EE-432
Systeme de Telecommunication

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Linear Passband (Carrier) Modulation
AM and Quadrature

Week 3: Table of Contents

Baseband Signals

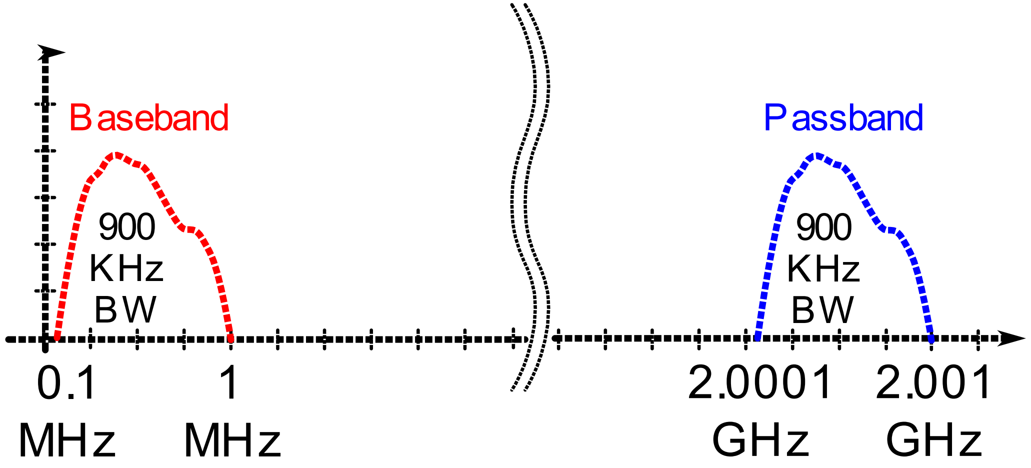
- **The baseband is the frequency band of the “original” signal** $\mathrm{m}(t)$
  - “Original signals” may be physical signals OR artificially constructed signals to represent data
- **Examples:**
  - Voice: 300 – 3700 Hz
  - High-fidelity audio: 0.001 – 20 KHz
  - Analog television (NTSC) video: 0.001 – 4.3 MHz
  - Digital ethernet (10 Mbps): 0.001 – 20 MHz
  - Any digital signal in general (X Mbps): 0.001 – 2*X MHz
- **Most baseband signals start at (or very close to) DC (0 Hz)**
  - and have a naturally or artificially limited bandwidth
- **Baseband signals are not necessarily real-valued**
  - We will see that we actually often prefer complex-valued baseband signals

Baseband Communication Limitations (1)

The low frequency band of baseband signals limits the available transmission media and transmission type.

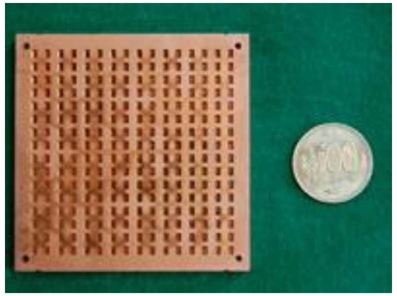
- **Impedance of the antenna must be matched to the medium**
  - Antenna size d proportional to the wavelength λ and therefore inversely proportional to the frequency
  - Good matching must be achieved over the entire frequency band

$$d \approx \frac{1}{f_{min}} \approx \frac{1}{f_{max}}$$



$29.9\text{m} < \lambda < 299\text{m}$ $\lambda \approx 149.425287\text{m}$

350 kHz antenna



16x16 60 GHz antennas

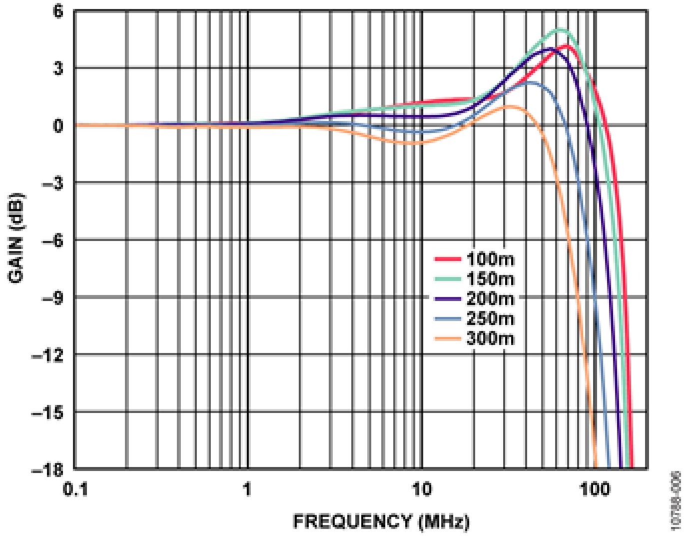
Baseband Communication Limitations (2)

The very similar (overlapping) frequency bands of different baseband signals causes interference between different transmissions

- **Every baseband signal needs to have its own channel with a suitable frequency characteristics (low attenuation for low frequencies)**
- **Only real-valued baseband signals can be transmitted**

Passband signals are used in wireline channels:

- **Wires confine the signal and isolate links from each other**
- **Wires have a low-pass frequency characteristic**



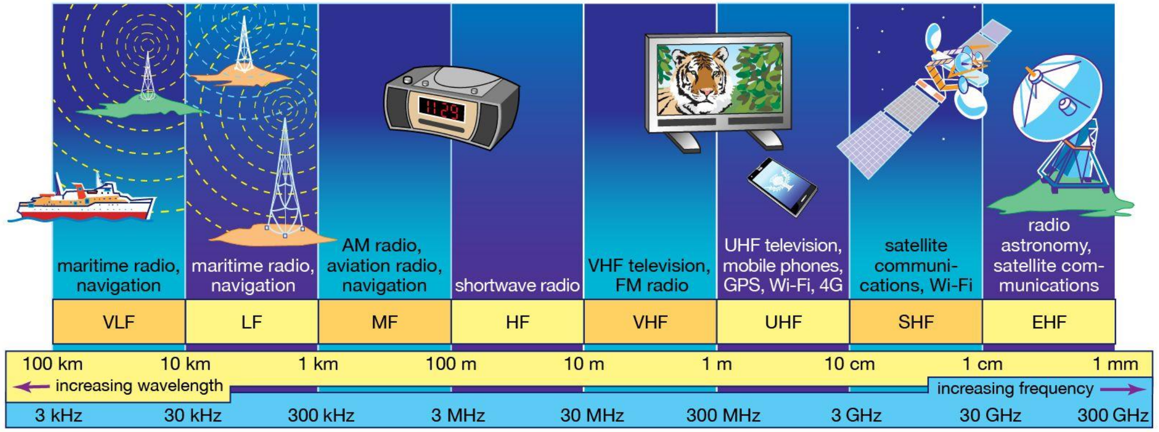
Cat-6 Cable Freq. Characteristics

Passband Frequencies and Radio Spectrum (1)

"Passband Signals" or "Carrier Signals" are located around a given "carrier frequency", often referred to as f_c or f_0

- **Wireless communication requires frequencies higher than the baseband BW**
- **The occupied bandwidth of a single passband signal is typically much less than the carrier frequency**
- **Multiple Passband signals can be sent at same**
  - Frequency division multiple access

- **Radio spectrum is coarsely divided into ranges corresponding to different**
  - Amounts of available bandwidth
  - The target reach of the radio links
  - The application



Passband Frequencies and Radio Spectrum (2)

- **The use of radio frequencies is regulated to avoid interference**
  - Regulation on national level (government) with international coordination

- **Process and involved institutions**
  - **International Telecommunication Union (ITU)**
    - Allocates frequency bands to different services at the global level
    - World Radiocommunication Congress (WRC) every 3 to 4 years
    - (e.g., WRC-23 initial discussion on 6G)
  - European Conference of Postal and Telecommunications Administrations (CEPT)
    - Refines ITU guidelines (on European level)
  - **National authorities** (CH: Federal Office of Communications (OFCOM - BAKOM))
    - Create national frequency allocation tables based on ITU guidelines
    - Issues licenses for frequency use and monitors compliance
    - Organizes spectrum auctions for mobile operators (e.g., auctions for 5G in 2019)

https://www.fedlex.admin.ch/filestore/fedlex.data.admin.ch/eli/fgae/2024/85/fr/pdf-a/fedlex-data-admin-ch-eli-fgae-2024-85-fr-pdf-a.pdf

Passband (Carrier) Signals

- **Passband signals are always real-valued signals**
- **Any passband signal can be described in the form of**

$$s(t) = A(t) \times \cos\left(2 \cdot \pi \cdot \left(f_0 + f(t)\right) \cdot t + \phi(t)\right)$$

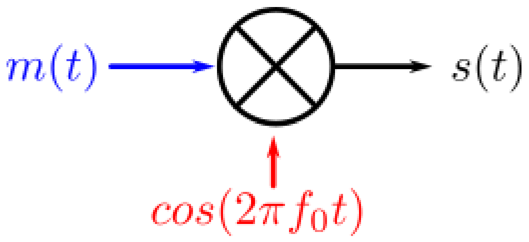
  - The sinusoid $\cos(2 \cdot \pi \cdot f_0 \cdot t)$ at the "carrier frequency" f_o is often referred to as the "carrier"
- **Modulation is the process of generating a carrier of this form from a "carrier"**
- $A(t)$, $f(t)$, **and** $\phi(t)$ **represent the original baseband signal** $\mathbf{m}(\boldsymbol{t})$ **to be transmitted and "modulate" the properties of the carrier, individually or jointly.**
  - **Amplitude modulation**: $A(t)$ is proportional to the baseband signal $\mathrm{m}(t)$
  - **Frequency modulation**: $f(t)$ is proportional to the baseband signal $\mathrm{m}(t)$
  - **Phase modulation**: $\phi(t)$ is proportional to the baseband signal $\mathrm{m}(t)$

(Frequency modulation and phase modulation: Angle modulation)

AM: Amplitude Modulation

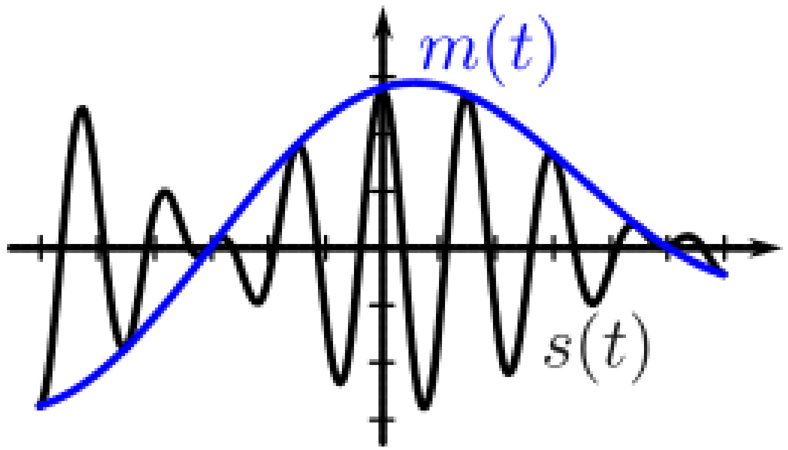
- **Consider a baseband signal** $m(t)$ **with FT** $M(f)$

- **Amplitude modulation of a carrier** $\cos(2 \cdot \pi \cdot f_0 \cdot t)$ **at** f_0



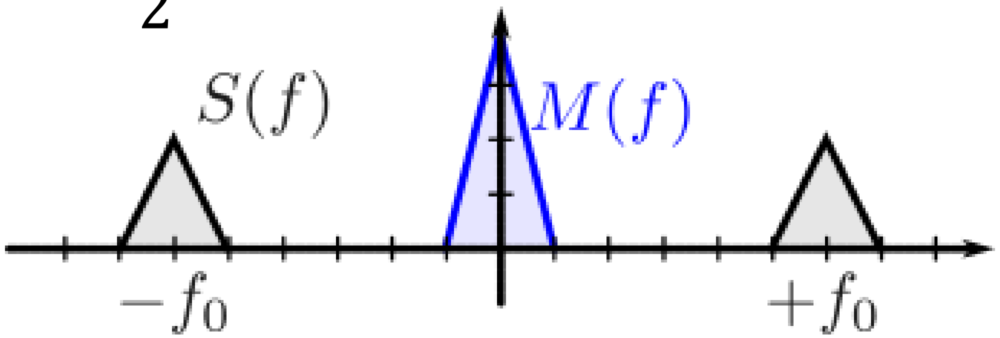
Time domain

$$s(t) = m(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t)$$



Frequency domain

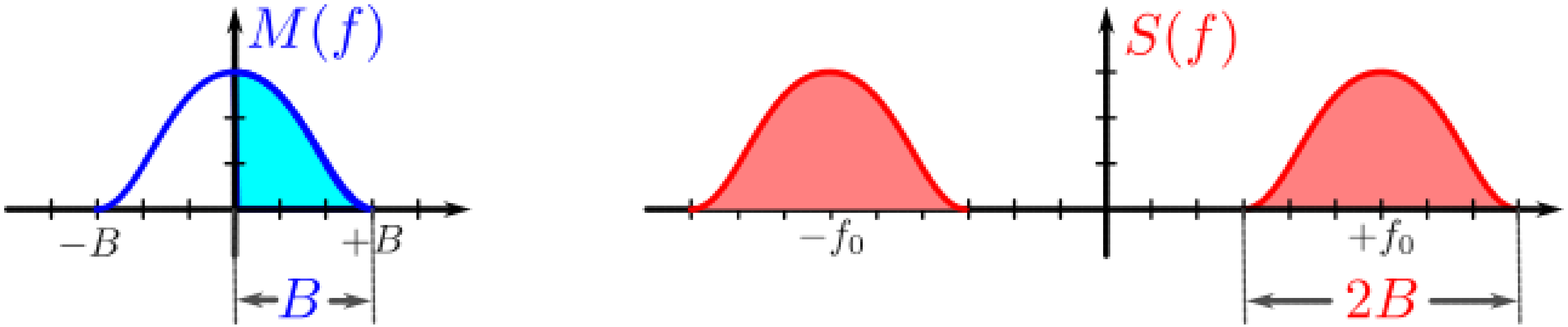
$$S(f) = M(f) \times \frac{1}{2}[\delta(f + f_0) + \delta(f - f_0)]$$
$$= \frac{1}{2}[M(f + f_0) + M(f - f_0)]$$



AM: Baseband vs. Passband Bandwidth (1)

- **The bandwidth of a signal describes the occupied frequency range, considering only the positive frequencies**
  - Bandwidth of a baseband (BB) signal: given by the highest frequency component
  - Bandwidth of a passband (PB) signal: the occupied frequency range around the carrier

- **Compare the baseband bandwidth of a BB signal $M(f)$ with the passband bandwidth of the corresponding PB signal $S(f)$**

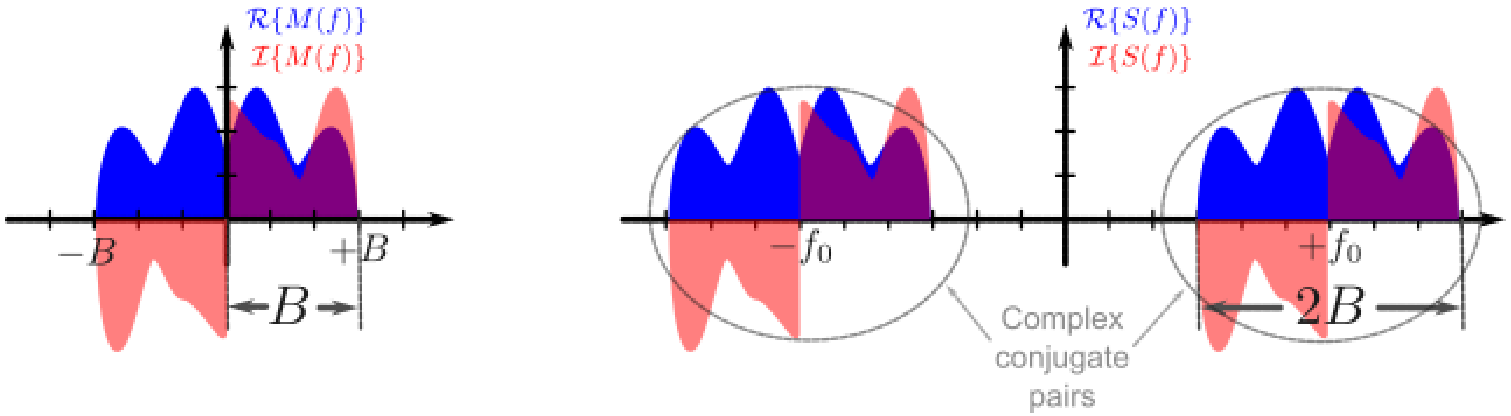


AM modulated passband signal occupies twice the bandwidth of the BB signal

AM: Baseband vs. Passband Bandwidth (2)

- **AM modulation starts from a real-valued BB signal**
  - Spectrum of the real-valued BB signal is complex conjugate symmetric $\boldsymbol{M}(-\boldsymbol{f}) = \boldsymbol{M}^*(\boldsymbol{f})$
  - The negative frequencies contain no additional information (are redundant)

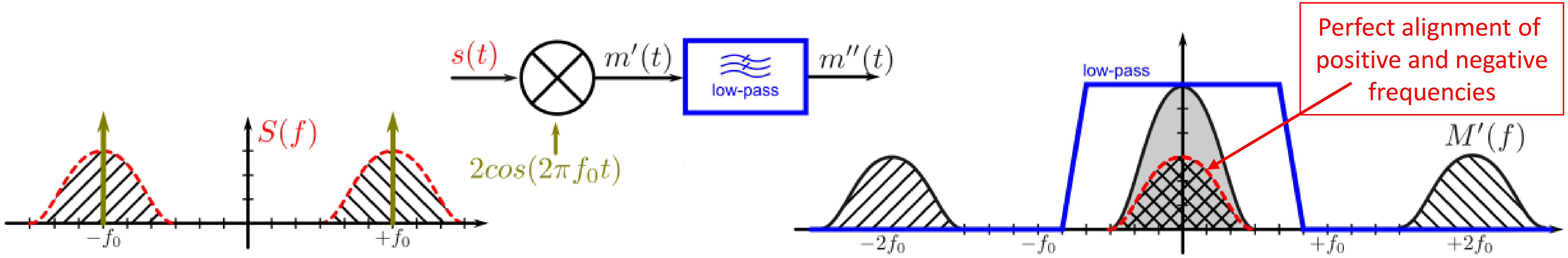
- **In the AM modulated signal, both positive and negative frequencies are "visible" and occupy spectrum around the carrier → Redundancy!**



- **AM modulated spectrum is again complex conjugate symmetric → Real-Valued Signal!**

Coherent AM Demodulation

- **To demodulate an AM signal, we need to shift it back to the baseband and remove spectral components other than the baseband***
  - **Coherent**: demodulating and modulating carriers are phase and frequency aligned



$$m''(t) = \mathrm{LPF}\{s(t) \cdot 2\cos 2\pi f_0 t\}$$

$$\frac{1}{2}[M(f+f_0) + M(f-f_0)] \times [\delta(f+f_0) + \delta(f-f_0)] =$$

$$= \frac{1}{2}M(f+2f_0) + \underbrace{\frac{1}{2}M(f) + \frac{1}{2}M(f)}_{M(f)} + \frac{1}{2}M(f-2f_0)$$

* How would you choose the bandwidth of this filter?

AM Demodulation with Phase Offset

- **Transmitter and receiver do not have a common time reference and are anyway an unknown distance apart** (signal needs time to propagate*)
- **Consider a phase difference ϕ between modulation and demodulation**

$$s(t) = m(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t)$$

$$m''(t) = \mathrm{LPF}\{s(t) \cdot 2\cos(2\pi f_0 t + \phi)\}$$

$$\mathrm{LPF}\left\{\frac{1}{2}[M(f+f_0) + M(f-f_0)] \times \left[\delta(f+f_0)e^{-j\phi} + \delta(f-f_0)e^{+j\phi}\right]\right\} =$$

$$= \mathrm{LPF}\left\{\frac{1}{2}M(f+2f_0)e^{-j\phi} + \underbrace{\frac{1}{2}M(f)e^{-j\phi} + \frac{1}{2}M(f)\,e^{+j\phi}}_{\frac{1}{2}M(f)\cdot\underbrace{\left(e^{-j\phi}+e^{+j\phi}\right)}_{\cos\phi}} + \frac{1}{2}M(f-2f_0)e^{+j\phi}\right\}$$

$$\frac{1}{2}M(f) \cdot \cos\phi$$

Attenuation depending on the phase γ

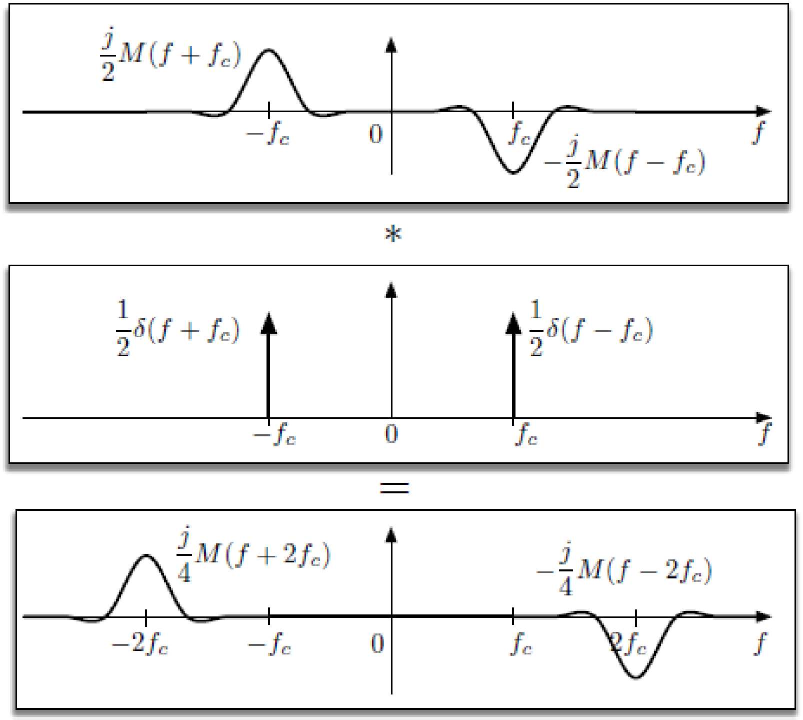
* How much time will the signal need to propagate and how does this translate into phase (assuming a common time 0)

AM Demodulation with Phase Offset

- **In practice, the phase difference ϕ is unknown and varies rapidly***
- Consider the example of $\gamma = -\frac{\pi}{2}$

$$\frac{1}{2} M(f) \cdot \cos \phi$$

  - Received signal vanishes

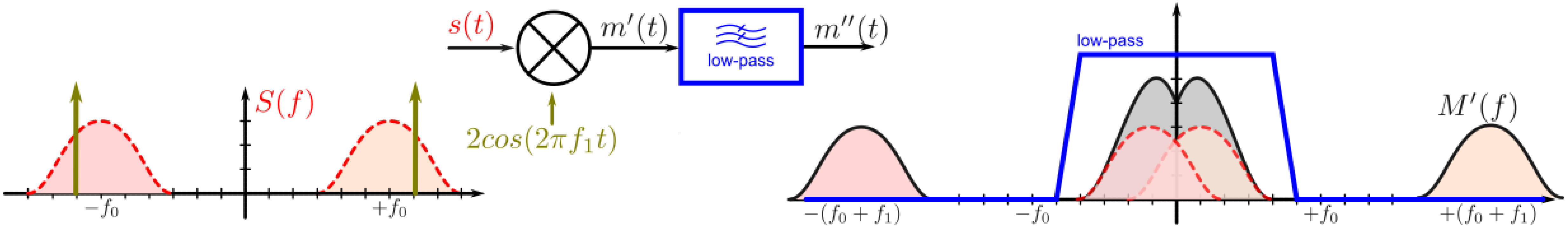


Note: in the figure, the demodulating carrier is $\cos(2\pi f_0 t + \gamma)$ which explains the additional factor of 1/2

* What happens when transmitted and receiver have slightly different frequencies (~<1Hz difference)

AM Demodulation with Significant Frequency Offset

- **Usually, it is not only difficult to align the phase of transmitter and receiver, but it is also difficult to align the frequencies**
  - Need to expect a significant frequency offset between TX f_0 and RX f_1
  - This offset can easily be a considerable percentage of the BB bandwidth

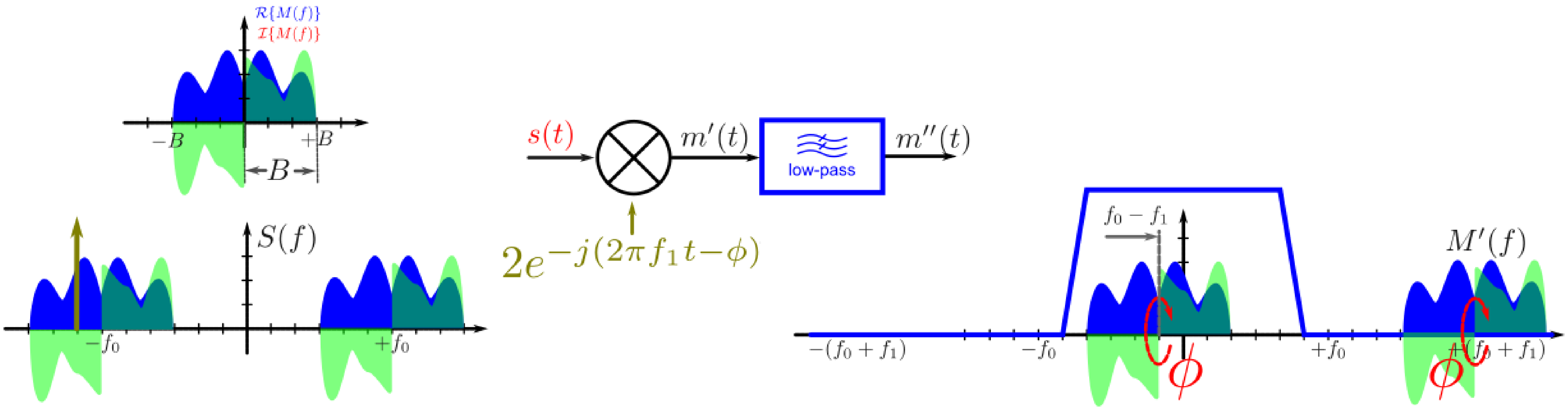


- **Positive and negative passband spectra no longer overlap perfectly in the baseband (around DC) → Aliasing!**

Frequency offsets between TX and RX distort the received signal

AM Demodulation with a Complex Sinusoid (1)

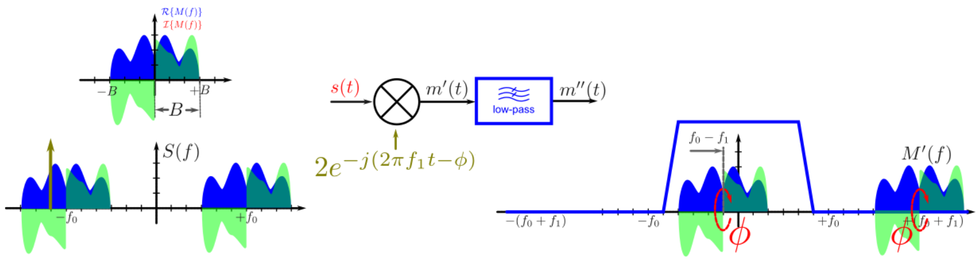
- **Goal: avoid aliasing (overlap from negative and positive frequencies)**
- **Idea: a complex sinusoid only <u>shifts</u> the signal in frequency domain**
  - Only the negative or positive passband signal lands around DC



$$m''(t) = \mathrm{LPF}\left\{s(t) \cdot 2e^{-j(2\pi f_1 t - \phi)}\right\}$$

AM Demodulation with a Complex Sinusoid (2)

- **Demodulation with a complex sinusoid with mismatched frequency f_1 and phase offset ϕ**



$$m''(t) = \mathrm{LPF}\left\{s(t)\cdot 2e^{-j(2\pi f_1 t-\phi)}\right\} = ??$$

$$\Delta f = f_0 - f_1$$

  - We find the solution in the frequency domain first:

$$\mathrm{LPF}\left\{\frac{1}{2}[M(f+f_0)+M(f-f_0)]\times\delta(f+f_1)e^{-j\phi}\right\} =$$

$$\mathrm{LPF}\left\{M\left(f+\underbrace{(f_0-f_1)}_{\Delta f}\right)e^{+j\phi}+M(f-f_0-f_1)e^{+j\phi}\right\} = M(f+\Delta f)e^{+j\phi}$$

$$m''(t) = m(t)\cdot e^{-j2\pi\Delta f\cdot t}\cdot e^{+j\phi}$$

- **Complex demodulation avoids aliasing, BUT the demodulated signal**
  - can be complex valued (if $\phi \neq 0$ or $\Delta f \neq 0$)
  - Has a (complex) phasor that changes with ϕ or changes even over time when $\Delta f \neq 0$

AM Demodulation with a Complex Sinusoid (3)

- **Consider the special case of coherent demodulation with $\phi = 0$ and $\Delta f = 0$**

$$m''(t) = m(t) \cdot e^{-j2\pi\Delta f \cdot t} \cdot e^{+j\phi} = m(t)$$

- **Note that:**

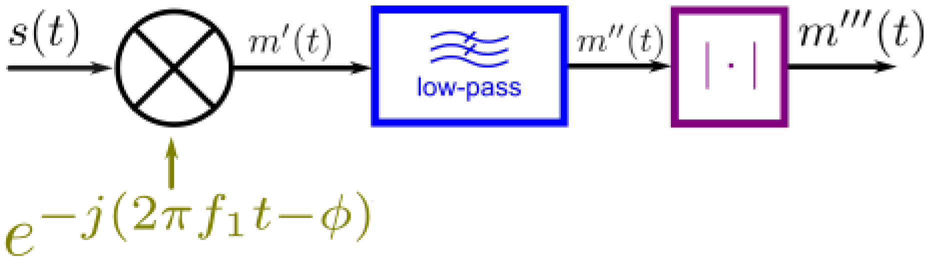
$$\mathcal{R}\{m''(t)\} = m(t)$$

$$\mathcal{J}\{m''(t)\} \equiv 0$$

We receive the real-valued transmitted signal (modulated with a cosine) on the real part of the complex-valued demodulated signal $m''(t)$

Non-Coherent AM Demodulation

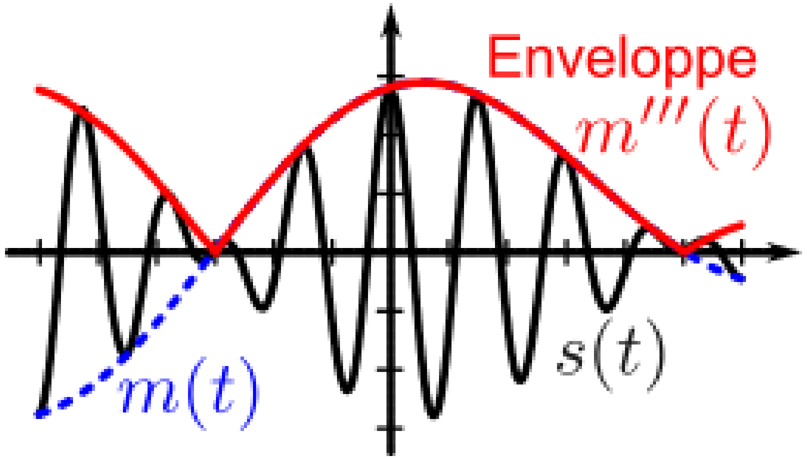
- **Non-coherent modulation: modulate a signal in a way that it can be recovered independently from the phase at the demodulator**



- **Idea for non-coherent demodulation**
  - Envelope demodulation:

$$m'''(t) = \left|\mathrm{LPF}\left\{s(t)\cdot 2e^{-j(2\pi f_1 t-\phi)}\right\}\right| = \underbrace{\left|m(t)\cdot e^{-j(2\pi \Delta \mathrm{f}\cdot t-\phi)}\right|}_{\textbf{ENVELOPE}}$$

- **Unfortunately, this demodulator recovers only $|m(t)|$ which leads to distortions for $m(t) < 0$**



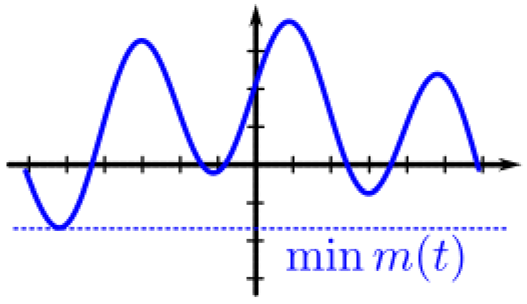
AM Modulation with Carrier

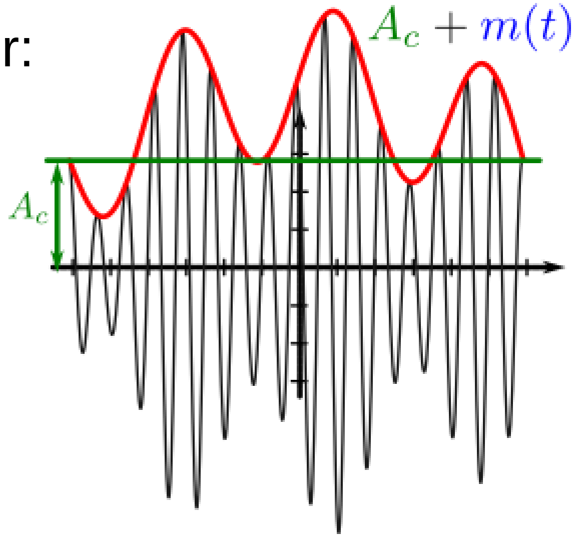
- **To enable non-coherent modulation, we must ensure that the modulated signal is always positive** $A(t) \geq 0$

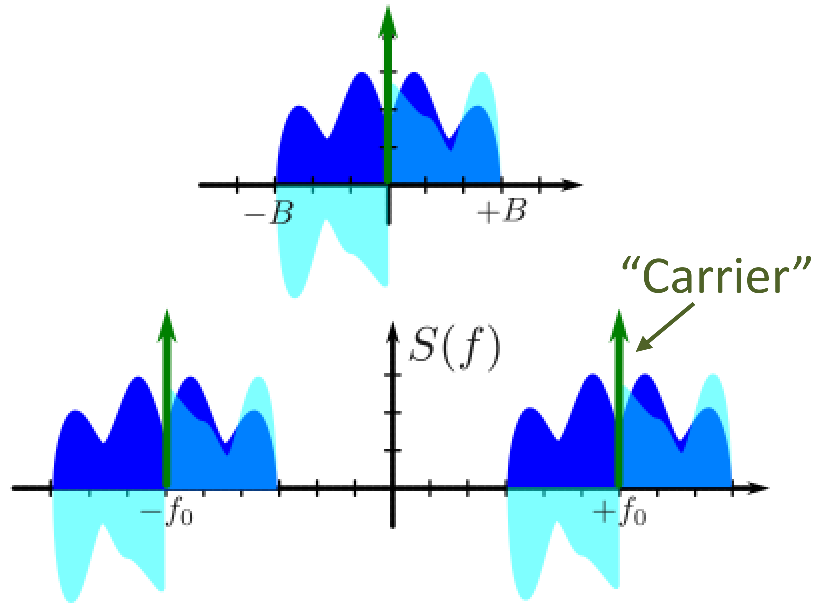
$$s(t) = \underbrace{\left(A_c + m(t)\right)}_{A(t)} \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) \text{ with } A_c \geq -\min_t m(t)$$

$$S(f) = \frac{1}{2}[M(f + f_0) + M(f - f_0)] + \frac{A_c}{2}[\delta(f + f_0) + \delta(f - f_0)]$$

  - Double side band with carrier:







AM with Carrier Demodulation

- **Demodulate an AM signal with carrier using the envelope demodulator:**
  - Complex valued with frequency and phase offset:

$$A'''(t) = \left|\mathrm{LPF}\left\{s(t)\cdot 2e^{-j(2\pi f_1 t-\phi)}\right\}\right| = \underbrace{\left|A(t)\cdot e^{-j(2\pi\Delta\mathrm{f}\cdot t-\phi)}\right|}_{\textbf{ENVELOPE}}$$

$$= |A(t)|\cdot\underbrace{\left|e^{-j(2\pi\Delta\mathrm{f}\cdot t-\phi)}\right|}_{1} = |A(t)| \text{ only since } m(t)\geq 0$$

  - Real valued (without frequency or phase offset):

$$A'''(t) = |\mathrm{LPF}\{s(t)\cdot\cos 2\pi f_0 t\}| = |\mathrm{LPF}\{A(t)\cdot\cos^2 2\pi f_0 t\}|$$

$$= \left|\mathrm{LPF}\left\{A(t)\cdot\frac{1}{2}(1+\cos(2\cdot 2\pi f_0 t))\right\}\right| = \frac{1}{2}|A(t)|$$

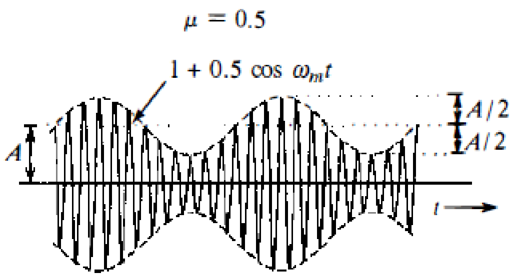
AM Modulation with Carrier (Properties)

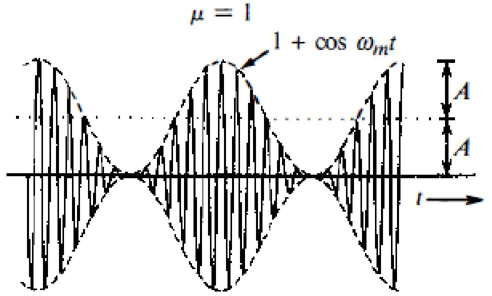
A few notes are in order:

- **The baseband signal is often symmetric ($\min_t m(t) = \max_t m(t) = \max_t |m(t)|$)**
- **You will sometimes see a notation of the transmitted signal as**

$$s(t) = \left(A_c + \mu \cdot m(t)\right) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t)$$

  - $A_c \geq -\min_t m(t)$
  - **μ: is the modulation index**. We must choose $0 < \mu < 1$ to avoid distortion.





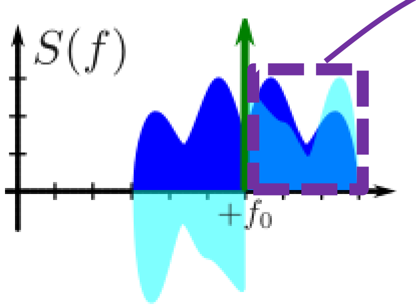
- **We can calculate μ from the maximum baseband amplitude**

$$\mu = \frac{\max|m(t)|}{A_c}$$

AM Modulation with Carrier (Properties)

A few notes are in order:

- **The choice of μ determines the power efficiency ρ of the transmission** (i.e., the useful power in the signal vs. the power spent on the carrier)



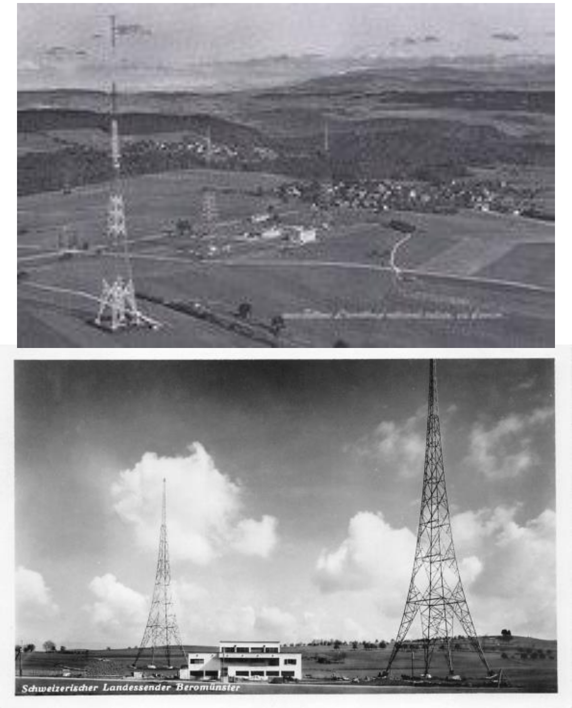
$$\rho = \frac{\mu^2 \cdot \frac{P_m}{2}}{\mu^2 \cdot P_m + \frac{A_c^2}{2}} = \frac{1}{2 + \frac{A_c^2}{\mu^2 \cdot P_m}} < \frac{1}{2 + \frac{1}{\mu^2}} < \frac{1}{3}$$

Bound based on $P_m \leq A_c^2, \mu \leq 1$

- If the maximum amplitude of the baseband signal is not well known, AM is quite power-inefficient since modulation index μ must be chosen conservatively (small)

AM Radio Broadcast

- **Early radio broadcast was based on AM since**
  - AM modulation is relatively simple to realize with basic components
  - Low bandwidth of audio signals occupy only a limited spectrum
  - Enough space even at low frequencies for many radio AM stations



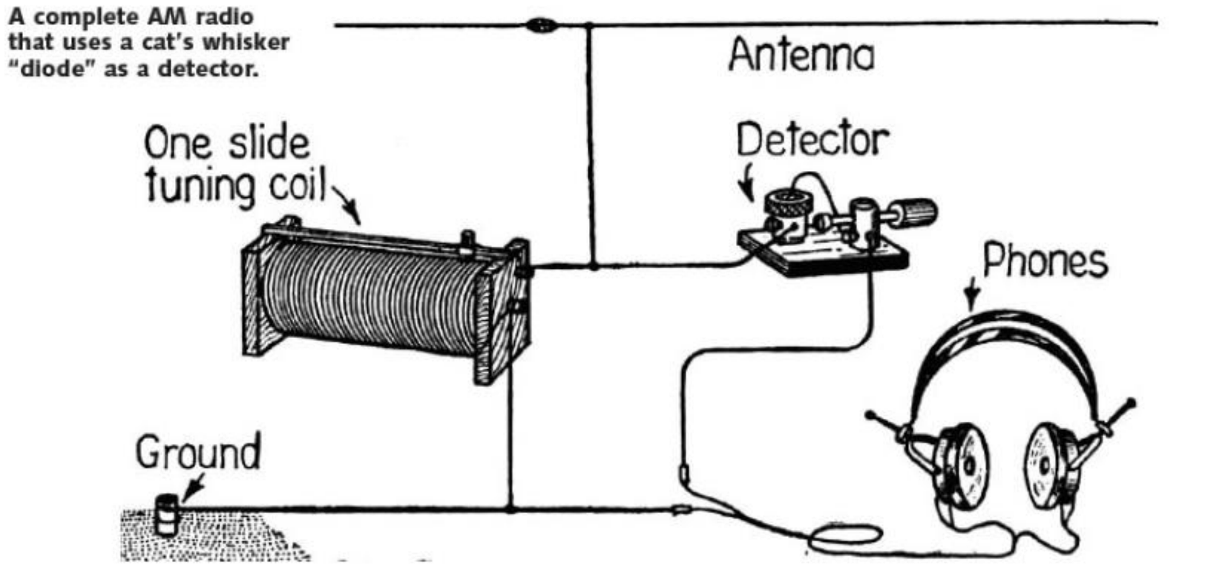
Sottens
& Beromünster

- **First commercial AM radio stations in the 1920s**
  - Switzerland: commercial AM radio started in 1930s
  - Three major medium-wave transmitters:
    - Radio Sottens: French speaking population
    - Radio Beromünster: German speaking population
    - Monte Ceneri: Italian speaking population
  - AM radio broadcast started to loose popularity after introduction of FM radio in the 1950s/1960s
  - Last AM radio station in Switzerland operation in 2010

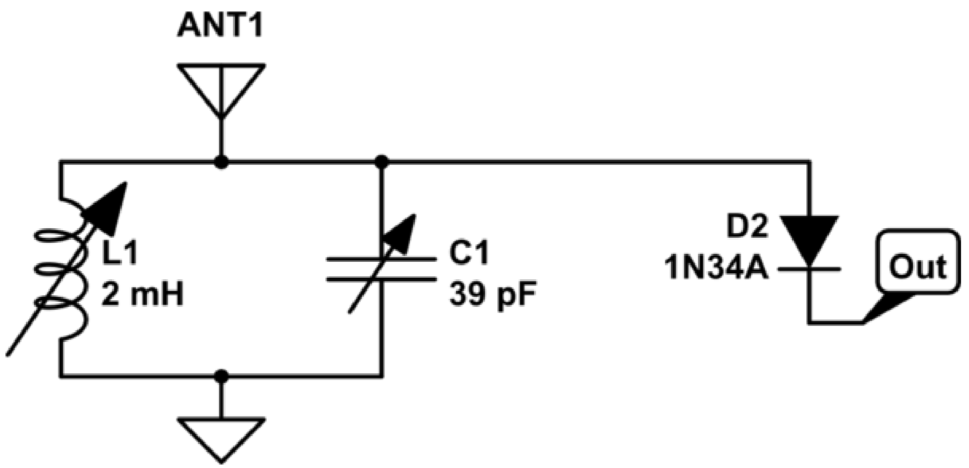
Practical AM Radio Receiver (1)

- **AM radio was possible due to the simplicity of the receiver**
  - Receiver can be realized even without active components (with sufficient received power)

- **Nonlinearity based AM receiver**

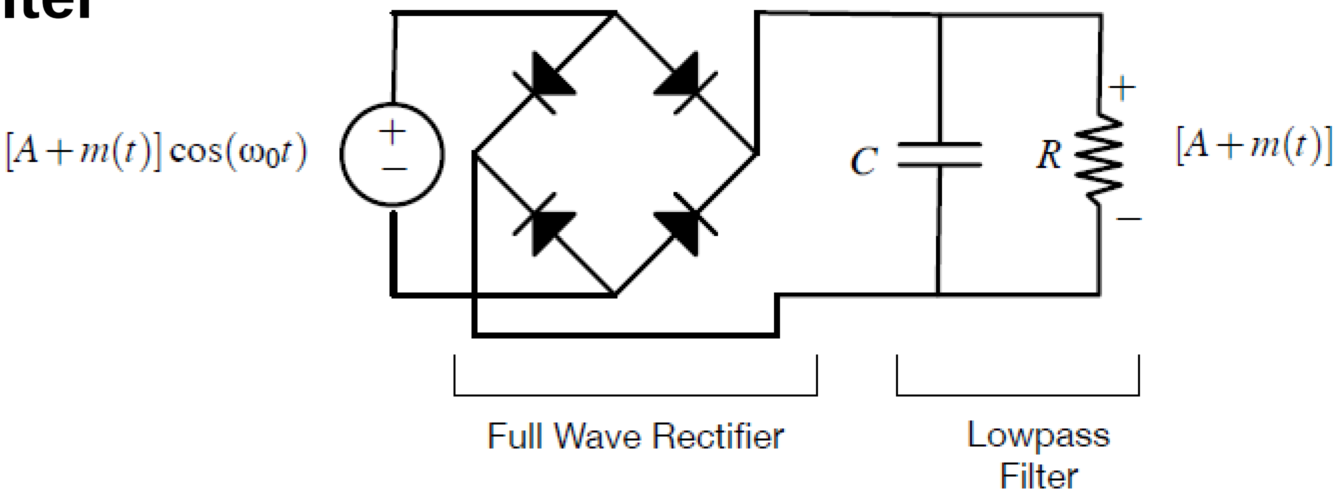


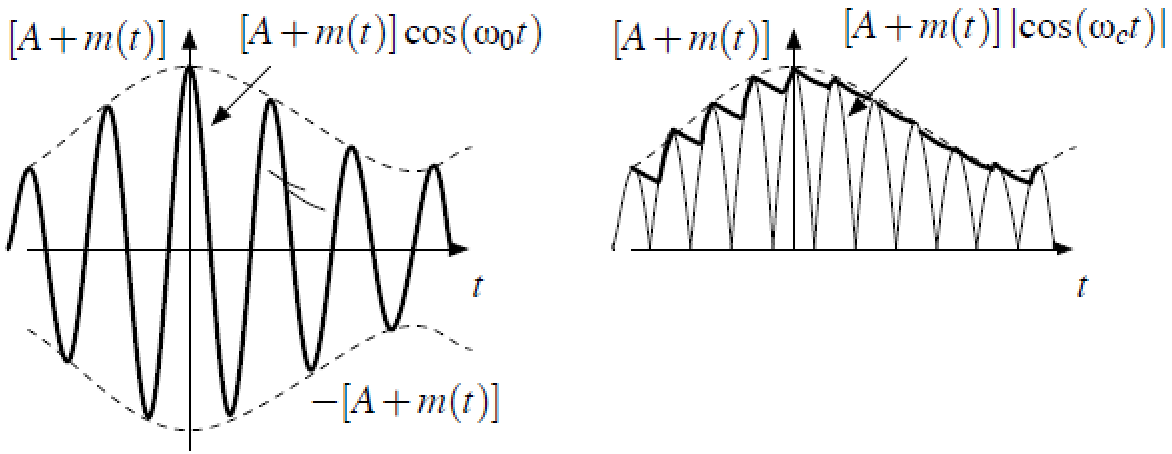
Wikipedia: Crystal radio wiring pictorial based on Figure 33 in Gernsback's 1922 book *Radio For All* (copyright expired) with "Aerial" changed to Antenna by J.A. Davidson.



Practical AM Radio Receiver (2)

- **A slightly more sophisticated receiver can be realized with a "rectifier" and a low-pass filter**



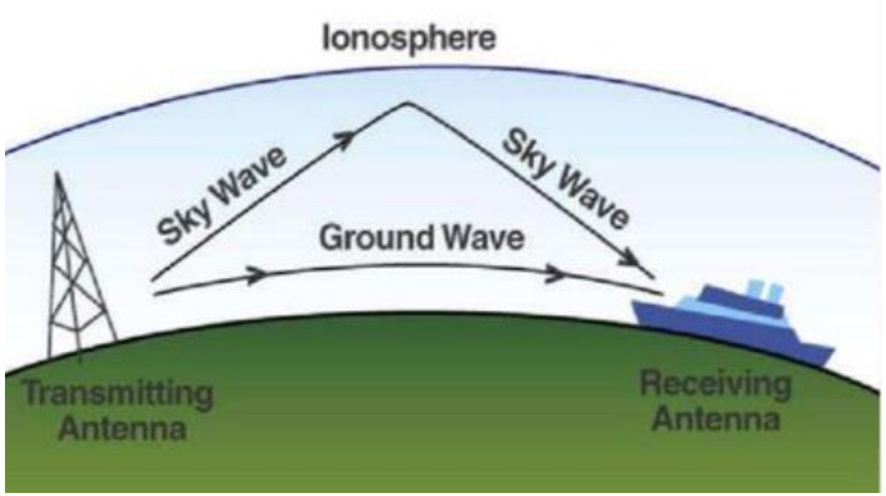


AM Radio Propagation

- **Frequency bands allocated to AM radio in the 100s of kHz regime**
  - Channel spacing ~9-10 kHz, supporting audio with 4.5kHz BW
  - Some countries allowed for 10.2kHz audio (~20.4kHz spacing) in higher frequency bands

[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
Long Wave (LW)	153 kHz – 279 kHz	9 kHz	~126 kHz	Europe, Africa, and parts of Asia.
Medium Wave (MW)	530 kHz – 1,700 kHz	9-10 kHz	~1,170 kHz	Primary AM broadcast band worldwide.
Short Wave (SW)	1.6 MHz – 30 MHz	Variable	~28.4 MHz	International broadcasting.

- **Low frequencies are good for long-range**
  - **Ground waves** propagate along the surface and therefore reach beyond the horizon (100km – 500km)
  - **Sky waves** are reflected on the outer layer of the atmosphere during night time as during day time a lower layer absorbes the radio signal (up to 2000km)



Touching an AM Radio "Tower"

- **AM radio transmitters operate with a transmit power of 50k – 600k Watt**

Extracted from: https://www.youtube.com/watch?v=GgDxXDV4_hc

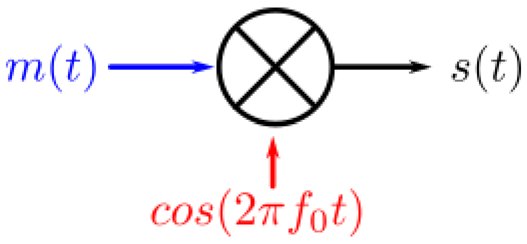
Recap from Week-3

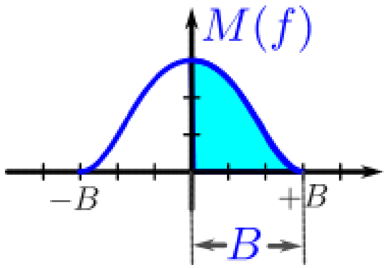
- **Baseband signals, centered around DC are used for wireline transmission**
- **Passband signals, are centered at a carrier frequency, used for wireless**

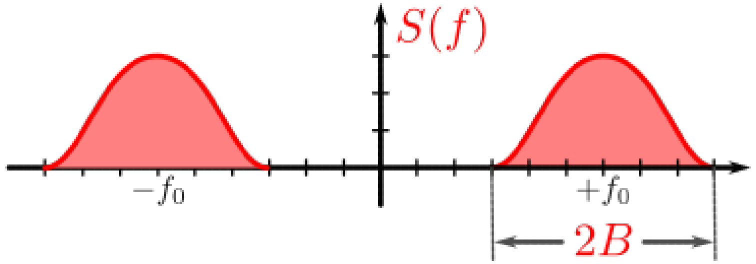
- **Modulation translates a baseband signal by altering amplitude or phase of a sinusoid carrier signal**

$$s(t) = A(t) \times \cos\left(2 \cdot \pi \cdot f_0 \cdot t + \phi(t)\right)$$

- **Amplitude modulation with sinusoidal carrier with a real-valued signal of bandwidth B (only positive frequencies) uses a bandwidth $2B$ in Passband**
  - Very bandwidth inefficient

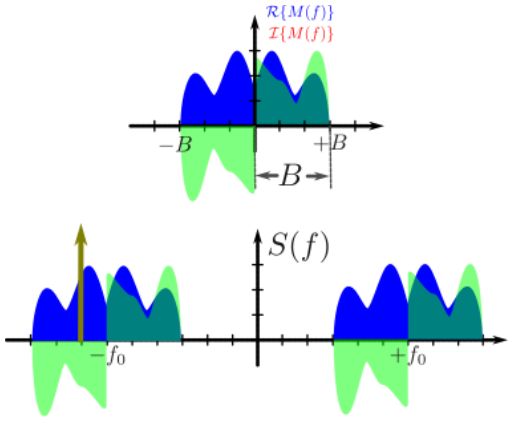


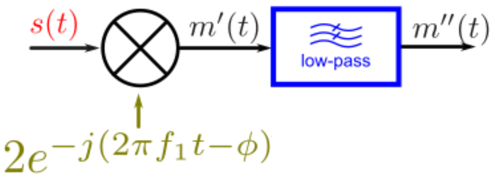


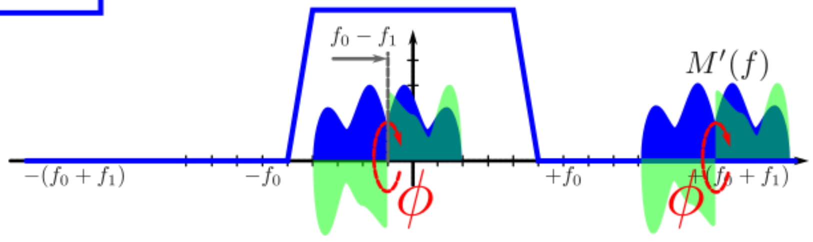


Recap from Week-3

- **AM signals can be demodulated with a complex sinusoid and a LP-filter**





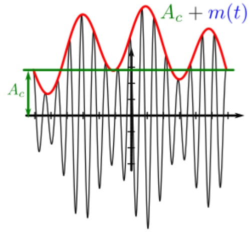


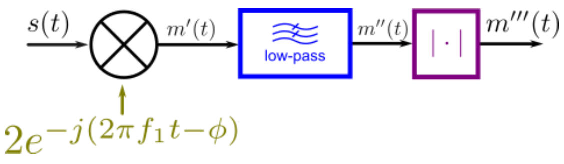
- **Phase and frequency offsets cause the resulting BB signal to "rotate"**

$$m''(t) = m(t) \cdot e^{-j2\pi\Delta f \cdot t} \cdot e^{+j\phi}$$

- **Non-Coherent AM Modulation creates an always positive signal that can be demodulated based on the magnitude (Envelope)**

$$s(t) = \left(A_c + m(t)\right) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t)$$





Quadrature Modulation (Motivation)

- **Reminder: AM modulation is very inefficient in terms of its bandwidth**
  - The bandwidth of the passband signal is 2x wider than the bandwidth of the baseband signal

Fundamentally, there should be enough space in the passband signal of bandwidth $2 \cdot B$ for two baseband signals of bandwidth B

Quadrature Modulation (Reminders)

- **How can we "squeeze" a second real-valued baseband signal $m_2(t)$ of baseband bandwidth B into the same real-valued passband signal $s(t)$ of bandwidth $2B$?**

- **Reminders:**
  - The first real-valued baseband signal $m_1(t)$ is modulated with a cosine

$$s_1(t) = m_1(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t)$$

  - We could recover the signal with a coherent receiver as

$$\mathcal{R}\left\{\mathrm{LPF}\left\{s(t) \cdot 2e^{-j(2\pi f_0 t)}\right\}\right\} = \mathrm{LPF}\{s_1(t) \cdot 2\cos(2\pi f_0 t)\} = m_1(t)$$

$$\mathcal{I}\left\{\mathrm{LPF}\left\{s(t) \cdot 2e^{-j(2\pi f_0 t)}\right\}\right\} = \mathrm{LPF}\{s_1(t) \cdot 2\sin(2\pi f_0 t)\} \equiv 0$$

Demodulation with sin shows no trace of the signal modulated with a cos

Quadrature Modulation

- **Real and Imaginary part are orthogonal (can carry independent signals)**
- **Idea: look for a carrier at the same carrier frequency that is orthogonal to the cosine carrier that "carries" the first signal $m_1(t)$**
  - Reminder: at f_0, we have **two orthogonal sinusoids**

$$\cos(2 \cdot \pi \cdot f_0 \cdot t) \text{ and } \sin(2 \cdot \pi \cdot f_0 \cdot t)$$

$$\langle \cos(2 \cdot \pi \cdot f_0 \cdot t), \sin(2 \cdot \pi \cdot f_0 \cdot t) \rangle = 0$$

- **Modulation and demodulation of only $m_2(t)$ with sine (instead of cos) yields**

$$s_2(t) = m_2(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)$$

$$\mathcal{R}\left\{\text{LPF}\left\{s(t) \cdot 2e^{-j(2\pi f_0 t)}\right\}\right\} = \text{LPF}\{s_2(t) \cdot 2\cos(2\pi f_0 t)\} \equiv 0$$

$$\mathcal{I}\left\{\text{LPF}\left\{s(t) \cdot 2e^{-j(2\pi f_0 t)}\right\}\right\} = \text{LPF}\{s_2(t) \cdot 2\sin(2\pi f_0 t)\} = m_2(t)$$

Demodulation with cos shows no trace of the signal modulated with a cos

Quadrature Modulation

- **Quadrature modulation modulates two real-valued signals with two orthogonal carriers: sin and cos**

$$s(t) = m_1(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) + m_2(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)$$

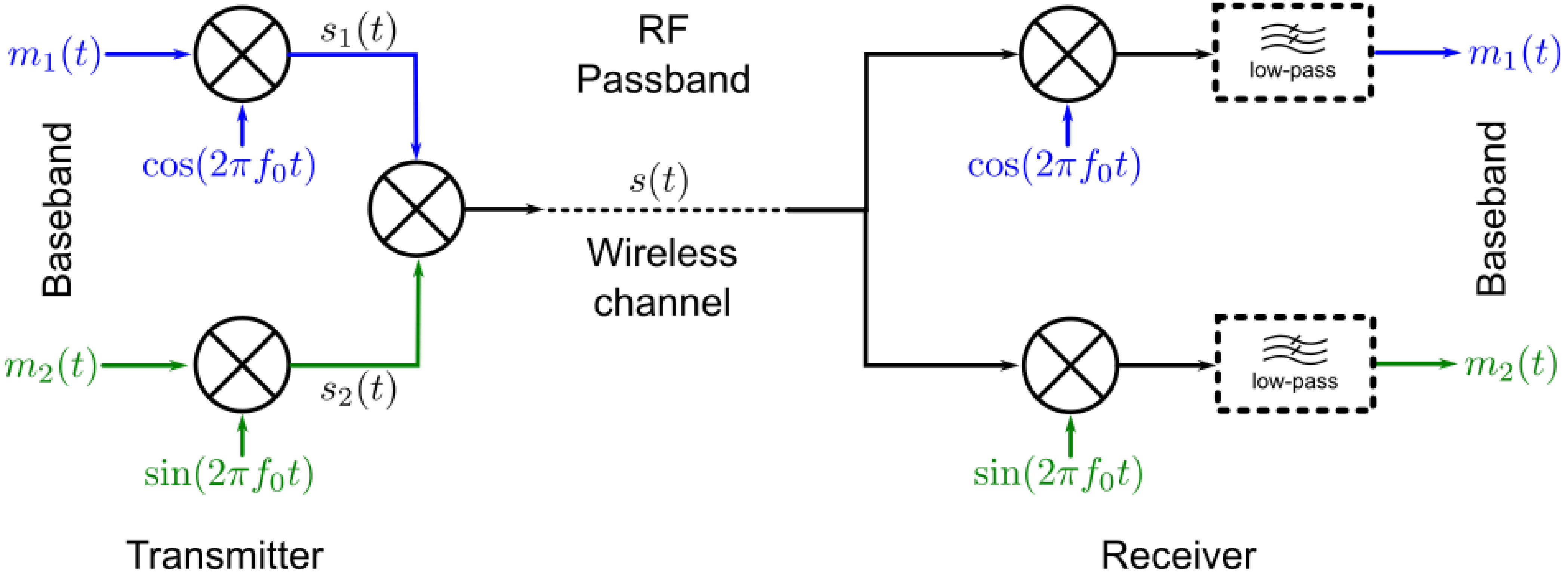
- **Demodulation (using linearity)**

$$\text{LPF}\{[m_1(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) + m_2(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)] \cdot 2\cos(2\pi f_0 t)\} =$$
$$= \mathcal{R}\left\{\text{LPF}\left\{s(t) \cdot 2e^{-j(2\pi f_0 t)}\right\}\right\}$$
$$= m_1(t) + 0$$

$$\text{LPF}\{[m_1(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) + m_2(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)] \cdot 2\sin(2\pi f_0 t)\} =$$
$$= \mathcal{R}\left\{\text{LPF}\left\{s(t) \cdot 2e^{-j(2\pi f_0 t)}\right\}\right\}$$
$$= 0 + m_2(t)$$

Quadrature Modulation

- **Quadrature transmitter / receiver transmitting two real-valued signals (both baseband bandwidth B in the same $2B$ wide passband around f_0**



Quadrature Modulation in Complex Domain

- **We can also think of a quadrature modulator in the complex domain:**
  - Think of the two baseband signals $m_1(t)$ and $-m_2(t)$ as the real and imaginary values of a complex-valued baseband signal $\mathrm{m}(t)$

$$m(t) = m_1(t) - j \cdot m_2(t)$$

  - Modulation is performed with a complex-valued sinusoid $e^{j(2\pi f_0 t)}$, keeping only the real value

$$s(t) = \mathcal{R}\left\{m(t) \cdot e^{j(2\pi f_0 t)}\right\} =$$

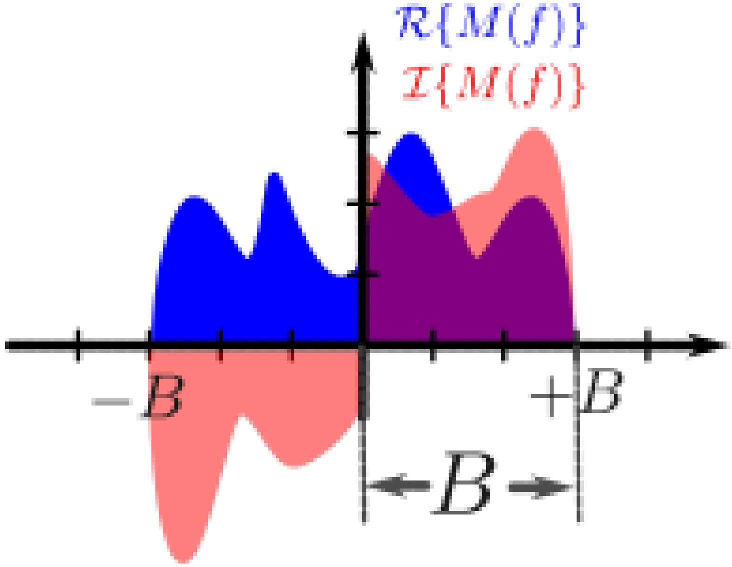
$$\mathcal{R}\{[m_1(t) - j \cdot m_2(t)] \cdot [\cos(2\pi f_0 t) + j \cdot \sin(2\pi f_0 t)]\} =$$

$$\mathcal{R}\{[m_1(t) \cdot \cos(2\pi f_0 t) + m_2(t) \cdot \sin(2\pi f_0 t)] + j \cdot [m_1(t) \cdot \sin(2\pi f_0 t) - m_2(t) \cdot \cos(2\pi f_0 t)]\} =$$

$$[m_1(t) \cdot \cos(2\pi f_0 t) + m_2(t) \cdot \sin(2\pi f_0 t)]$$

Quadrature Modulation (Spectral Interpretation)

- **We start from a complex-valued signal:** $m(t) = m_1(t) - j \cdot m_2(t)$
  - The spectrum between $f = -B \ldots + B$ does not need to be complex-symmetric. Any arbitrary spectrum is allowed



- **The complex-valued transmitted signal uses both sides of the spectrum with no redundancy!**

Quadrature Modulation (Spectral Interpretation)

- **The complex-valued formulation of the quadrature modulation is convenient to interpret in the frequency domain as a two step process**

$$s(t) = \mathcal{R}\{m(t) \cdot e^{j(2\pi f_0 t)}\}$$

1. Shifting the complex-valued signal up to the carrier: results in a purely positive spectrum, but in a complex-valued signal
2. Taking the real part: "creates" the "negative" frequency components around $-f_0$ as the complex-conjugate of the spectrum around $+f_0$