Telecommunications Systems Exercises 4

April 2025

4.1 The Need for Pulse Shaping

In the last exercise session, you have observed that a perfectly rectangular pulse occupies an infinite band and that a perfectly limited spectrum occupies an infinite time. You also showed that a signal cannot be both time limited and band limited. As you might have guessed, there are alternatives to the latter extremes. We therefore have to make a trade-off between spectrum occupancy and duration of the pulses (which increase the risk of Inter-Symbol Interference (ISI)). To avoid ISI, we design pulses that are zero-valued on each multiple of the sampling period. Hence a signal will have no ISI if perfectly sampled at the right times. This is the **second Nyquist criterion or the Nyquist ISI criterion**.

Consider the digital signal $s[n]$ that is oversampled and the pulse-shaping filter impulse response $h[n]$ shown in Figure 1. The output of the pulse-shaping filter is the convolution of the signal with the filter's impulse response.

1. Draw the output of the pulse-shaping filter.
2. Does the pulse satisfy the Nyquist ISI criterion ?
3. Is the output signal ISI-free ?
4. Bonus : Show that the Nyquist ISI criterion can be derived as:

$$\frac{1}{T_s} \sum_{k=-\infty}^{+\infty} H\left(f - \frac{k}{T_s}\right) = 1$$

Hint: Start by stating that the Nyquist ISI criterion forces the impulse response to be 0 at every multiple of the sampling period except 0:

$$h(t) = \begin{cases} 1, & \text{if } t = 0; \\ 0, & \text{if } t = kT_s,\ k \in \mathbb{Z}^* \end{cases}$$

This means that when the impulse response is sampled at period T_s (i.e. a train of Dirac pulses spaced of T_s), the result is a single Dirac pulse at $t = 0$.

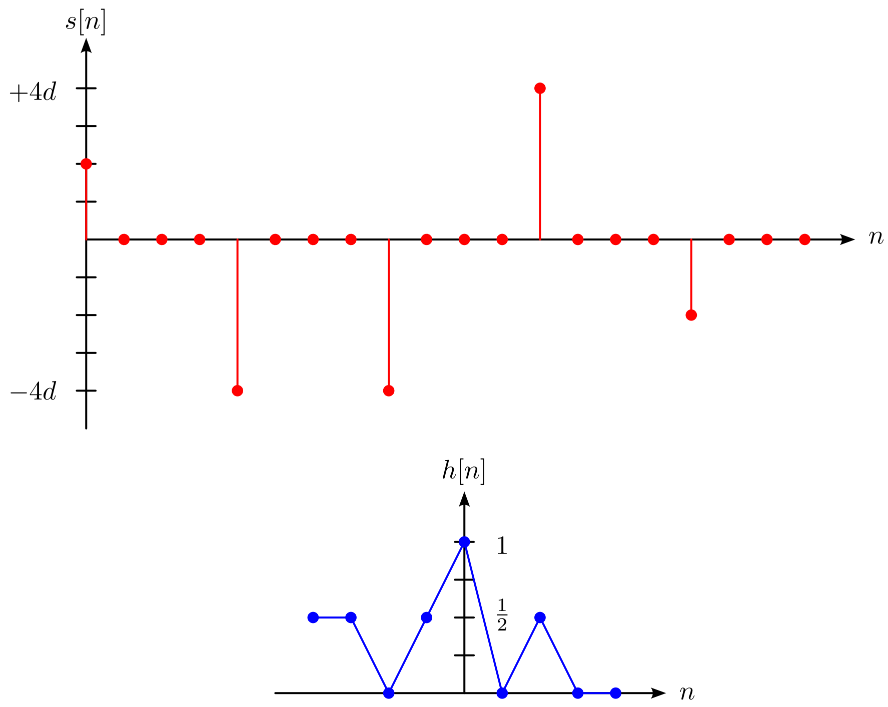

Figure 1: A signal $s[n]$ and the impulse response of the pulse shaping filter $h[n]$

4.2 The Nyquist ISI Criterion

The Fourier transform $P(f)$ of the pulse-shaping filter impulse response $p(t)$ used in a certain binary communication system is shown in Figure 2

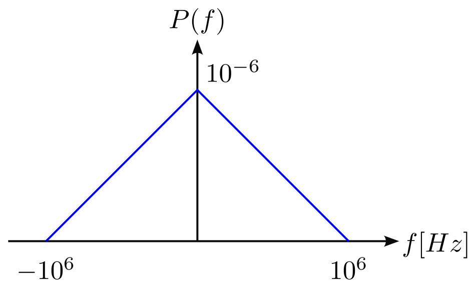

Figure 2: The Fourier transform $P(f)$ of $p(t)$

1. From the shape of $P(f)$, explain at what pulse rate this pulse would satisfy the Nyquist ISI criterion.
2. Find $p(t)$ and verify if this pulse does satisfy Nyquist's second criterion.
3. If the pulse does satisfy the Nyquist criterion, what is the transmission rate (in bits per second) and what is the roll-off factor?

4.3 PAM Constellation

Construct an 8-PAM constellation that:

- Has a unit power.
- Is bipolar.
- Has equal spacing.
- Minimizes the erroneous bit should a symbol by falsely read as its neighbor by assigning binary code that only differ of 1 bit from a symbol to the other.

4.4 Error Rate for 3-PAM Constellation

For some strange reason, we would like to use a constellation for 3-PAM with 3 constellation points. The following three options are available:

$$\mathcal{O}_3' = -1, 0, 1$$
$$\mathcal{O}_3' = -1/2, 1/2, 3/2$$
$$\mathcal{O}_3' = -3/2, -1/2, 1/2$$

- Which one of the three options is the best in terms of error rate performance?

4.5 Error Rate for M-PAM Constellations

We are interested in the error rate for M-PAM constellation as a function of the power of the signal $P_{\mathcal{O}_M}$ and the noise variance σ. The M-PAM constellation alphabet is given by $\mathcal{O}_M = \{\pm\frac{d}{2}, \pm 3\frac{d}{2}, ..., \pm(M-1)\frac{d}{2}\}$. For this type of constellation the power is given by

$$P_{\mathcal{O}_M} = \frac{M^2 - 1}{2}\left(\frac{d}{2}\right)^2$$

- Derive the symbol error rate for this constellation as a function of M and as a function of the signal-to-noise ration (SNR).
- What penalty in terms of SNR do you expect when increasing from 4-PAM to 8-PAM
- What penalty in terms of SNR do you expect when increasing from 8-PAM to 16-PAM
- When M is sufficiently large, what penalty do you expect for each additional bit?

4.6 Link between Power, Symbol/Bit Duration, Rate, and Error Rate

Consider a system that has a noise power spectral density of $N_0 = -104 dBm/Hz$ (note that this value is different from the PSD of Thermal noise, here due to interference). The received power is $P_r = -25\, dBm$ and we would like to have a symbol error rate of $\epsilon = 10^{-5}$ with Bi-polar 2-PAM modulation.

- What is the E_b/N_0 that is required to achieve the error rate target (use the plot below)?
- What is the bit duration (and bit rate)?
- What is the required signal bandwidth assuming an ideal SINC filter?