Telecommunications Systems Exercises 6

May 2025

6.1 Free-Space Path Loss and Antenna Gain

With the free-space path loss formula:

$$P_r(R) = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2}$$

1. A transmitter operating at $f = 2.4$ GHz is placed 50 meters from the receiver. Both antennas are isotropic ($G_t = G_r = 1$). Calculate the received power, assuming $P_t = 0$ dBm. Use $\lambda = \frac{c}{f}$, where $c = 3 \times 10^8$ m/s.

2. Now assume the transmit antenna has a directional gain of 6 dBi. Recalculate the received power. What does this tell you about the impact of antenna directivity?

3. Suppose the minimum required received power (receiver sensitivity) is -90 dBm. With $P_t = 0$ dBm and both antenna gains equal to 1 (0 dBi), calculate the maximum distance $R_{\max}$ at which communication is still possible in free-space.

4. Repeat the calculation above with a transmit antenna gain of 6 dBi and a receive antenna gain of 3 dBi. What is the new $R_{\max}$? Comment on how directional antennas extend communication range.

6.2 Cellular System Design & User Capacity

You work for a Cellular Operator and your task is to design and evaluate a GSM cellular deployment. Ultimately, we are interested in the user density (users per km^2) that can be supported. The operator has licensed a total bandwidth of 25 MHz, which is split 50/50 into uplink and downlink bands (each 12.5 MHz). The GSM channel spacing is 200 kHz and each channel supports up to 8 users (in TDMA fashion). The cellular system we design is based on hexagonal channels.

6.2.1 Cell Size Based on Sensitivity Requirement

We first calculate the maximum cell size to ensure we can meet the sensitivity requirement at the cell edge. To account for obstacles and fading, we include a margin of 23 dB of additional attenuation. The sensitivity for GSM in the 900 MHz band is -104 dBm and the transmit power is +30 dBm.

1. For a free space path-loss coefficient of $\beta = 2$ calculate the maximum cell radius R_2 to meet the sensitivity requirement.
2. For a free space path-loss coefficient of $\beta = 3$ calculate the maximum cell radius R_3 to meet the sensitivity requirement.

6.2.2 Architecture and Reuse

Since the signal quality is also limited by the Signal-to-Interference ratio (SIR) we reduce the interference with clusters of N cells in which we distribute the available channels across the N cells in the same cluster. The SIR is approximately $SIR \approx (\sqrt{3N})^\beta$ (see slides). The target for the SIR is $SIR > 11.5\,dB$.

1. What is the cluster site that is required (choose from $N = 3, 4, 7$) for $\beta = 2$
2. What is the cluster site that is required (choose from $N = 3, 4, 7$) for $\beta = 3$

6.2.3 Supported User Density for Largest Possible Cell

We are interested in the number of users we can support per square-km (km^2) IF we choose the cell radius as large as possible (to reduce the number of cells) to meet the sensitivity requirement (see first sub-task). The area of one cell with Radius R (distance from outermost point in the cell to its BTS in the center) is given by $A_{\text{cell}} = \frac{3\sqrt{3}}{2}R^2$.

1. What is the supported user density (for the largest possible cell) for $\beta = 2$
2. What is the supported user density (for the largest possible cell) for $\beta = 3$
3. What do you observe when comparing the two scenarios for the case where the cell size is chosen as large as possible (to keep the number of cells as small as possible)?

6.2.4 Supported User Density for Fixed Cell Size

Assume the cell size is given by the cell size (radius) is given by what is required for $\beta = 3$.

1. Will this cell size also support $\beta = 2$ (in terms of sensitivity?
2. If the answer to the question above is YES, what is the number of cells per cluster N for this cell size when $\beta = 2$ and what is the supported user density?

6.3 Two-Ray Interference

In wireless communication, signals often reach the receiver via multiple propagation paths:

- A line-of-sight (LOS) path
- A ground-reflected path

Due to differences in the path lengths, the signals may arrive with different phases, leading to either constructive or destructive interference at the receiver.

Given the following parameters:

- Transmitter height: $h_t = 1$ m
- Receiver height: $h_r = 3$ m
- Carrier frequency: $f = 1.8$ GHz
- Speed of light: $c = 3 \times 10^8$ m/s
- Horizontal distance d: logarithmically spaced from 1 m to a large value

1. Derive expressions for:

- The ground-reflected path length d_1
- The direct path length d_2

Hint: Use the Pythagorean theorem with $(h_t \pm h_r)^2 + d^2$.

2. Compute the Phase Difference Calculate the phase of each path using:

$$\theta = \frac{2\pi}{\lambda} \times \text{path length}$$

Compute the phase difference $\Delta\theta = \theta_1 - \theta_2$.

3. Simulate the Electric Field Assume each received field component is attenuated by distance and phase shifted:

$$E = E_1 + E_2, \quad E_1 \propto \frac{1}{d_1} e^{j\theta_1}, \quad E_2 \propto \frac{1}{d_2} e^{j\theta_2}$$

Simulate the magnitude of the total received field $|E|$ as a function of d.

4. Plot the Results

- Plot the phase difference vs. distance

- Plot the received field magnitude $|E|$ vs. distance on a log-log scale

Questions

1. How does the interference pattern evolve with increasing distance?
2. Where do deep nulls (signal drops) appear? What causes them?
3. How does changing the transmitter or receiver height affect the pattern?
4. What happens to the interference if the frequency increases (e.g., to 2.4 GHz)?

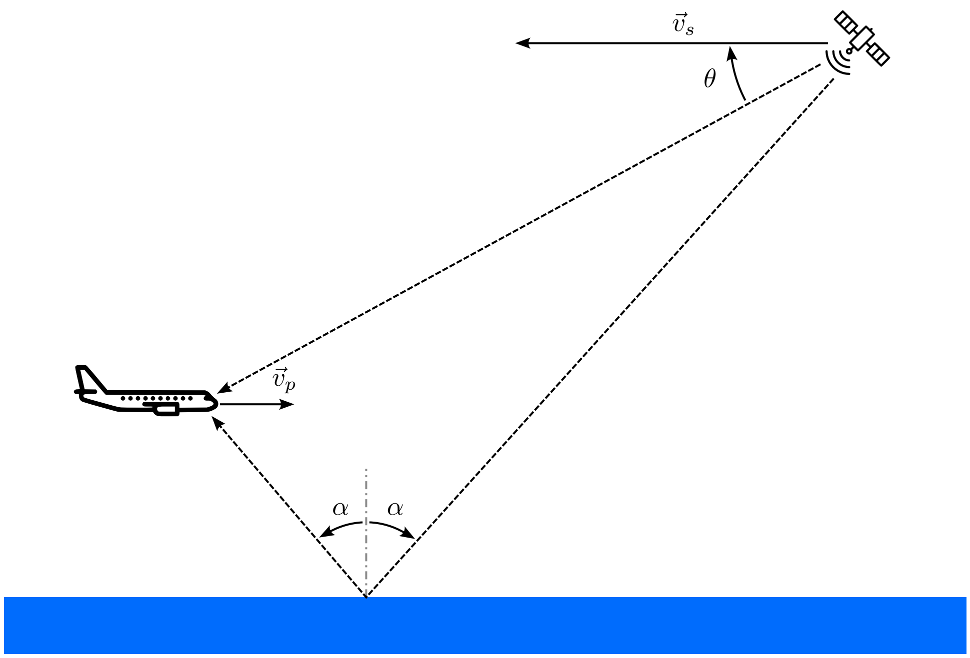
6.4 Noise Figure

Consider a Variable Gain Amplifier (VGA) that supports different amounts of gain. The internal noise is independent of the programmed gain. Which of the following options is true:

1. The noise figure is independent of the gain
2. The noise figure increases when the gain is higher
3. The noise figure decreases when the gain is higher

6.5 Doppler Shift

You are on a long flight and still have many hours before reaching your destination. Fortunately, telecommunication satellite constellations such as Starlink and Oneweb are now used to provide high-speed internet to planes at a much cheaper price than the previously so you buy a data packet to get access to the plane wifi and enjoy your trip. But before you could enjoy such comfort, a lot of challenges were overcome to make such telecommunication systems work. Consider your plane at velocity v_p traveling above the ocean as depicted below. A satellite is downlinking data to the plane from its orbit at velocity v_s on the carrier frequency f_c in Ku-band, the angle between the satellite and the plane is θ. The ocean surface reflects the signal at incidence α on a second path towards the plane.



Questions

1. What is the Doppler shift of the received signal from the satellite ?
2. Is the Doppler shift an issue for the receiver ? Explain why.
3. What is the Doppler shift of the received signal from the reflection ?
4. Consider that the transmitted signal has bandwidth B, sketch the emitted signal spectrum, and the received signals spectra and explain what will the reflected signal cause.
5. Can you imagine a simple solution on the receiver side to avoid this second path ?
6. Only focusing on the signal received from the satellite, assume that after a few minutes, the satellite flew above the plane and further and we now are in a symmetric situation as depicted above but with the satellite behind the plane. What is the Doppler shift now ?
7. Sketch how the Doppler shift of the received signal from the satellite evolves over time and discuss the consequences for the receiver.

Numerical values

$f_c = 15$ GHz

$B = 1$ MHz

$v_p = 250$ m/s

$v_s = 8000$ m/s

$\theta = 40°$

$\alpha = 60°$