Midterm - 16/04/2025

Given Equations

Semiconductors at thermal equilibrium (Boltzmann and Fermi-Dirac formulas)

$n_0 = N_c \cdot e^{-\frac{E_C - E_f}{kT}}$
$p_0 = N_v \cdot e^{-\frac{E_f - E_V}{kT}}$
$n_0 = N_c \cdot \frac{1}{1+e^{\frac{E_C - E_f}{kT}}}$
$p_0 = N_v \cdot \frac{1}{1+e^{\frac{E_f - E_V}{kT}}}$
$n_i^2 = n \cdot p$
$n_i^2 = N_c N_v e^{-\frac{E_g}{kT}}$

Carrier transport

$\sigma = q \cdot (\mu_n n + \mu_p p)$
$L_n = \sqrt{D_n \tau_n}$
$L_p = \sqrt{D_p \tau_p}$

PN junction

$\phi_b = \frac{kT}{q} ln\left(\frac{N_a N_d}{n_i^2}\right)$
$\phi_n = \frac{kT}{q} ln\left(\frac{N_d}{n_i}\right)$
$\phi_p = -\frac{kT}{q} ln\left(\frac{N_a}{n_i}\right)$
$x_d(V) = \sqrt{\frac{2\epsilon_{Si}(N_a + N_d)}{qN_a N_d}\left(\phi_b - V\right)}$

MOS transistor

$V_{FB} = \phi_{ms} - \frac{qQ_{ss}}{C_{ox}}$
$V_{th} = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p}$
$\gamma = \frac{\sqrt{2\epsilon_{Si} q N_a}}{C_{ox}}$
$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - \frac{V_{DS}}{2} - V_{th}\right)V_{DS}$

Given Constants

$k = 8.62 \cdot 10^{-5}\,[eV/K] = 1.38 \cdot 10^{-23}\,[J/K]$
$q = 1.60 \cdot 10^{-19}\,[C]$
$\epsilon_0 = 8.85 \cdot 10^{-14}\,[F/cm]$
$\phi_m(Al) = 3.2\,[V]$

Si properties

$n_i = 1.5 \cdot 10^{10}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$E_g = 1.12\,[eV] \quad @ \quad T = 300\,[K]$
$N_v = 1.04 \cdot 10^{19}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$N_c = 2.8 \cdot 10^{19}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$\chi_{Si} = 3.25\ eV$
$\epsilon_{Si} = 11.7 \cdot \epsilon_0$
$\epsilon_{SiO_2} = 3.9 \cdot \epsilon_0$

GaN properties

$E_g = 3.39\,[eV] \quad @ \quad T = 300\,[K]$
$N_v = 4.6 \cdot 10^{19}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$N_c = 2.3 \cdot 10^{18}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$

Exercise 01

Consider a sample of gallium nitride (GaN) in a wurtzite crystal structure. The valence and conduction bands effective density of states follow a $T^{3/2}$ thermal dependency law. At $0\,[K]$, the band gap energy is $E_g(0) = 3.47\,[eV]$. Assume that the energy gap depends on the temperature by this law:

$$E_g = E_g(0) - 7.7 \cdot 10^{-4} \cdot \frac{T^2}{T + 600} \tag{1}$$

- Calculate the intrinsic carrier concentration at $300\,[K]$.
- Calculate the intrinsic carrier concentration at $600\,[°C]$: is this higher or lower than the room temperature value in silicon?
- Propose one application for this material, where silicon is inappropriate.

Exercise 02

Consider a sample of silicon (Si) at $300\,[K]$, doped with a concentration of boron (B) such that the Fermi level is $10\,[meV]$ higher than the dopant level. Consider a dopant ionization energy of $45\,[meV]$.

- Draw a band diagram of this sample of silicon, highlighting the zero-energy reference.
- Calculate the charge carrier concentration using both the Boltzmann approximation and the Fermi-Dirac distribution, and calculate the percentage error of the Boltzmann approximation over the full formula.
- Comment on the result: what is the condition that is not satisfied in this case for the use of the Boltzmann approximation?

Exercise 03

Consider a piece of lightly p-doped Si of length $1\,[cm]$ and section $1\,\left[mm^2\right]$ at $300\,[K]$. Upon application of $4\,[V]$ across the two extremities via ohmic contacts, a current of approximately $1\,[mA]$ is measured. Neglect any contact resistance.

- Based on the plot provided in figure 1, estimate the doping concentration.
- You want to modify the doping of this sample, in order to obtain a current one order of magnitude higher, either by increasing the B concentration, or by introducing some phosphorus (P). Which is more convenient to design? Give the required dopant concentration in the two cases.

Exercise 04

Consider an abrupt Si PN junction at $T = 300\,[K]$ with doping concentrations $N_a = 8 \cdot 10^{15}\left[cm^{-3}\right]$ and $N_d = 3 \cdot 10^{16}\left[cm^{-3}\right]$.

- Calculate the widths of the depleted regions in the p-side and n-side for the following cases: 1) thermal equilibrium; 2) $V_D = 0.5\,[V]$ (forward bias); 3) $V_D = -1\,[V]$ (reverse bias).
- Calculate and draw the space charge density $\rho(x)$ for the three cases.
- Calculate and draw the electric field $E(x)$ for the three cases. Indicate each time the value of E_{max} in $[V/cm]$.

Exercise 05

Consider the same junction as the previous exercise. The junction parameters are: $W_n = W_p = 150\,[um]$, $\tau_{n0} = \tau_{p0} = 1 \cdot 10^{-7}\,[s]$, $D_n = 27\left[cm^2/s\right]$, $D_p = 11\left[cm^2/s\right]$, $A = 1\left[mm^2\right]$.

- Check whether the device has short neutral sides or long neutral sides compared to the minority carriers diffusion lengths. Write the corresponding formula for the reverse saturation current I_S.
- Calculate I_S at the two temperatures $T_1 = 300\,[K]$ and $T_2 = 250\,[K]$. The valence and conduction bands effective density of states follow a thermal dependency $N_v \propto T^{3/2}$, $N_c \propto T^{3/2}$. Consider $E_g(250\,[K]) \approx E_g(300\,[K]) = 1.12\,[eV]$.
- Calculate $I(0.5\,[V])$ at T_1 and T_2.
- Draw in a single plot the $log|I(V)|$ curves at T_1 and T_2 and give a brief comparison of them.

Exercise 06

Consider a Si PN diode at $T = 300\,[K]$ with parameters: $N_a = N_d = 10^{15}\left[cm^{-3}\right]$, $I_S = 2 \cdot 10^{-13}\,[A]$, $A = 0.1\left[mm^2\right]$, $\tau_T = 2 \cdot 10^{-6}\,[s]$ (weighted average transit time).

- Draw the small-signal equivalent circuit of the diode.
- Calculate the small-signal admittance g_d, the depletion capacitance C_j and the diffusion capacitance C_d at the DC working points $V_D = 0.3\,[V]$ and $V_D = -2\,[V]$.
- In which of the two operating points is best to bias the diode to realize a variable capacitor? Why?

Exercise 07

Consider an NPN BJT with parameters: $N_{dE} = 10^{17}\left[cm^{-3}\right]$, $N_{aB} = 10^{16}\left[cm^{-3}\right]$, $D_n = 27\left[cm^2/s\right]$, $D_p = 9\left[cm^2/s\right]$, $\mu_{nE} = 900\left[cm^2V^{-1}s^{-1}\right]$, $A_E = 100\left[um^2\right]$.

- Design the emitter width W_E to have an emitter resistance $R_E = 5\,[\Omega]$.
- The minimum base width achievable with this technology is $W_B = 300\,[nm]$. Calculate the current gain β_F. Assume that $W_B << L_{nB}$ and $W_E << L_{pE}$ and that we can neglect the width of the depletion region of the B-E junction.
- The BJT has $\tau_{n0} = \tau_{p0} = 5 \cdot 10^{-7}\,[s]$. Are the short base and emitter assumptions verified?

Exercise 08

Consider a planar MOSFET structure with a $10 \times 10\left[\mu m^2\right]$ aluminum gate on a p-doped Si substrate with $N_a = 10^{15}\left[cm^{-3}\right]$ at $300\,[K]$. Let us first focus on the gate stack. Upon acquiring a capacitance-voltage (C-V) curve at high frequency, the capacitance of the MOS capacitor in accumulation is measured to be $5.0 \cdot 10^{-13}\,[F]$, and drops to $1.2 \cdot 10^{-14}\,[F]$ in inversion.

- Calculate the thickness of SiO_2 and of the depletion region.
- The C-V plot also shows a flat-band voltage $V_{FB} = -2.0\ [V]$: calculate the interfacial charge density.
- Based on the data obtained in the previous questions, calculate the threshold voltage V_T of this transistor.
- Assume you are operating the transistor in linear regime, at $V_{GS} = V_{th} + 0.10\,[V]$ and $V_{DS} = 10\,[mV]$. Assume an electron mobility of $1.3{\cdot}10^3\left[cm^2 \cdot V^{-1} \cdot s^{-1}\right]$. Calculate the current I_D.

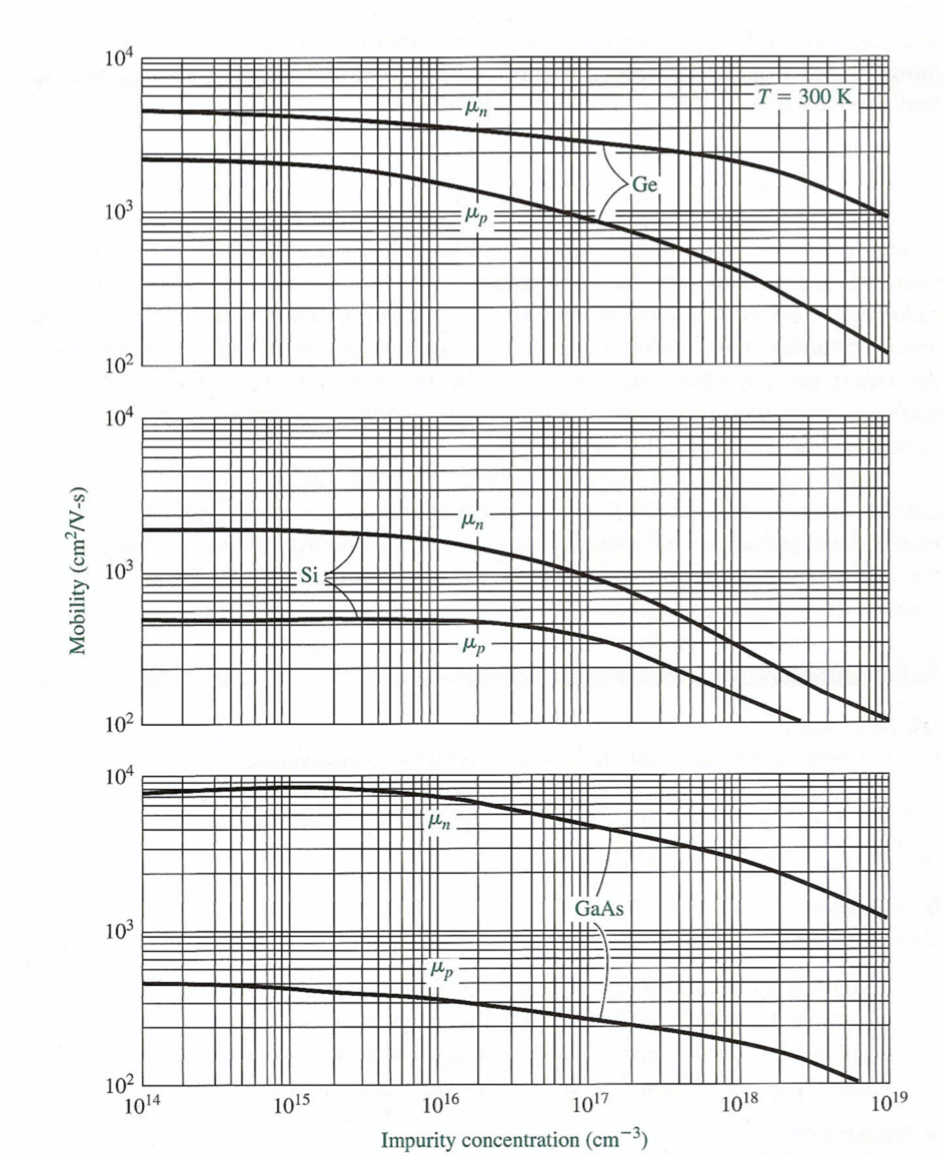

Figure 1: Electron and hole mobilities versus impurity concentrations for germanium, silicon, and gallium arsenide at $T = 300\,[K]$.