Telecommunications Systems Exercise 3

Spring semester 2025

1 AD Convertion and Quantization

A digital PCM signal must be transmitted through a line with analog amplifiers before being regenerated by a digital repeater. The line is made of two segments of attenuation α both terminated by analog amplifiers of gain G. Each amplifier introduces thermal noise at its own input at N dBm.

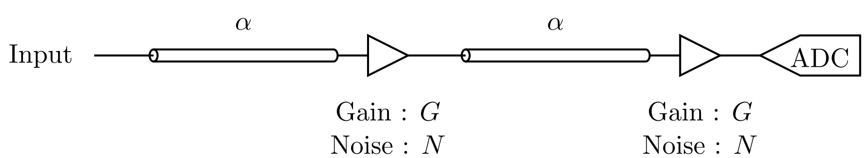

Figure 1: Line with amplifiers followed by the repeater's ADC

The repeater is made of an ADC and a modulator. The ADC has Q bits and requires that the SNR is at least 3 dB above the SQR to perfectly regenerate the signal.

At the input of the line, a noiseless signal is injected at power level $P_s = 33$ dBm. At the input of the repeater, the measured SQR is 2 dB.

Using:

$$\alpha = 14 \text{ dB}$$

$$G = 14 \text{ dB}$$

$$N = 2 \text{ dBm}$$

$$Q = 8 \text{ bits}$$

Answer the following questions:

1. What is the SNR at the input of the ADC? Is the requirement fulfilled?
2. Discuss what happens when $SNR < SQR$ at the input of the ADC and why this is bad for signal reconstruction.
3. Since there is some margin, we decide to increase the resolution as much as possible. What is the maximum value of Q?
4. In consequence, what is the fractional increase of the channel bandwidth?

2 Audio Quantization

When recording a silly Reel for your social media, your smartphone front microphone records audio signal using PCM. Assume the audio signal bandwidth is 15 kHz.

1. What is the Nyquist rate?
2. If the samples are quantized on 65'536 levels, how many bits are needed to record a sample?
3. Determine the required bit rate (bits/s) to record to encode the audio signal;
4. For standardization purposes, audio signals are sampled well above the Nyquist rate. Common formats use 44'100 samples per second. If the samples are recorded on 65'536 levels, determine the number of bits per second needed to encode the signal, and the minimum bandwidth required to transmit the signal.

3 Frequency Multiplexing of PCM Signals

Five telemetry signals, each of bandwidth of 1 kHz are encoded in binary PCM. They are transmitted simultaneously using FDMA. The maximum tolerable error in sample amplitudes is 0.2% of the peak signal amplitude. The signals must be sampled at least 20% above the Nyquist rate. Framing and synchronizing requires an additional 0.5% extra bit. Determine the minimum possible data rate (bits/s) that must be transmitted, and the minimum bandwidth required to transmit the signal.

4 Hints Towards Pulse Shaping

An analog signal $s(t)$ of band $B = 5kHz$ have to be transmitted by PCM. Its maximum amplitude is $A = 0.8$ and the quantization error shall not be more than $e = 0.1$.

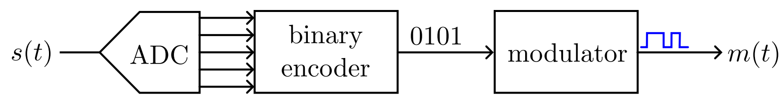

Figure 2: From analog signal to PCM signal

1. What is the minimum number of bits required to quantize each sample?

2. What is the minimum sampling rate, and in consequence the minimum bit rate?

3. Assuming signed binary values, what is the binary code to represent the sample $s(t_0) = 0.425$ that minimizes the quantization error?

4. Assuming that the PCM signal is made of perfectly rectangular pulses defined by:

$$\text{rect}(t) \;=\; \begin{cases} 1, & \text{if } 0 \le t \le 1; \\ 0, & \text{otherwise} \end{cases}$$

   The PCM signal is given by:

$$m(t) = \sum_k b_k \cdot \text{rect}\left(\frac{t - kT}{T}\right)$$

   Where:

   b_k is the binary digit: 1 or 0;

   T is the pulse duration.

   Answer the questions:

   - What is the largest possible pulse width T?
   - Express analytically the pulse train to transmit the sample $s(t_0)$ and sketch it.

5. Express analytically the spectrum of that pulse train and sketch its amplitude. (Hint: Use the Fourier transform pairs).

6. Is this spectrum band limited? Explain why this is a problem.

7. Now assume that instead of using perfectly rectangular pulses, the PCM modulator sends *sinc* pulses (or, pulses that are shaped by a *sinc*) where:

$$m(t) = \sum_k b_k \cdot sinc\left(\frac{\pi(t - kT)}{T}\right)$$

With this new pulse, express again, analytically, the pulse train to transmit the sample $s(t_0)$ and sketch it.

8. Express analytically the spectrum of that pulse train and sketch it. Is it band limited? What is its bandwidth?

9. Is this pulse train limited in the time domain? Explain why this is a problem.

10. Prove that a signal cannot be simultaneously time-limited and band-limited. Use a proof by contradiction. Assume a signal simultaneously time and band limited so that such as:

$$G(2\pi f) = 0 \text{ for } |f| > B$$

In this case:

$$G(2\pi f) = G(2\pi f) \cdot \text{rect}\left(\frac{|f|}{B'}\right)$$

for any $B' > B$. This means that:

$$g(t) = g(t) * B' sinc(B' t)$$

Show that the latter cannot be time-limited.