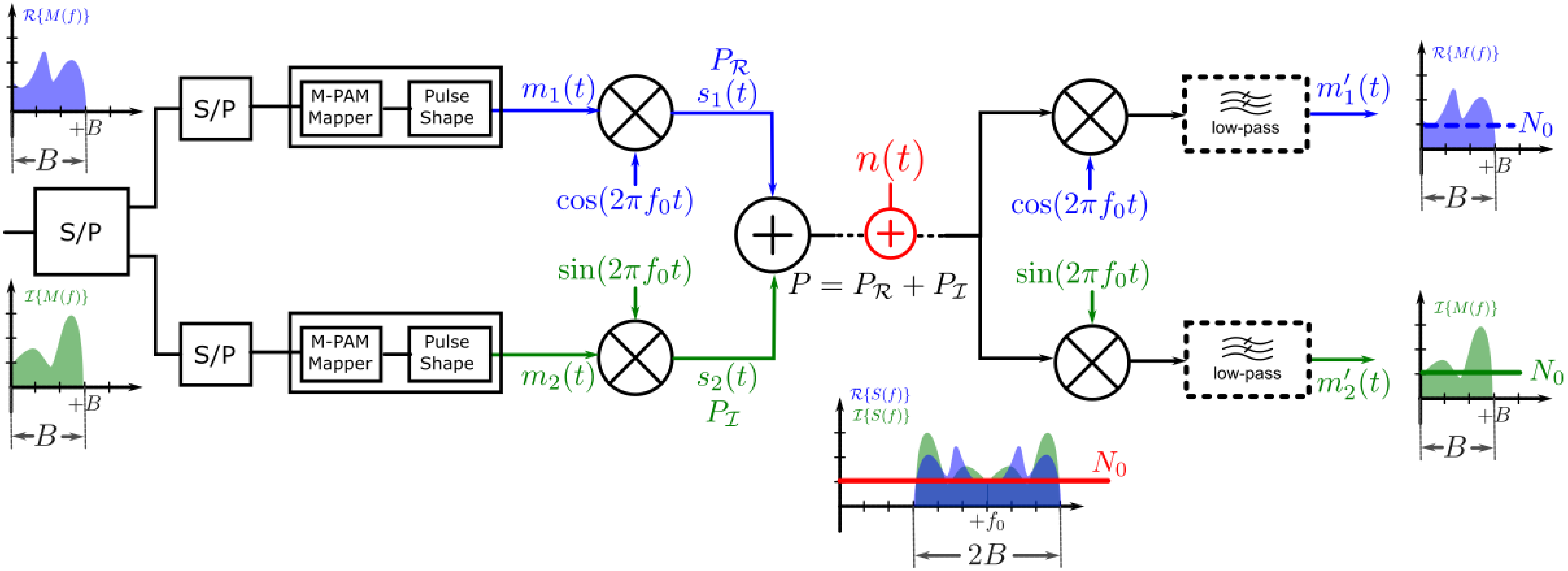
RF to Baseband (Carrier Demodulation)

- **The two orthogonal carriers can be demodulated separately and can be separated perfectly when the receiver is coherent (phase and frequency aligned) with the transmitter**



Noise and Signal Power in Carrier Modulation

- **To calculate error rates, we need**
  - the signal power (to calculate the distance between symbols)
  - the noise power (variance) in the direction of the neighbouring symbol



Noise and Signal Power in Carrier Modulation

- **Signal power**
  - The power of the RF signal is the sum of the powers of the two baseband signals

$$P = P_{\mathcal{R}} + P_{\mathcal{I}}$$

- **Noise power (variance of the noise)**
  - Noise is added in the RF signal with bandwidth $B_{RF} = 2 \cdot B$
  - Noise of the RF signal has power $\sigma_{RF}^2 = 2 \cdot B \cdot N_0$

  - At the receiver, the noise is split equally between the two branches
  - Each branch obtains 50% of the noise on the RF signal

$$\sigma_{\mathcal{R}}^2 = \sigma_{\mathcal{I}}^2 = \frac{\sigma_{RF}^2}{2} = B \cdot N_0$$

  - This is consistent with adding noise in the baseband with PSD N_0 and a bandwidth B
  - If we consider the demodulated noise as a complex-valued signal, the variance would be

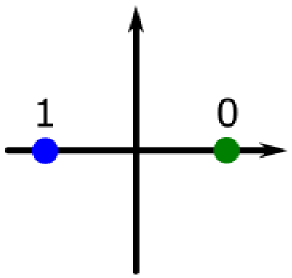
$$\sigma^2 = \sigma_{\mathcal{R}}^2 + \sigma_{\mathcal{I}}^2 = 2 \cdot B \cdot N_0$$

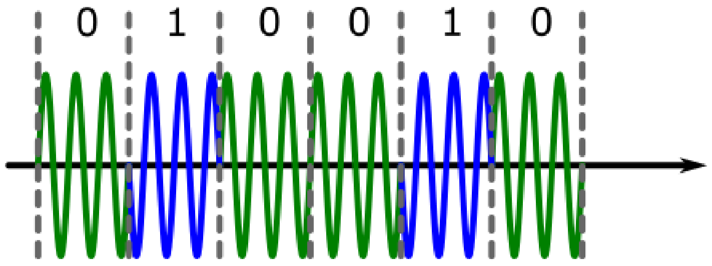
BPSK Modulation Error Rate

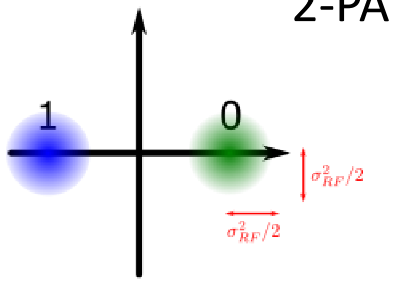
- **Binary modulation (Bi-polar 2-PAM) with $M = 2$ (1 bit per symbol)**
  - Uses only one "carrier" of the complex-valued signal (spectrally inefficient)

$$m_1[k] \in \{-\sqrt{P}, +\sqrt{P}\} \text{ and } m_2[k] = 0$$

Reminder from 2-PAM: $\varepsilon = Q\left(\frac{d}{2}\frac{1}{\sigma}\right)$







Noise in RF: $\sigma^2_{RF} = 2 \cdot \frac{f_s}{2} \cdot N_0$

Noise on real part (m_1'): $\sigma^2_{\mathcal{R}} = \frac{f_s}{2} \cdot N_0$

Half-distance: $\frac{d}{2} = \sqrt{P}$

$$\varepsilon_{BPSK} = Q\left(\frac{d}{2}\frac{1}{\sqrt{\sigma^2}}\right) = Q\left(\sqrt{\frac{P}{\sigma^2_{\mathcal{R}}}}\right) = Q\left(\sqrt{\frac{P}{\frac{f_s}{2} \cdot N_0}}\right) = Q\left(\sqrt{2\frac{P/f_s}{N_0}}\right) = Q\left(\sqrt{2\frac{E_s}{N_0}}\right) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$$

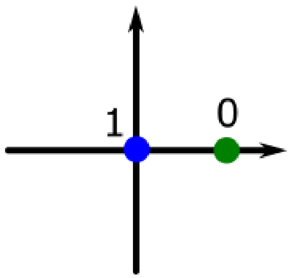
OOK Modulation and Error Rate

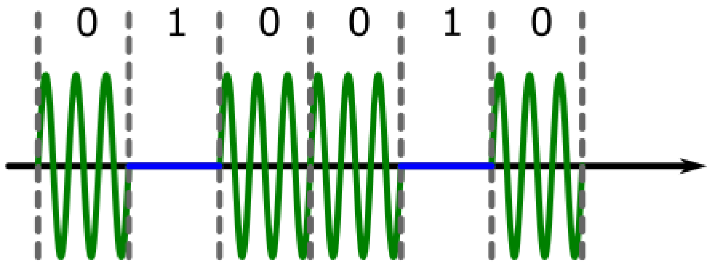
- **On-Off Keying (OOK) is the uni-polar version of BPSK**
  - Uses only one “carrier” of the complex-valued signal (spectrally inefficient)

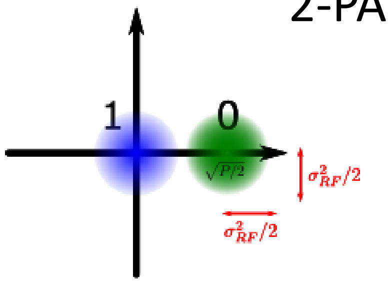
$$m_1[k] \in \{0, +\sqrt{2P}\} \text{ and } m_2[k] = 0$$

Reminder from

2-PAM: $\varepsilon = Q\left(\frac{d}{2}\frac{1}{\sigma}\right)$







Noise in RF: $\sigma^2_{RF} = 2 \cdot \frac{f_s}{2} \cdot N_0$ Noise on real part (m_1'): $\sigma^2_{\mathcal{R}} = \frac{f_s}{2} \cdot N_0$

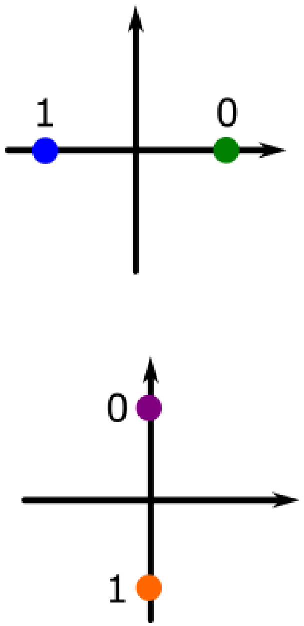
Half-distance: $\frac{d}{2} = \sqrt{P/2}$

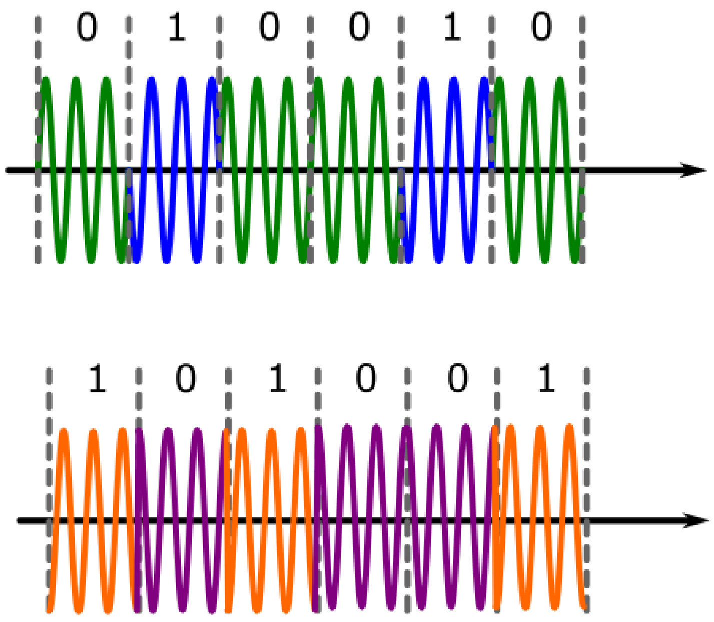
$$\varepsilon_{BPSK} = Q\left(\frac{d}{2}\frac{1}{\sqrt{\sigma^2}}\right) = Q\left(\sqrt{\frac{P/2}{\sigma^2_{\mathcal{R}}}}\right) = Q\left(\sqrt{\frac{P/2}{\frac{f_s}{2} \cdot N_0}}\right) = Q\left(\sqrt{\frac{P/f_s}{N_0}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

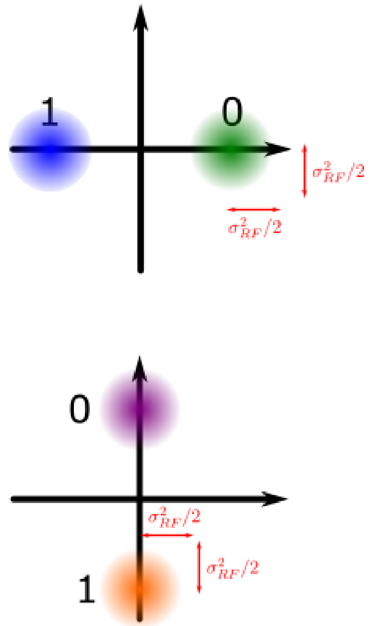
QPSK Modulation

- **QPSK uses both "carriers" (sin/cos) in parallel, each with Bi-polar 2-PAM to send 2 bits in parallel in the same bandwidth (more BW efficient)**
  - To maintain total power P after summing both carriers, the amplitude of each carrier is reduced

$$m_1[k] \in \{-\sqrt{P/2}, +\sqrt{P/2}\} \text{ and } m_2[k] \in \{-\sqrt{P/2}, +\sqrt{P/2}\}$$

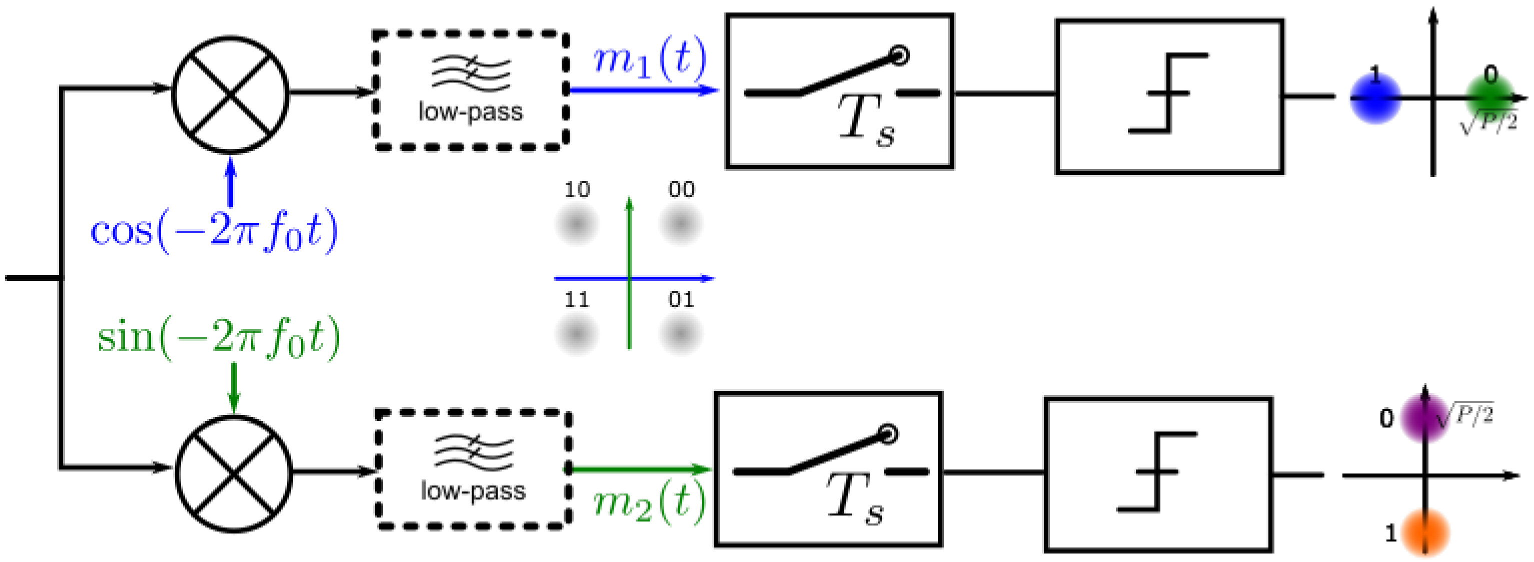






QPSK De-Modulation

- **QPSK can be de-modulated by considering the two branches (real- and imaginary parts) separately.**



QPSK Error Rate

- **Bit-Error rates can be computed separately on the two branches, similar to BPSK, but considering the scaled constellation (to maintain the power):**

$$m_{1,2}[k] \in \{-\sqrt{P/2}, +\sqrt{P/2}\}$$

Noise in RF: $\sigma_{RF}^2 = 2 \cdot \frac{f_s}{2} \cdot N_0$ Noise on real part (m_1'): $\sigma_{\mathcal{R}}^2 = \frac{f_s}{2} \cdot N_0$

Half-distance: $\frac{d}{2} = \sqrt{P/2}$

$$\varepsilon_{QPSK} = Q\left(\frac{d}{2}\frac{1}{\sqrt{\sigma^2}}\right) = Q\left(\sqrt{\frac{P/2}{\sigma_{\mathcal{R}}^2}}\right) = Q\left(\sqrt{\frac{P/2}{\frac{f_s}{2} \cdot N_0}}\right) = Q\left(\sqrt{\frac{P/f_s}{N_0}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

- Due to the reduced distance, the error rate in terms of E_s/N_0 is 3dB worse than BPSK
- HOWEVER, when considering E_b/N_0 with $E_s = 2 \cdot E_b$ we find

$$\varepsilon_{QPSK} = Q\left(\sqrt{2 \cdot \frac{E_b}{N_0}}\right) = \varepsilon_{BPSK}$$

For same E_b/N_0, the bit error rate of QPSK and BPSK are the same

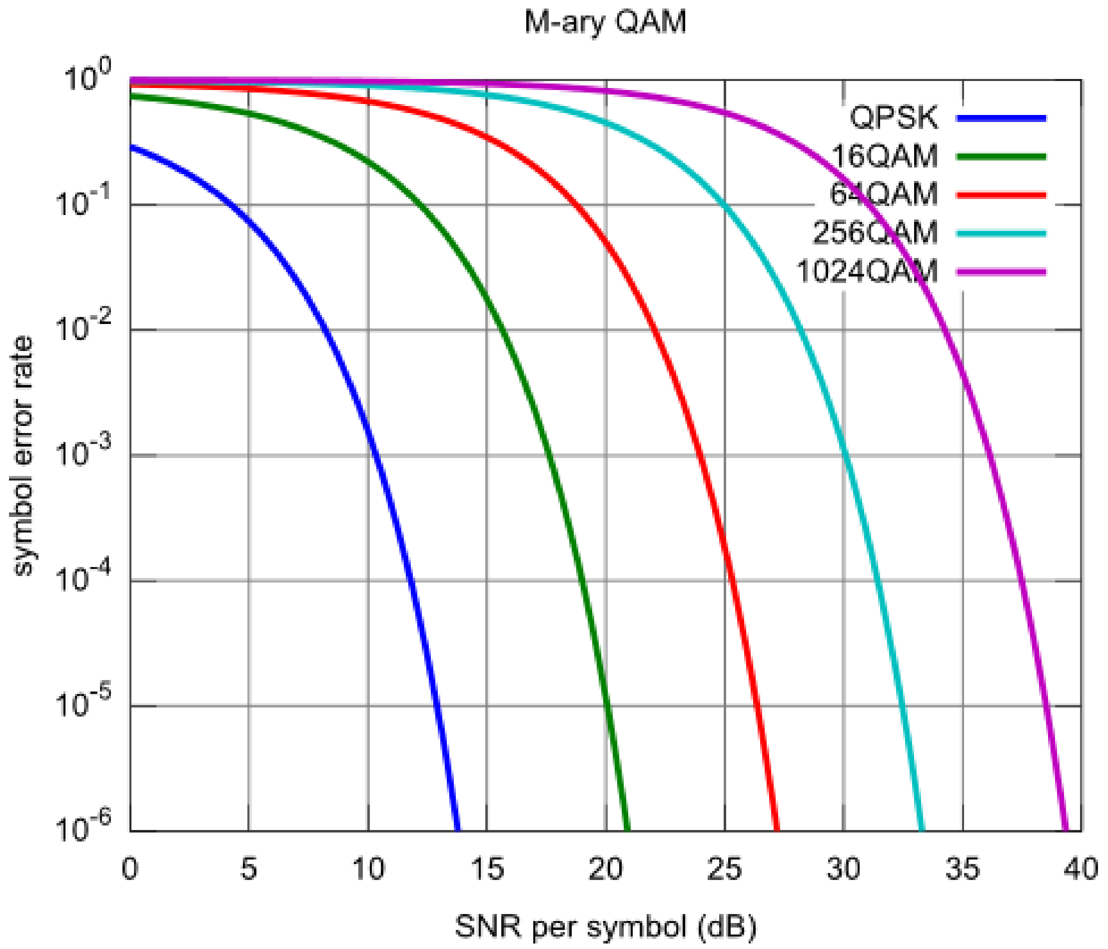
N-QAM Modulation

- **Similar to QPSK, N-QAM combines two orthogonal M-PAM modulations**
  - To maintain total power P after summing both carriers, the amplitude of each carrier is reduced
  - The number of M-PAM constellation points is $M = \sqrt{N}$

- **Start from the error rate of M-PAM:** $\varepsilon_{M-PAM} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2 \cdot 3}{M^2-1} \frac{E_s}{N_0}}\right)$
  - Substitute $E_s \rightarrow E_s/2$
  - Substitute $M = \sqrt{N}$

$$\varepsilon_{N-QAM} = \frac{2\left(\sqrt{N}-1\right)}{\sqrt{N}} Q\left(\sqrt{\frac{2 \cdot 3}{N-1} \frac{E_s/2}{N_0}}\right) = \frac{2\left(\sqrt{N}-1\right)}{\sqrt{N}} Q\left(\sqrt{\frac{3}{N-1} \frac{E_s}{N_0}}\right)$$

N-QAM Modulation Error Rate

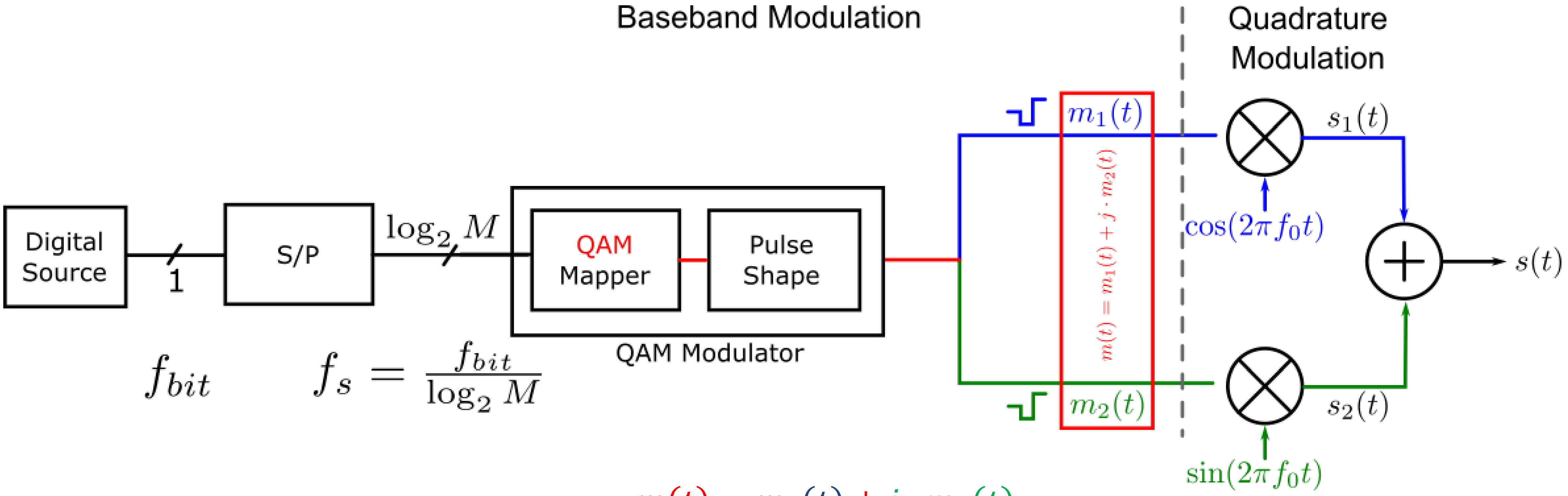


$$\varepsilon_{N-QAM} = \frac{2\left(\sqrt{N}-1\right)}{\sqrt{N}} Q\left(\sqrt{\frac{3}{N-1}\frac{E_s}{N_0}}\right)$$

- **Observation: For $N \rightarrow 4N$ we require approximately** $\frac{E_s}{N_0} \rightarrow 4 \cdot \frac{E_s}{N_0}$
  - Every 2 additional bits require +6dB SNR

General Complex-Valued Carrier Modulation

- **With the notion of complex-valued constellations, we can describe a complex-valued modulation, where only the choice of the constellation points allows for many degrees of freedom**



$$m(t) = m_1(t) + j \cdot m_2(t)$$

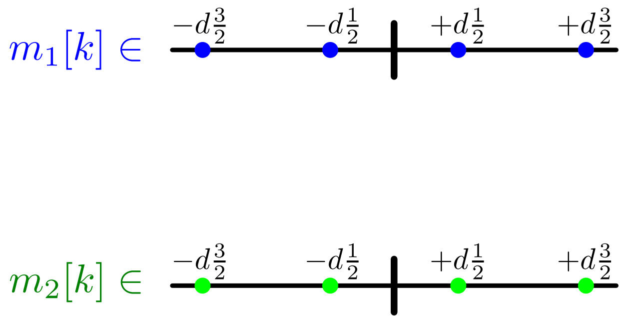
Complex-Valued QAM Constellations

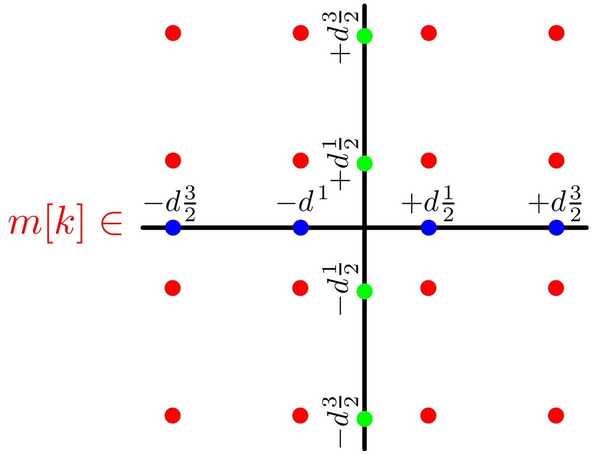
- **We can interpret the two real-valued inputs of the "Carrier Modulation" as a complex valued baseband signal**

$$m(t) = m_1(t) + j \cdot m_2(t)$$

$$m_1(t) = \sum_{k=-\infty}^{+\infty} m_1[k] \cdot p(t - k \cdot T_s) \qquad m_2(t) = \sum_{k=-\infty}^{+\infty} m_2[k] \cdot p(t - k \cdot T_s)$$

- **Interpret the two M-PAM constellation points $m_1[k]$ and $m_2[k]$ as a single complex-valued constellation point $m[k] = m_1[k] + j \cdot m_2[k]$**



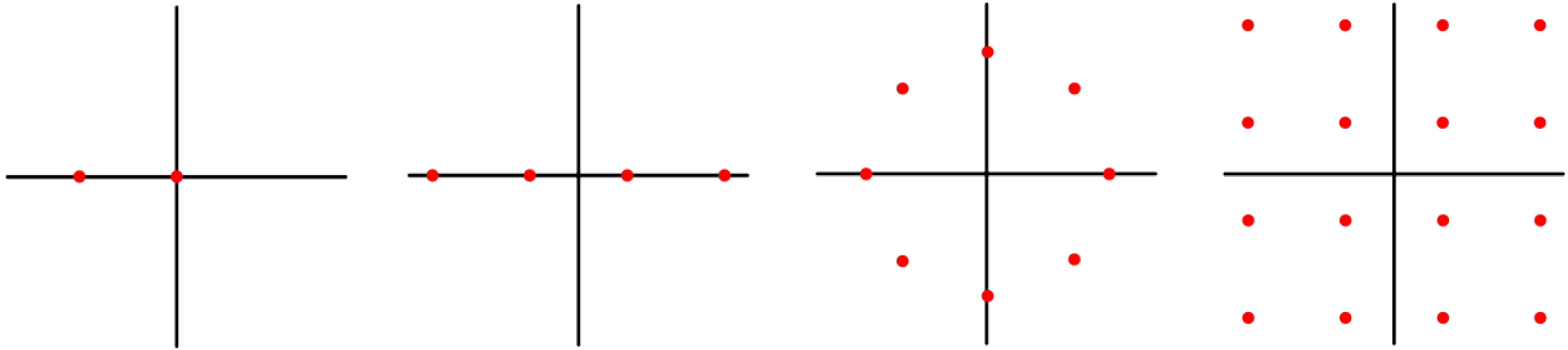


General Complex-Valued Constellations

- Instead of constructing complex constellation alphabets from two identical real-valued alphabets **we can define directly a single complex-valued alphabet**

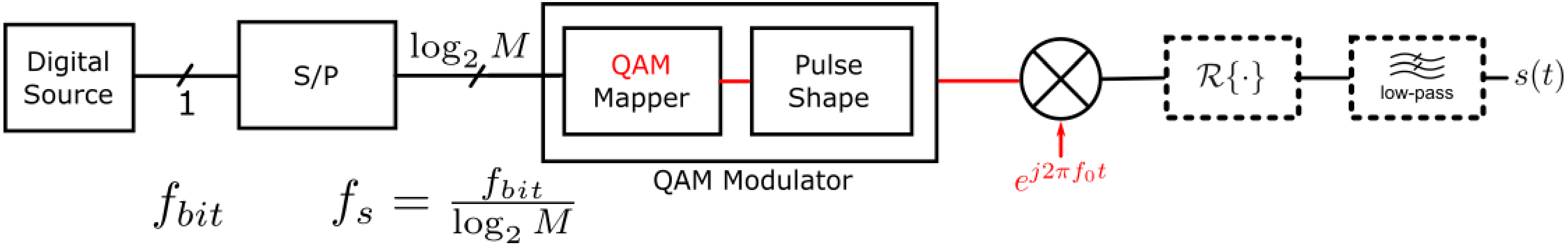
$$m(t) \in \mathcal{O}_M = \{x_1, x_2, \dots, x_M\} \text{ with } x_l \in \mathbb{C}$$

$$\bar{P}_{\mathcal{O}} = \frac{1}{M} \sum_{m=0}^{M-1} |x_m|^2$$



General Complex-Valued Carrier Modulation

- **With the notion of complex-valued constellations, we can describe a complex-valued modulation, where only the choice of the constellation points allows for many degrees of freedom**

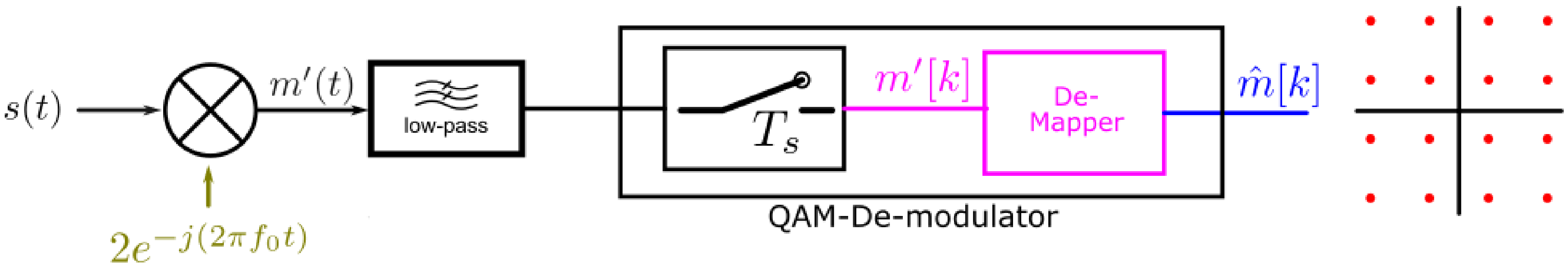


$$s(t) = \mathcal{R}\{m(t) \cdot e^{j(2\pi f_0 t)}\} =$$

$$[\mathcal{R}\{m(t)\} \cdot \cos(2\pi f_0 t) + \mathcal{I}\{m(t)\} \cdot \sin(2\pi f_0 t)]$$

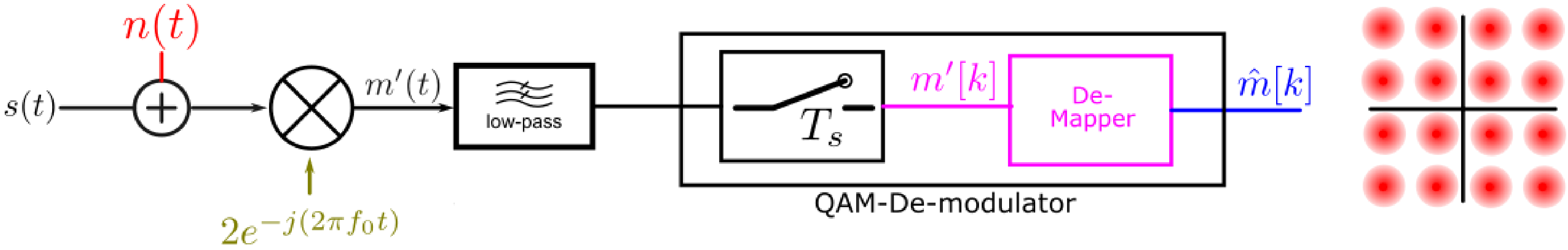
Coherent Receiver

- **Demodulation works with a Quadrature Demodulator (see WK4): no noise**



$$m'[k] = m[k] \cdot e^{j(2\pi f_0 k T_s)} \cdot e^{-j(2\pi f_0 k T_s)} = m[k]$$

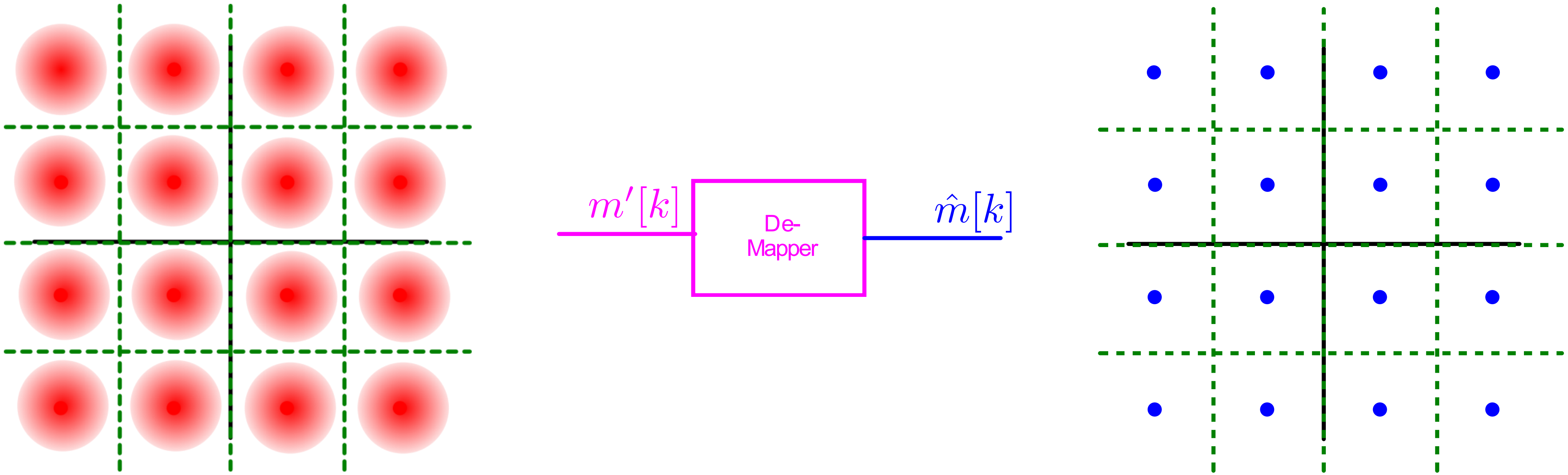
- **Demodulation with noise: signal displaced from ideal const. points by noise**



$$m'[k] = (m[k] + n[t]) \cdot e^{j(2\pi f_0 k T_s)} \cdot e^{-j(2\pi f_0 k T_s)} = m[k] + n[t]$$

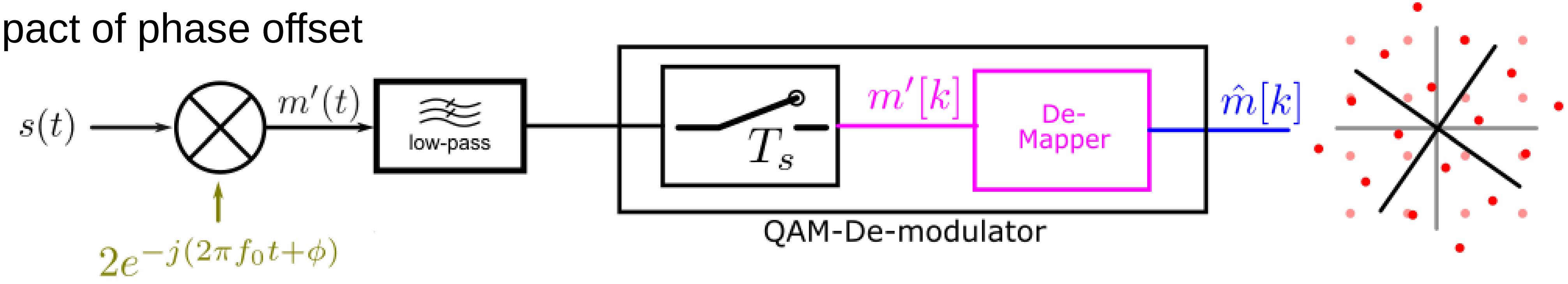
Coherent Receiver: Complex QAM Demapper

- **Demapping a (noisy) complex-valued symbol works in the same way as the slicer for real valued constellations**
  - We look for the closes constellation point from $\mathcal{O}_M$ to the received point $m'[k] = m[k] + n[t]$



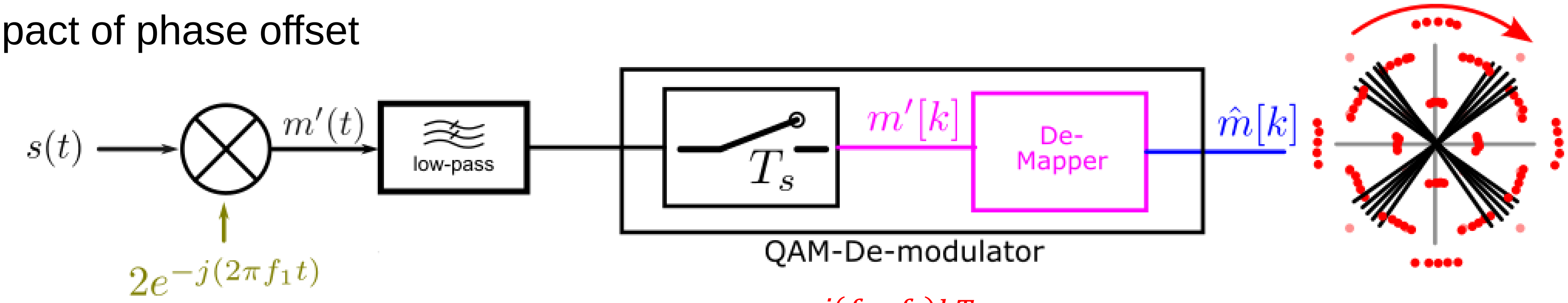
Impact of Non-Coherent QAM Receiver

- **A non-coherent receiver is not phase aligned with the transmitter of has even an offset in the frequency $f_1 \neq f_0$**
  - Impact of phase offset



$$m'[k] = m[k] \cdot e^{j\phi}$$

  - Impact of phase offset



$$m'[k] = m[k] \cdot e^{j(f_1 - f_2)kT_s}$$

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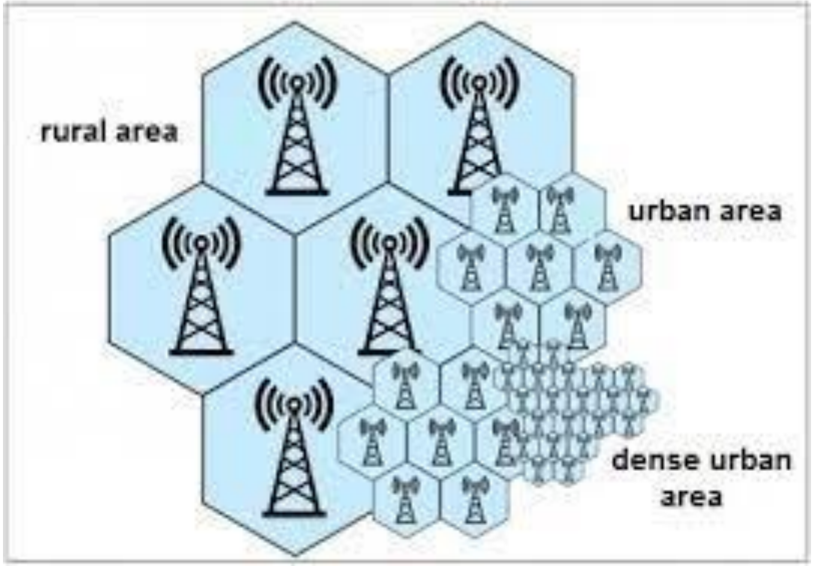
GSM

Cellular Networks

- **Cellular networks are the basis for all mobile communication standards**
  - Long range due to local connections on a global network (no point-to-point links)
  - Scalable capacity due to **frequency re-use** (densification)
  - Enable mobility over a large coverage area
  - Relatively simple manner to enable "**distributed radio access**" with manageable interference

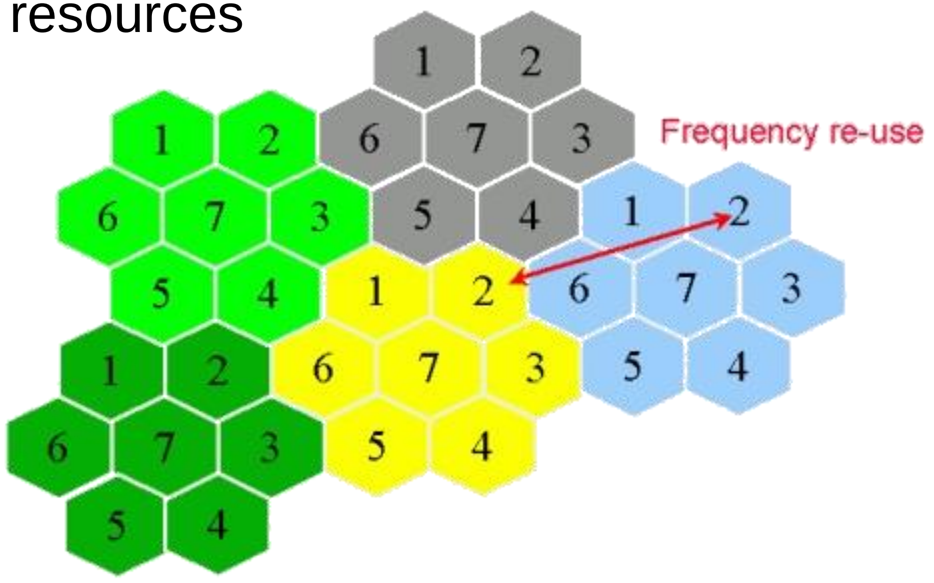
Point-to-point links
(limited range, high interference,
difficult to manage)



Cellular networks

Cellular Network Organization

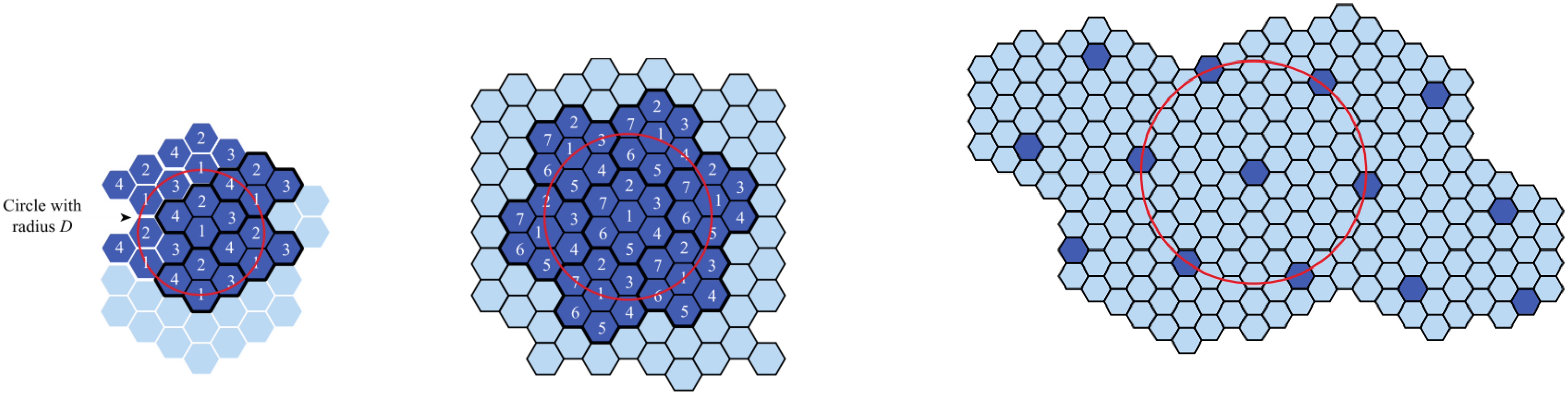
- **Objective:** high system capacity by re-using frequencies spatially
- **Cellular system**:
  - **World is partitioned into hexagonal cells** (approximately round)
    - Users in same area are assigned to a small set of frequencies and time slots (TDMA&FDMA)
    - Frequencies re-used in different cells, but adjacent cells must use different frequencies (interference)
  - **Cells** are **organized into clusters** to assign frequency resources
    - All cells in a cluster use different frequencies (numbers)
    - Different clusters re-use the same set of frequencies (frequency re-use)



Cellular Network Frequency Re-Use Patterns

- Geometrical considerations limit size of regular clusters to N cells where $N = i^2 + i \cdot j + j^2$ with $i, j = \mathbb{N}^+$

Cluster sizes : $N = 1,3,4,7,9,12, \ldots$



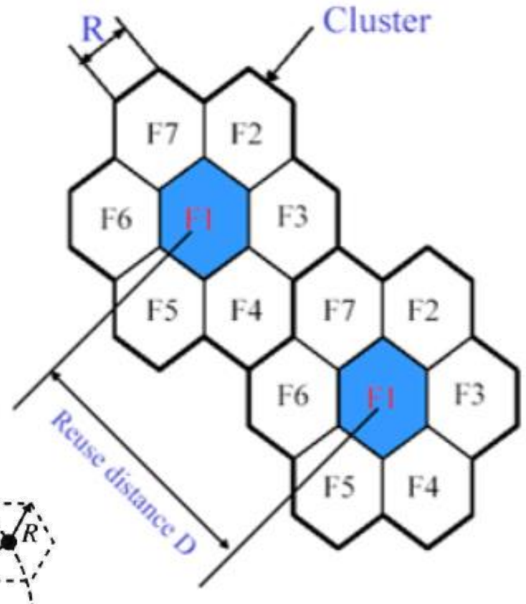
(a) Frequency reuse pattern for $N = 4$

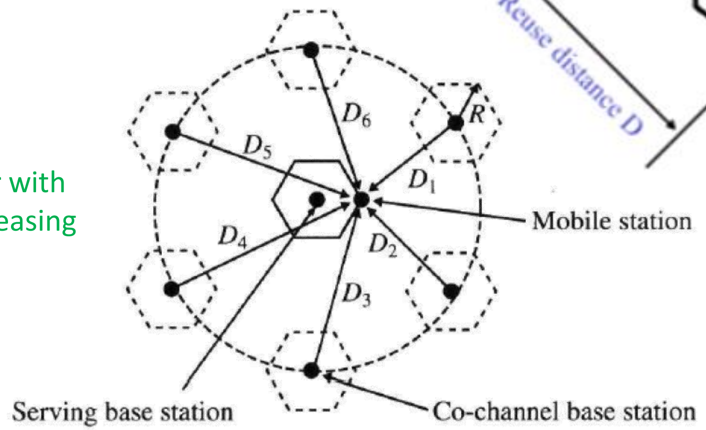
(b) Frequency reuse pattern for $N = 7$

(c) Black cells indicate a frequency reuse for $N = 19$

Frequency Re-Use and Interference (1)

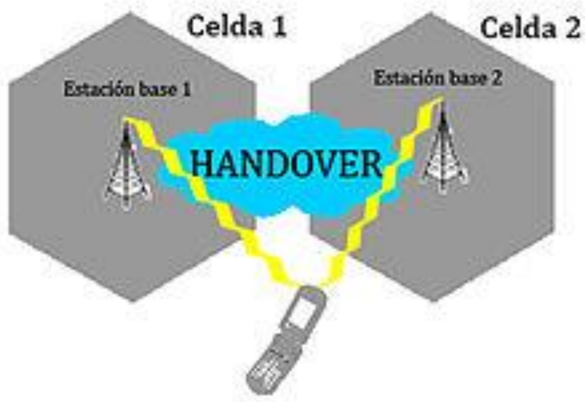
- **Organization into clusters is a trede-off** between
  - Re-use factor: how often frequencies can be re-used (less means lower capacity)
  - Interference: how much cells with the same frequency interfere with each other
  - Frequency re-use factor: $\frac{1}{N}$
  - #Channels (Bandwidth) per cell: $\frac{BW}{N}$ ← Worse with N increasing
  - Frequency re-use distance: $D = R \cdot \sqrt{3 \cdot N}$
  - Interference: $\sim D^{-\beta} = \left(R \cdot \sqrt{3 \cdot N}\right)^{-\beta}$
    - β is the pathloss exponent (2: free space, 3: more typical empirical urban)
    - Account only for the 1st tier of interferers (always 6)
  - Signal at cell edge: $\sim R^{-\beta}$
  - Signal-to-interference ratio: $\sim \frac{R^{-\beta}}{D^{-\beta}} = \left(\sqrt{3 \cdot N}\right)^{\beta}$ ← Better with N increasing

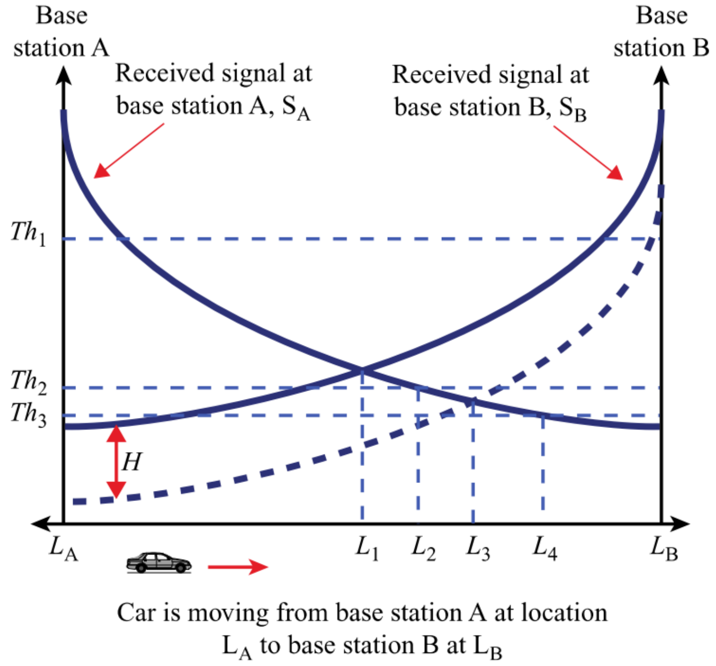




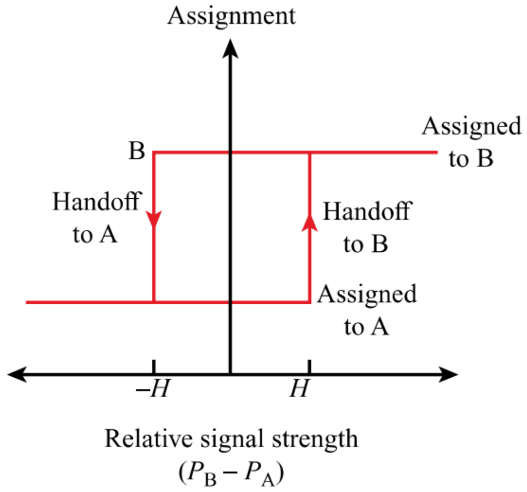
Hand-Over in Cellular GSM

- **Mobile users move across cell boundaries.**
  - When the signal gets weak, users need to be handed over to the next cell





(a) Handoff decision as a function of handoff scheme

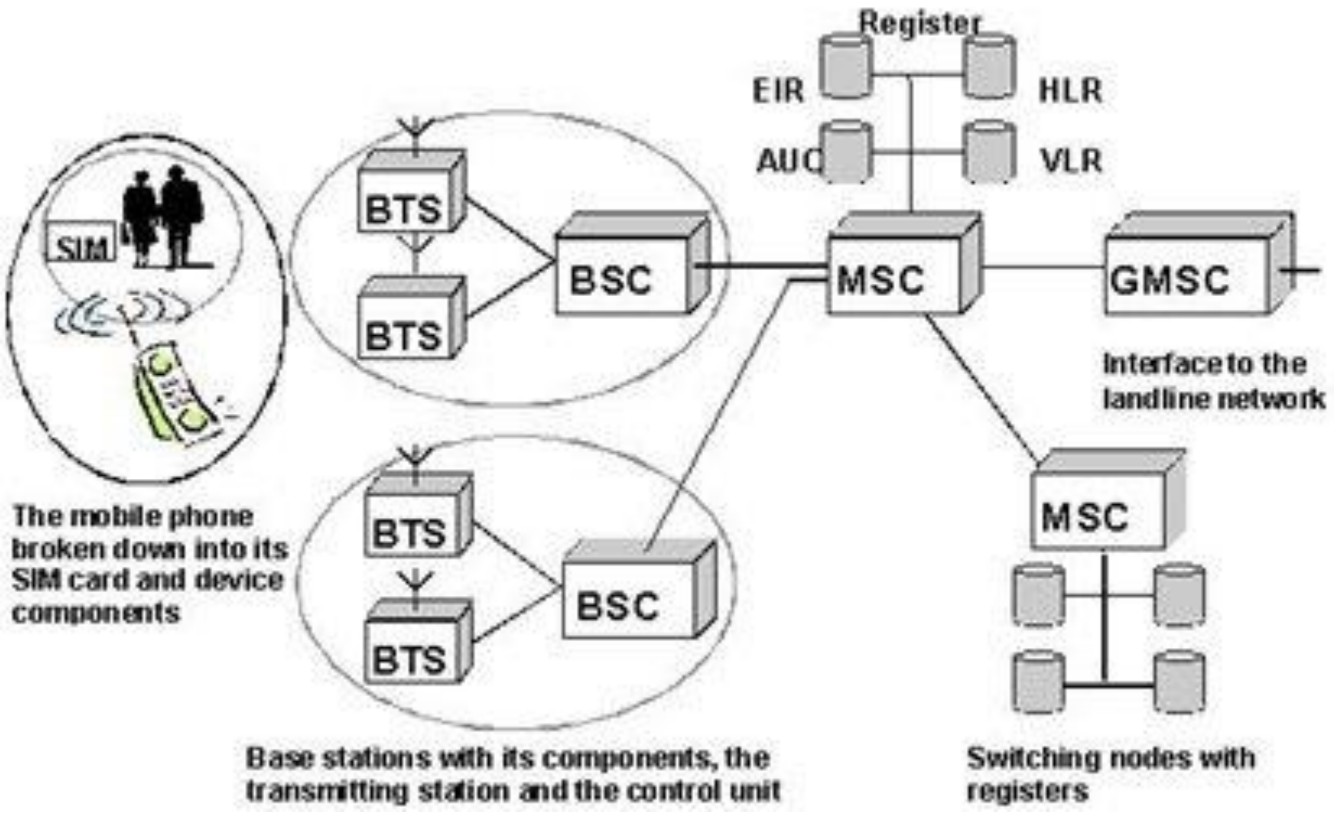


(b) Hysteresis mechanism

- GSM: Mobile assisted hand-over
  - Mobile measures channel strength
  - Mobile scans for better channels and sends measurements to Base Station
  - Base-station controller makes hand-over decisions

GSM Network Architecture

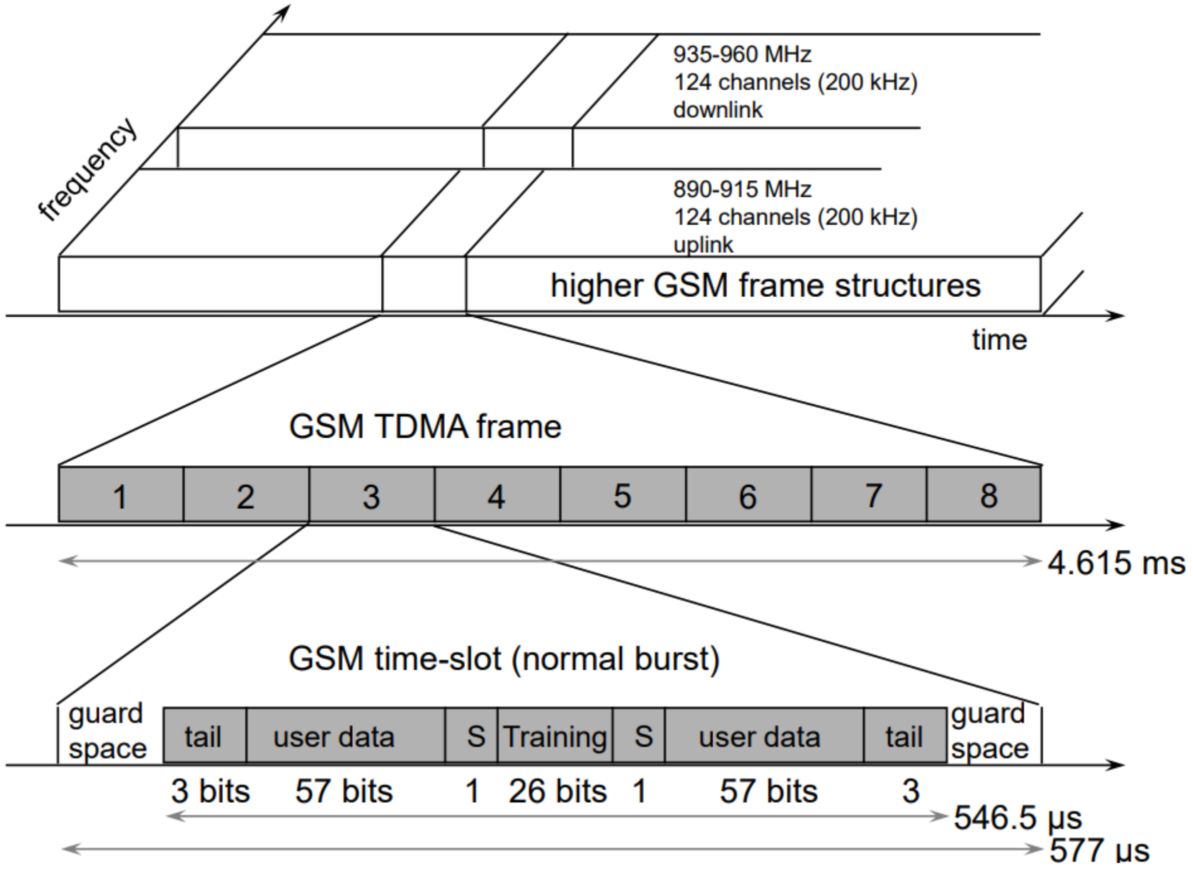
- **The GSM network is based on a number of components that manage calls**
  - GSM is a circuit-switched network: based on “connections” that are setup and maintained
  - Key network components:
    - **Mobile Station (MS)**: Handset and SIM
    - **Base Transceiver Station (BTS)**: Handles radio communication
    - **Base Station Controller (BSC)**: Manages multiple BTSs
    - **Mobile Switching Center (MSC)**: Switches calls, handles mobility
    - **Home Location Register (HLR)**: Permanent subscriber data
    - **Visitor Location Register (VLR)**: Temporary data for **roaming users**



+ Equipment Identity Register (EIR) & Authentication Center (AuC)

The GSM Radio Interface and Multiple Access

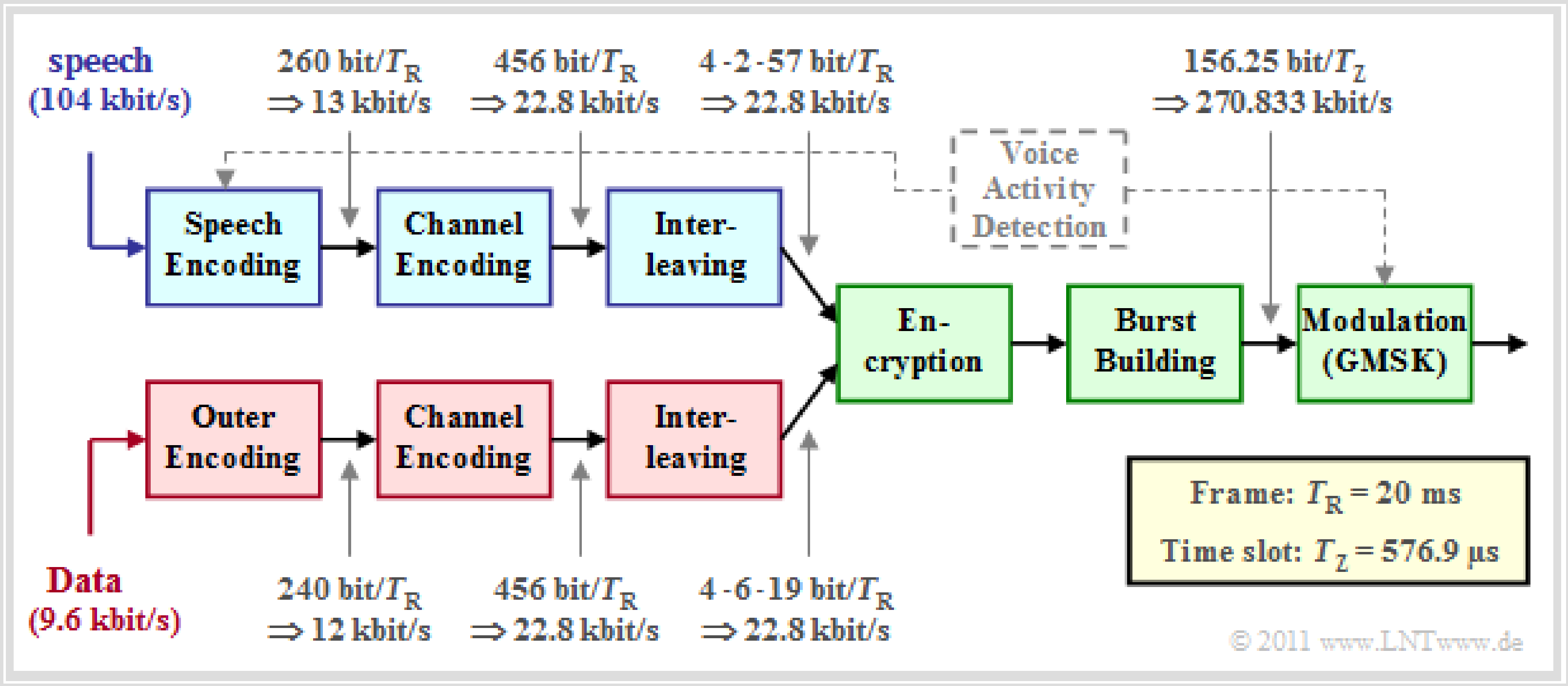
- **GSM user multiplexing is based on a combination of FDMA and TDMA**
  - Multiple carriers with bandwidth of 200 kHz (FDMA)
  - Each carrier supports 8 time-division multiplexed users (TDMA)
  - **Transmission organized in bursts**: Normal burst, frequency correction, synchronization, ...
  - Logical channels:
    - **Traffic Channels (TCH)**: Carry voice/data
    - **Control Channels**: SCH (sync), BCCH (broadcast), RACH (random access)
- **Up-link and down-link are based on frequency-division-duplex (FDD)**



GSM Physical Layer: Overview

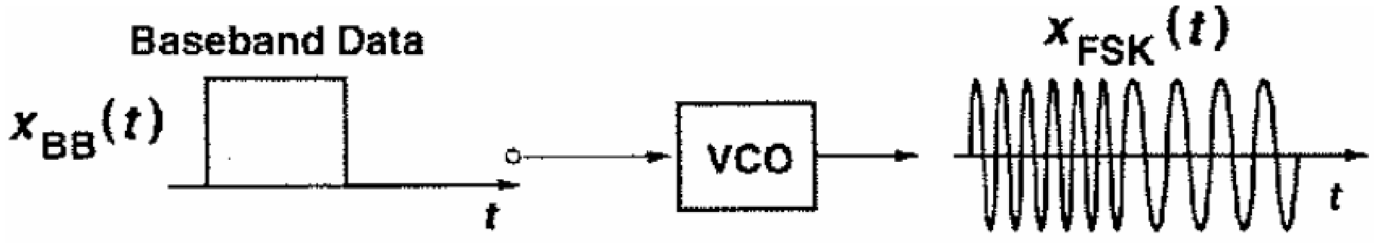
- **GSM was designed specifically for voice communication**
  - Frame structure and encoding organized specifically for compressed voice
  - Transmission is organized in frames of 20ms (suitable quantity for voice compression)

- **Key components:**
  - Speech encoding: compression
  - Channel encoding: error correction coding
  - Interleaving: shuffle bits for robustness
  - Burst-building: map to physical channels/slots
  - Modulation

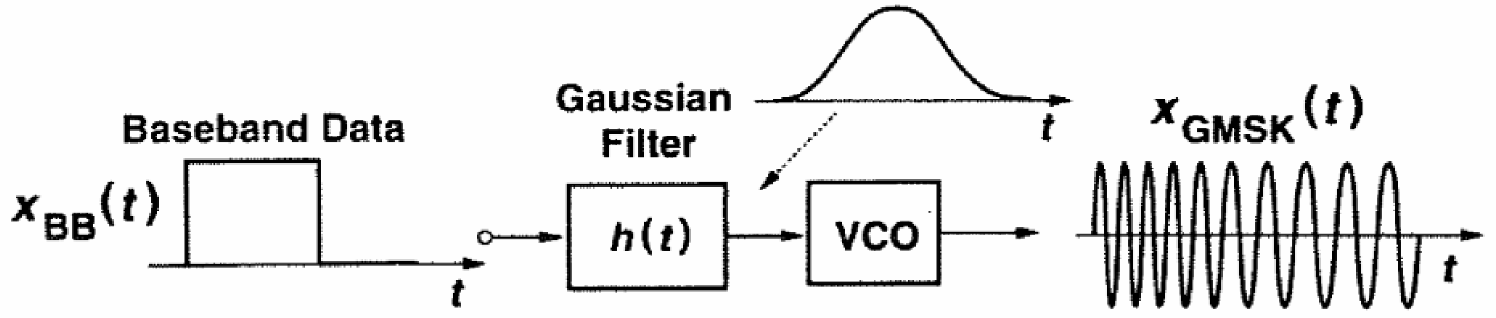


GSM Physical Layer: Modulation

- **GSM is based on Gaussian Minimum Shift Keying (GMSK), an improved (more narrow bandwidth) version of Minimum Shift Keying (MSK)**
  - MSK encodes data as two frequencies with minimum spacing to still be orthogonal (FSK: general version of MSK without enforcing minimum spacing between frequencies)



  - GMSK applies a filter to provide s smooth transition between frequencies:



Why GMSK Modulation

- **GMSK and QPSK are very comparable in terms of their BER with the same number of bits per symbol:**
  - Both have the same error probability: $\mathrm{P}_{GMSK} = \mathrm{P}_{QPSK} = Q\left(\sqrt{2 \cdot \frac{E_b}{N_0}}\right)$
- **Main difference lies in their overall spectrum and the overall bandwidth:**
  - Zero-crossing BW of MSK is wider then QPSK (with rectangular pulse-shape), but QPSK bandwidth decays slower (without suitable high-quality pulse shape)
  - MSK is often used in low-cost devices with very narrow BW requirement
  - GMSK has even lower BW then MSK
  - Further advantage: **(G)MSK has constant envelope**