Telecommunications Systems Exercises 8

April 2025

8.1 DSSS with Constant Energy/Symbol and Variable Bandwidth in AWGN

We transmit binary data $b_j \in \{-1, +1\}$ using DSSS. The symbol duration is T_s, the spreading sequence $c_j \in \{-1/\sqrt{G}, +1/\sqrt{G}\}$ has length G (spreading factor), and the chip duration is $T_c = T_s/G$. The signal energy per symbol is E_s and the additive noise is white Gaussian with PSD N_0.

(a) Derive the relationship between the signal bandwidth B_c, the symbol duration T_s, and the spreading factor G.

(b) What is the total noise energy (for white noise) and the useful-signal energy at the receiver (channel gain is 1) before de-spreading (considering the signal-energy and noise-energy for each chip individually)? Hint: signal energy per chip can be derived from E_s and G, for the noise, consider bandwidth, chip duration, and PSD.

(c) Draw the spectrum of the signal and the noise for $G = 2$ and $G = 4$ before and after de-spreading.

(d) What is the total noise energy (for white noise) and the useful-signal energy (over one T_s period) at the receiver (channel gain is 1) after de-spreading and filtering as appropriate (keeping only the useful bandwidth)?

(e) Calculate the SNR before and after de-spreading

(f) Explain why only a fraction of the noise appears to affect the SNR after de-spreading (Hint: consider the bandwidth that you actually need).

8.2 DSSS with Constant Energy/Symbol and Variable Bandwidth with Narrow Band Interferer

We transmit binary data $b_j \in \{-1, +1\}$ using DSSS. The symbol duration is T_s, the spreading sequence $c_j \in \{-1/\sqrt{G}, +1/\sqrt{G}\}$ has length G (spreading factor), and the chip duration is $T_c = T_s/G$. The signal energy per symbol is E_s. We add an interferer with fixed bandwidth $B_I = 1/T_s$ and PSD N_I.

(b) What is the total energy per chip of the Interferer (over one T_c, i.e., before de-spreading)?

(c) Draw the spectrum of the signal and the interferer for $G = 2$ and $G = 4$ before and after de-spreading (assuming no correlation between the interfering signal and the spreading sequence)

(d) What is the PSD density, the bandwidth, and the total energy of the interferer after de-spreading (multiplication with the spreading sequence) for $G = 2$ and $G = 4$?

(e) Calculate the SIR before and after de-spreading and filtering as appropriate (keeping only the useful bandwidth) for $G = 2$ and $G = 4$?

8.3 Numerical Example

Assume:

$$R_s = 1\,\text{kbit/s}, \quad G = 10, \quad N_0 = 10^{-9}\,\text{W/Hz}, \quad E_s = 10^{-6}\,\text{J}.$$

(a) Compute the chip rate R_c and the signal bandwidth B_c.

(b) Compute the SNR per symbol after despreading.

(c) Assume a narrowband interferer with total energy $E_I = 10^{-6}$ J. Calculate the signal-to-interference ratio (SIR) after despreading.

8.4 DSSS with Constant Power and Fixed Bandwidth in AWGN

We transmit binary data $b_j \in \{-1, +1\}$ using DSSS. The chip duration T_c and corresponding bandwidth $B_c = 1/T_c$ are now given and fixed. The symbol duration is then adjusted according to G such that $T_s = G \cdot T_c$ (i.e., larger spreading factor results in longer symbols), the spreading sequence $c_j \in \{-1, +1\}$ has length G (spreading factor). The transmit power is P_s and the additive noise is white Gaussian with PSD N_0.

(a) Write the symbol energy per chip as a function of the transmit power P_s, the chip duration T_c, and the spreading factor G

(b) Write the noise energy per chip as a function of N_0 and the chip duration T_c

(c) Calculate the SNR before de-spreading

(d) Draw the spectrum of the signal and the noise for $G = 2$ and $G = 4$ before and after de-spreading and filtering as appropriate (keeping only the useful bandwidth)?

(e) What signal bandwidth do you need to consider after de-spreading?

(a) Write the symbol energy (over T_s) as a function of the transmit power P_s, the chip duration T_c, and the spreading factor G after de-spreading and filtering as appropriate (keeping only the useful bandwidth)

(b) Write the noise energy (over T_s) as a function of N_0, the chip duration T_c, and the spreading factor G after de-spreading and filtering as appropriate (keeping only the useful bandwidth). Hint: consider the useful bandwidth expressed as $T_s = G \cdot T_c$

(e) Calculate the SNR before and after de-spreading

(f) Explain why only a fraction of the noise appears to affect the SNR after de-spreading.

(g) Compare how the SNR behaves compared to the case where the signal energy and the bandwidth was constant.

8.5 Despreading with Two Users

Consider two users transmitting simultaneously over the same channel using DSSS. The received signal (no noise) over one symbol period is:

$$y_i = b_i^{(1)} c_i^{(1)} + b_i^{(2)} c_i^{(2)}, \quad i = 1, \ldots, G$$

where $b^{(1)}, b^{(2)} \in \{-1, +1\}$ and $c_i^{(1)}, c_i^{(2)} \in \{-1/\sqrt{G}, +1/\sqrt{G}\}$.

The spreading sequences are:

$$c^{(1)} = \left[+\frac{1}{\sqrt{4}}, +\frac{1}{\sqrt{4}}, -\frac{1}{\sqrt{4}}, -\frac{1}{\sqrt{4}}\right]$$

$$c^{(2)} = \left[+\frac{1}{\sqrt{4}}, -\frac{1}{\sqrt{4}}, +\frac{1}{\sqrt{4}}, -\frac{1}{\sqrt{4}}\right]$$

(a) Write the mathematical expression for recovering $\hat{b}^{(1)}$ using the correlator matched to $c^{(1)}$.

(b) Calculate $\hat{b}^{(1)}$ explicitly in terms of $b^{(1)}$ and $b^{(2)}$, assuming $G = 4$.

(c) Evaluate the result when $c^{(1)}$ and $c^{(2)}$ are orthogonal. What happens to the interference from user 2?

(d) Propose a third spreading sequence of length 4 that remains orthogonal to both users.

(d) How many such orthogonal spreading sequences can you find?