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Comparing Uni-Polar vs. Bi-Polar

Uni-Polar: Given „distance" d: $O_u = \{0 \, ; \, +d\}$

$$P_u = \frac{1}{2}\cdot 0^2 + \frac{1}{2} d^2 = \frac{d^2}{2} \rightarrow \frac{d}{2} = \sqrt{\frac{P_u}{2}}$$

$$\varepsilon_u = Q\left(\frac{d}{2}\cdot\frac{1}{\sigma}\right) = Q\left(\sqrt{\frac{P_u}{2}}\sqrt{\frac{1}{\sigma^2}}\right) = Q\left(\sqrt{\frac{1}{2}\frac{P_u}{\sigma^2}}\right)$$

Bi-Polar: Given „distance" d: $O_B = \{-\frac{d}{2} \, ; \, +\frac{d}{2}\}$

$$P_B = \frac{1}{2}\left(-\frac{d}{2}\right)^2 + \frac{1}{2}\left(\frac{d}{2}\right)^2 = \frac{d^2}{4} \rightarrow \frac{d}{2} = \sqrt{P_B}$$

$$\varepsilon_B = Q\left(\frac{d}{2}\cdot\frac{1}{\sigma}\right) = Q\left(\sqrt{P_B}\cdot\sqrt{\frac{1}{\sigma^2}}\right) = Q\left(\sqrt{\frac{P_B}{\sigma^2}}\right)$$

For $\quad \varepsilon_u = \varepsilon_B$: $Q\left(\sqrt{\frac{1}{2}\frac{P_u}{\sigma^2}}\right) \overset{!}{=} Q\left(\sqrt{\frac{P_B}{\sigma^2}}\right)$

$$\Rightarrow \frac{1}{2}\underbrace{\frac{P_u}{\sigma^2}}_{SNR_u} = \underbrace{\frac{P_B}{\sigma^2}}_{SNR_B}$$

$$\Rightarrow SNR_u = SNR_B$$

Unipolar requires 2x (3dB) better SNR then Bipolar for same Error-Rate!

M-PAM Error-Rate scaling with M for fixed Bandwidth

$$\sqrt{\frac{1}{\sigma^2}}$$

$$\mathcal{E}_M = 2\,\frac{M-1}{M}\,Pr\left(n_k > \tfrac{1}{2}d\right) = 2\cdot\frac{M-1}{M}\,Q\left(\frac{d}{2}\,\frac{1}{\sigma}\right)$$

$$P_{\varnothing M} = \frac{M^2-1}{3}\left(\frac{d}{2}\right)^2 \rightarrow \frac{d}{2} = \sqrt{\frac{3}{M^2-1}\cdot P_{\varnothing M}}$$

$$\mathcal{E}_M = 2\cdot\frac{M-1}{M}\,Q\left(\sqrt{\frac{3}{M^2-1}}\,\underbrace{\sqrt{\frac{P_{\varnothing M}}{\sigma^2}}}_{SNR}\right)$$

M-PAM Error rate with PSD N_0

$$f_s = \frac{f_{bit}}{2\log_2 M} \qquad T_{sym} = \frac{\log_2 M}{f_{bit}}$$

$$\mathcal{E}_M = 2\cdot\frac{M-1}{M}\,Q\left(\sqrt{\frac{3}{M^2-1}}\,\sqrt{\frac{P_{\varnothing M}\,\log_2 M}{N_0\,f_{bit}}}\right)$$

T_{sym}

$$= 2\cdot\frac{M-1}{M}\,Q\left(\sqrt{\frac{3}{M^2-1}}\,\sqrt{\frac{E_s}{N_0}}\right)$$

$$= 2\cdot\frac{M-1}{M}\,Q\left(\sqrt{\frac{3}{M^2-1}}\,\sqrt{\frac{1}{N_0}\cdot\frac{P_{\varnothing M}}{f_{bit}}\cdot\log_2 M}\right)$$

E_b

$$= 2\,\frac{M-1}{M}\,Q\left(\sqrt{\frac{3}{M^2-1}}\,\sqrt{\frac{E_b}{N_0}\,\log_2 M}\right)$$