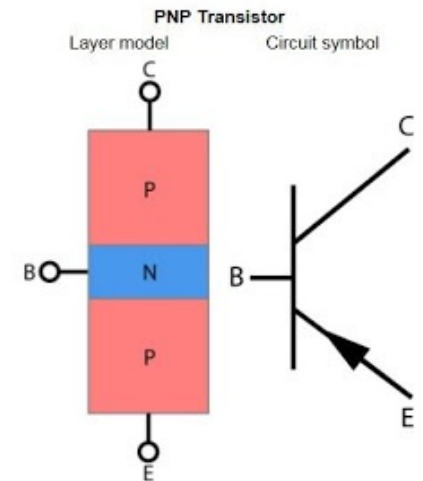
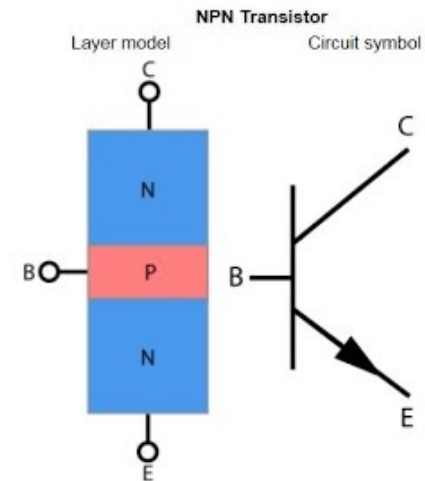


# Lecture 6

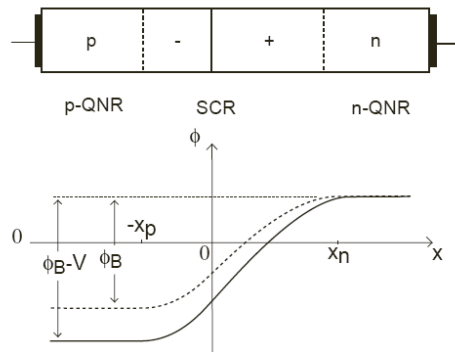
## Microelectronic Devices

### Bipolar Junction Transistor (BJT)

- BJT: structure and basic operation
- BJT operation in principle regimes
- Equivalent circuits, signal amplification



# Revisiting pn junctions (addition to L4-L5)

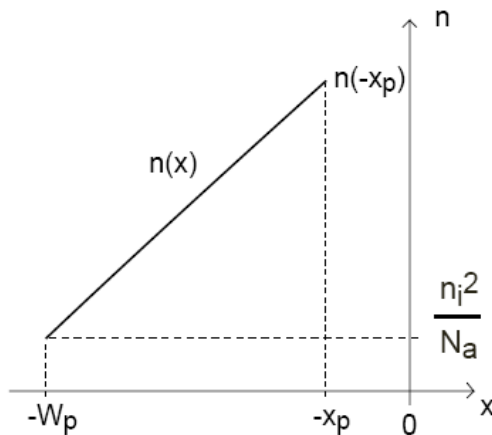


Approximation of “short” QNR:

- Electron/hole recombination is neglected
- Recombination occurs at the interface only

$$J_n = qD_n \frac{dn}{dx}$$

Zoom in to QNR (p-doped)



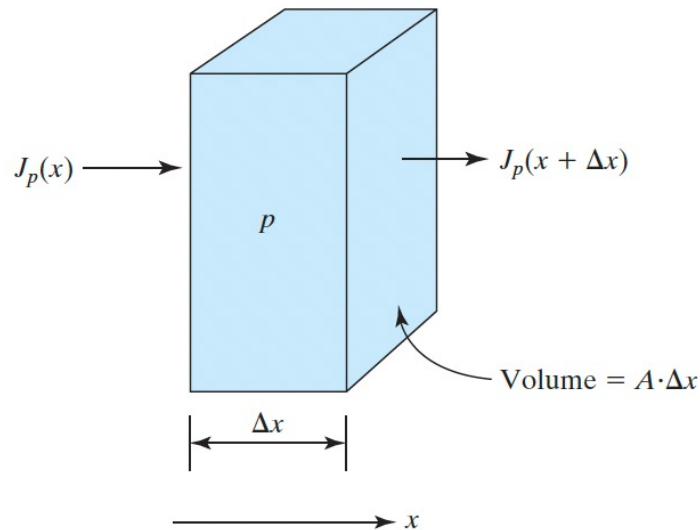
$$n(x = -W_p) = n_o = \frac{n_i^2}{N_a}$$

$$n(-x_p) = \frac{n_i^2}{N_a} \exp \frac{qV}{kT}$$

$$n_p(x) = n_p(-x_p) + \frac{n_p(-x_p) - n_p(-W_p)}{-x_p + W_p}(x + x_p)$$

**$n(x)$  is linear! (continuity of diffusion current)**

## Revisiting pn junctions (addition to L4-L5)



Approximation of “long” QNR:

- Recombination occurs within the QNR
- Recombination is characterized by the lifetime of carriers  $\tau$

### Continuity of the flux of holes:

the number of holes flowing into the box per second =  
number of holes flowing out of the box per second +  
number of holes recombining in the box per second

$$A \cdot \frac{J_p(x)}{q} = A \cdot \frac{J_p(x + \Delta x)}{q} + A \cdot \Delta x \cdot \frac{p'}{\tau}$$

$$-\frac{J_p(x + \Delta x) - J_p(x)}{\Delta x} = q \frac{p'}{\tau}$$

$$-\frac{dJ_p}{dx} = q \frac{p'}{\tau}$$

The differential equation describing  $p(x)$

$$qD_p \frac{d^2 p}{dx^2} = q \frac{p'}{\tau_p}$$

## Revisiting pn junctions (addition to L4-L5)

“long” QNR: constant lifetime approximation

- Recombination occurs within the QNR
- Recombination is characterized by the lifetime of carriers  $\tau$

$$qD_p \frac{d^2 p}{dx^2} = q \frac{p'}{\tau_p}$$

$$\frac{d^2 p'}{dx^2} = \frac{p'}{D_p \tau_p} = \frac{p'}{L_p^2}$$

$$L_p \equiv \sqrt{D_p \tau_p}$$

$$\frac{d^2 n'}{dx^2} = \frac{n'}{L_n^2}$$

$$L_n \equiv \sqrt{D_n \tau_n}$$

$$p'(x) = p_{N0}(e^{qV/kT} - 1)e^{-(x-x_N)/L_p}$$

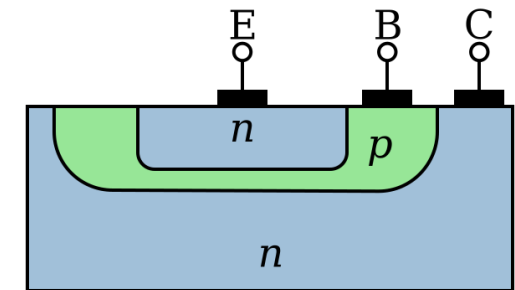
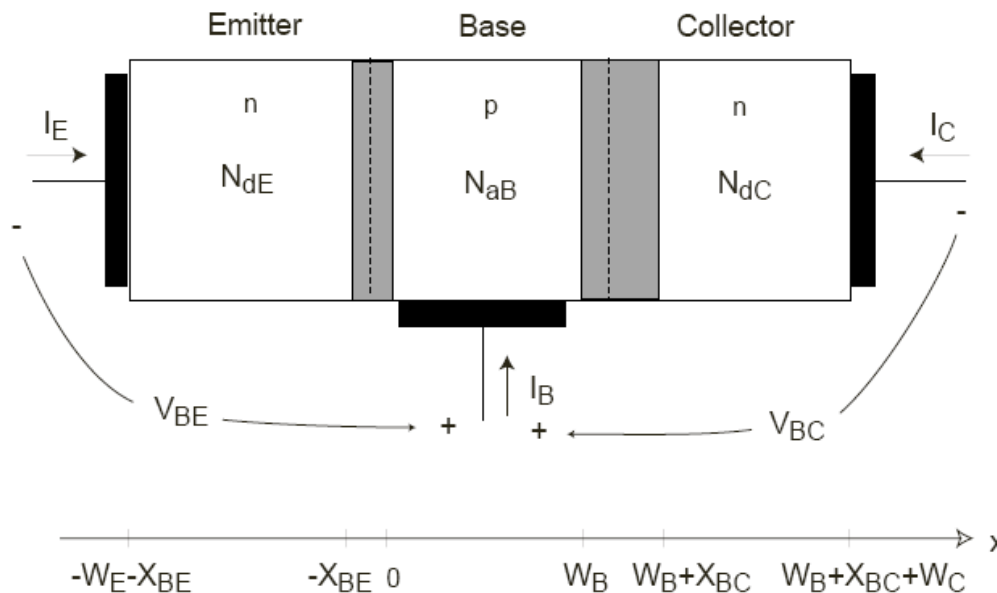
$$n'(x) = n_{P0}(e^{qV/kT} - 1)e^{(x-x_P)/L_n}$$

**The constant lifetime approximation results in an exponential decrease of the minority carriers with the characteristic lengths  $L_n$ ,  $L_p$**

## Bipolar junction transistors: key questions

- What does a bipolar junction transistor look like?
- How does a bipolar junction transistor operate?
- What are the leading dependencies of the terminal currents of a BJT in the forward active regime?
- Equivalent circuits, small signal amplification

# BJT: 1D simplified model



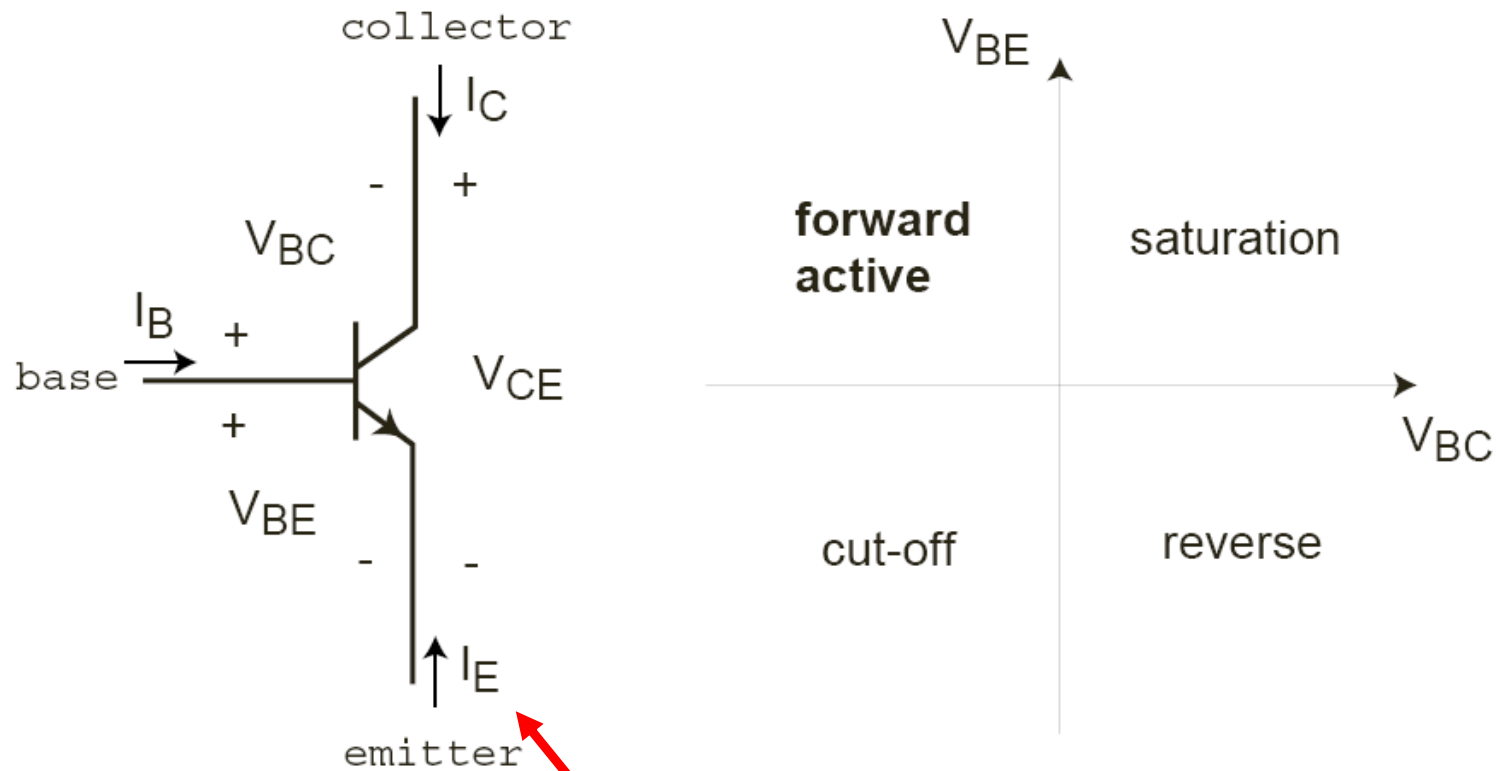
Simplified cross section of a planar *NPN* bipolar junction transistor

BJT = two neighbouring pn junctions back-to-back:

- close enough for minority carriers to interact (can diffuse quickly through base)
- far apart enough for depletion regions not to interact (prevent "punchthrough")

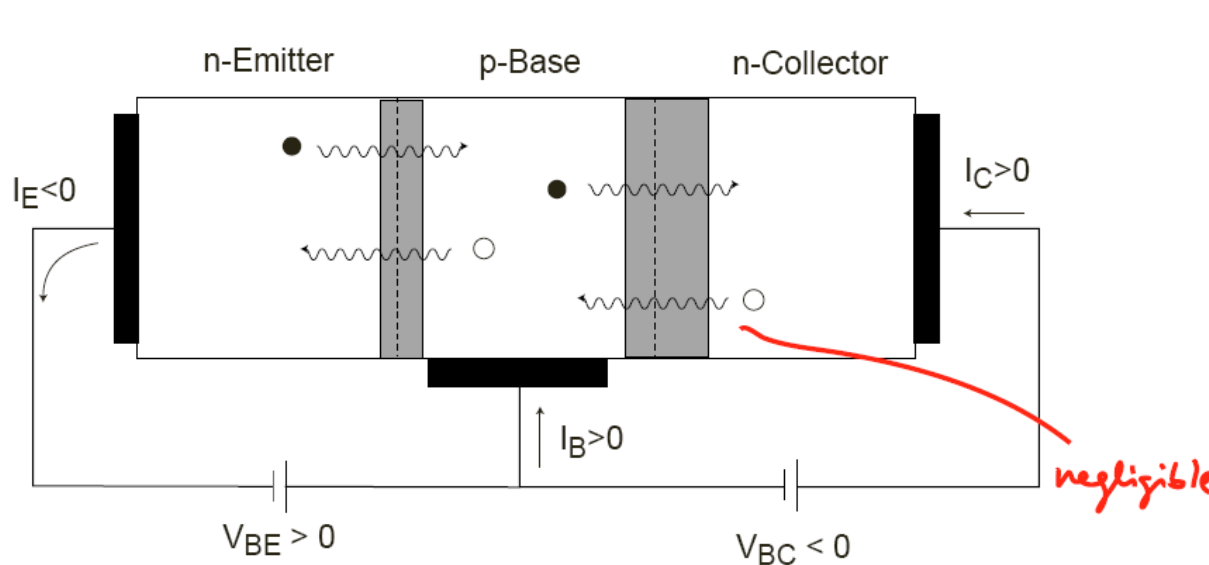
**IN ANY CASE: NOT EQUIVALENT TO TWO BACK-TO-BACK CONNECTED DIODE CIRCUIT!**

## BJT operation (NPN): symbol & the regime chart



Physical  $I_E$  is opposite (shown by arrow)

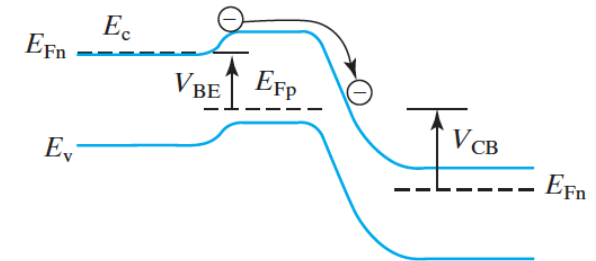
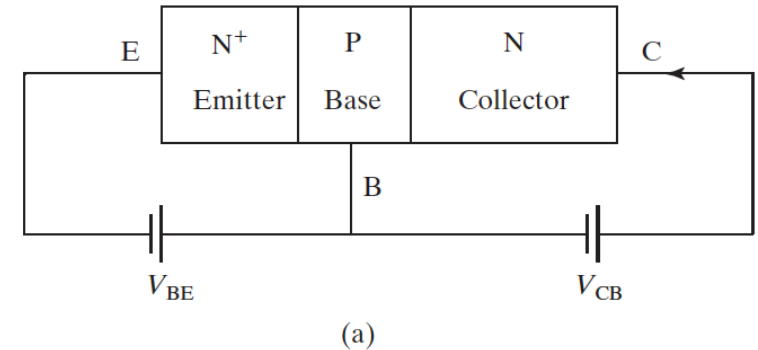
# BJT basic operation in forward mode



- $V_{BE} > 0 \Rightarrow$  injection of electrons from E to B  
injection of holes from B to E
- $V_{BC} < 0 \Rightarrow$  extraction of electrons from B to C  
extraction of holes from C to B

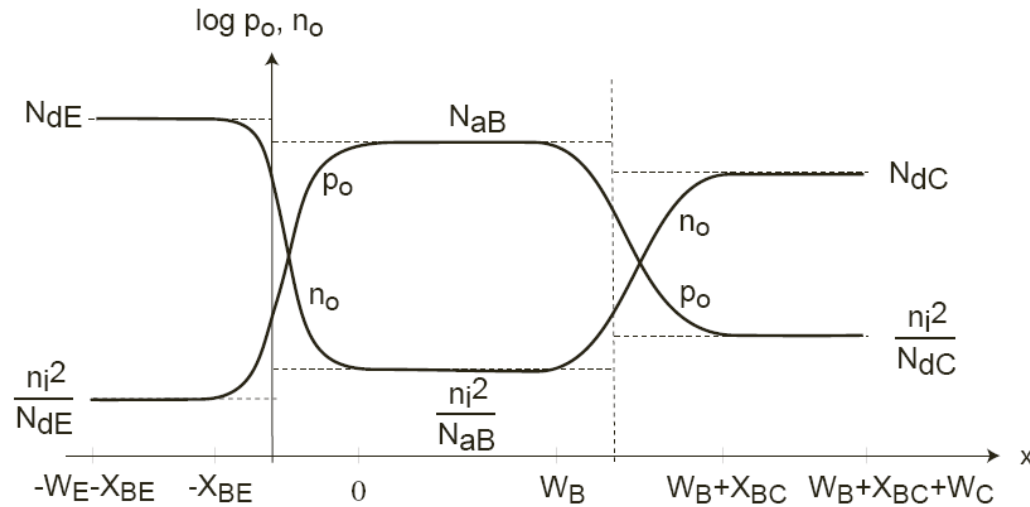
**Transistor effect: electrons injected from E to B, extracted by C!**

Transistor: nearly same current in the emitter and collector junctions,  
resistance (higher in BC): power amplification!

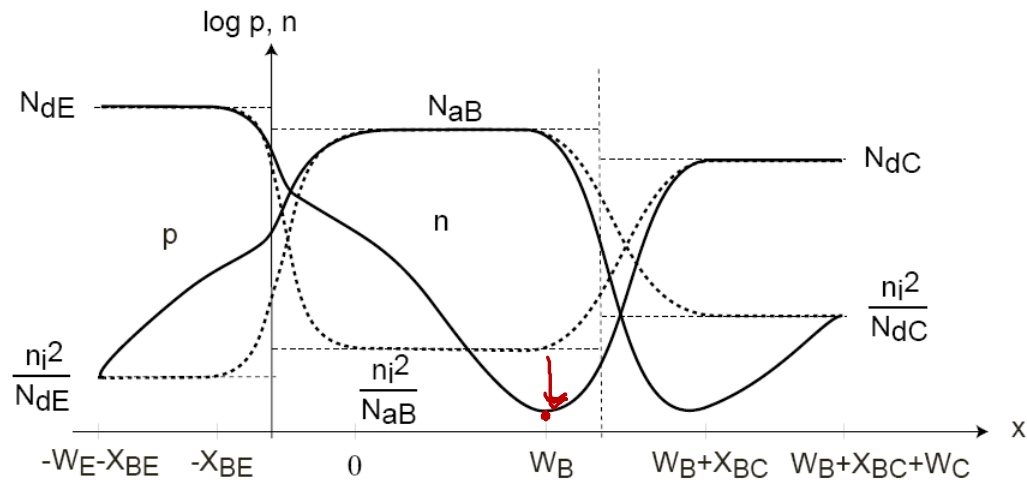




# Carrier profiles

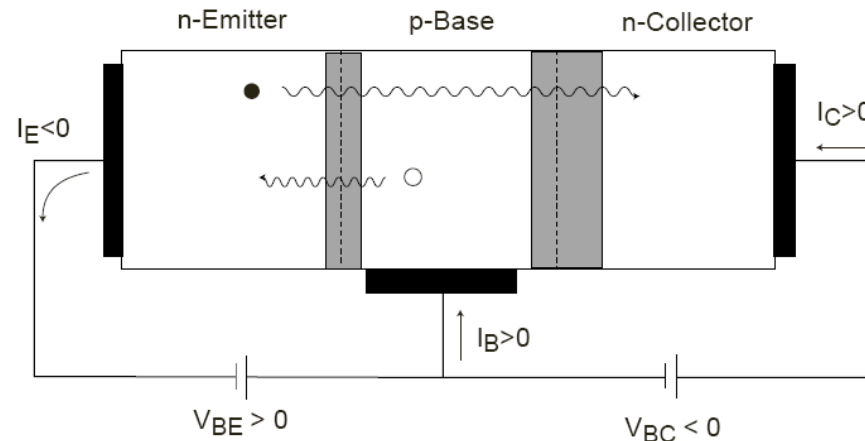


**Thermal  
equilibrium**



**Forward  
operation**

# Dominant current in forward BJT operation



$I_C$  : electron injection from E to B and collection into C

$I_B$  : hole injection from B to E

$$I_E = -I_C - I_B$$

Key dependencies:

$I_C$  on  $V_{BE}$  :  $\exp(qV_{BE}/kT)$

$I_C$  on  $V_{BC}$  : none

$I_B$  on  $V_{BE}$  :  $\exp(qV_{BE}/kT)$

$I_B$  on  $V_{BC}$  : none

$I_C$  on  $I_B$  : LINEAR ( $\beta$  factor)!

## Forward mode BJT operation

In forward-active regime:

- $V_{BE}$  controls  $I_C$ : "transistor effect"
- $I_C$  independent of  $V_{BC}$ : "isolation"
- price to pay for control:  $I_B \neq 0$

BJT figure of merit:  
current gain

$$\beta_F = \frac{I_C}{I_B}$$

# BJT: Current gain, $I_C$ , $I_B$ vs $V_{BE}$ , general discussion

Current gain

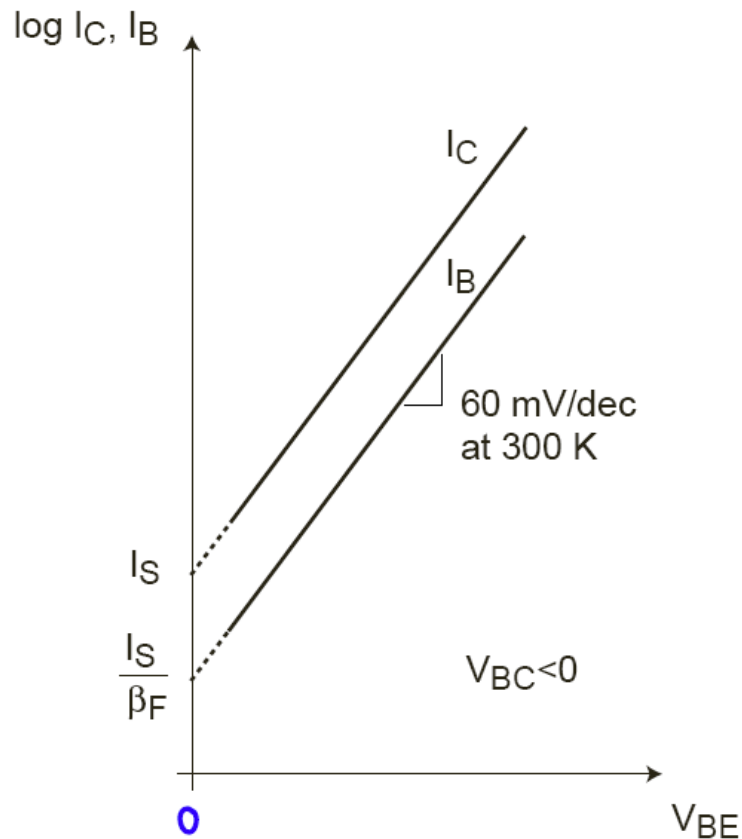
$$\beta_F = \frac{I_C}{I_B} = \frac{n_{pB0} \frac{D_n}{W_B}}{p_{nE0} \frac{D_p}{W_E}} = \frac{N_{dE} D_n W_E}{N_{aB} D_p W_B}$$

To maximize  $\beta_F$  :

- $N_{dE} \gg N_{aB}$
- $W_E \gg W_B$
- in most of cases preferred is npn, rather than pnp design because  $D_n > D_p$

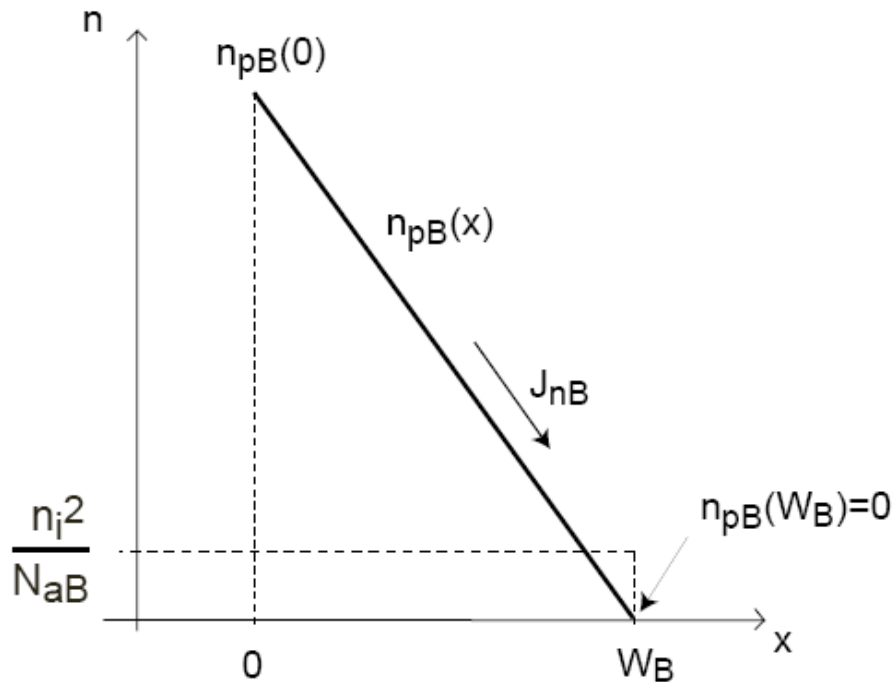
State-of-the-art IC BJT's today:

$I_C \sim 0.1-1 \text{ mA}$ ,  $\beta_F \sim 50 - 300$



# Calculation of collector current, $I_C$ (1)

Forward bias mode: focus on electron diffusion in base

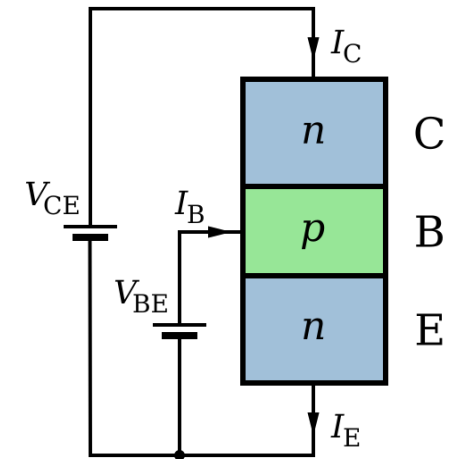


Boundary conditions:

$$n_{pB}(0) = n_{pBo} \exp \frac{qV_{BE}}{kT}, \quad n_{pB}(W_B) = 0$$

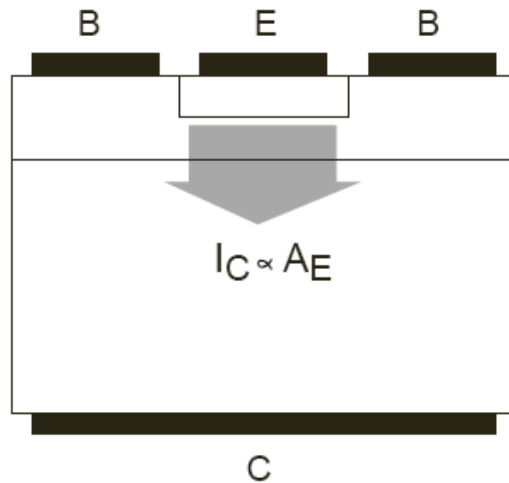
Electron profile:

$$n_{pB}(x) = n_{pB}(0) \left(1 - \frac{x}{W_B}\right)$$



## Calculation of collector current, $I_C$ (2)

$$J_{nB} = qD_n \frac{dn_{pB}}{dx} = -qD_n \frac{n_{pB}(0)}{W_B} \quad \text{Electron current density}$$



For the current calculation:

- consider the collector area - collector current scales with the base-emitter junction area,  $A_E$ , so that:

$$I_C = I_S \exp \frac{qV_{BE}}{kT}$$

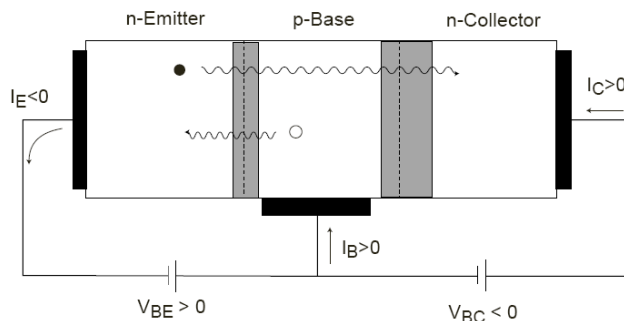
$$I_C = -J_{nB}A_E = qA_E \frac{D_n}{W_B} n_{pB0} \exp \frac{qV_{BE}}{kT}$$

Where  $I_S$  is called collector saturation current

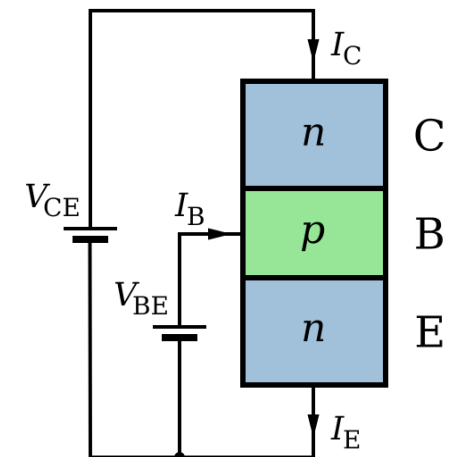
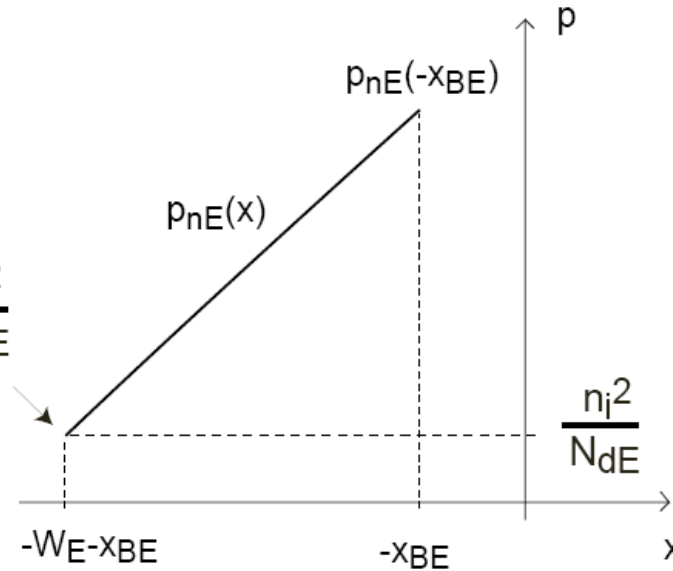
## Calculation of base current, $I_B$ (1)

Base current: focus on hole injection and recombination in emitter

Boundary conditions:  $p_{nE}(-x_{BE}) = p_{nE0} \exp \frac{qV_{BE}}{kT}$ ,  $p_{nE}(-W_E - x_{BE}) = p_{nE0}$



$$p_{nE}(-W_E - x_{BE}) = \frac{n_i^2}{N_{dE}}$$

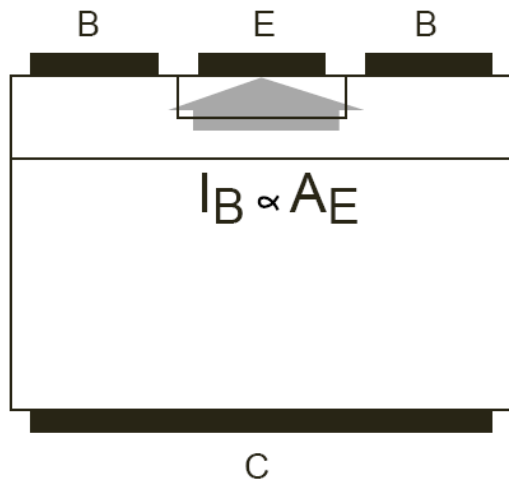


Hole  
Profile:

$$p_{nE}(x) = [p_{nE}(-x_{BE}) - p_{nE0}]\left(1 + \frac{x + x_{BE}}{W_E}\right) + p_{nE0}$$

## Calculation of base current, $I_B$ (2)

Hole current density:  $J_{pE} = -qD_p \frac{dp_{nE}}{dx} = -qD_p \frac{p_{nE}(-x_{BE}) - p_{nE0}}{W_E}$



- current scales with base-emitter area,  $A_E$

$$I_B = -J_{pE}A_E = qA_E \frac{D_p}{W_E} p_{nE0} \left( \exp \frac{qV_{BE}}{kT} - 1 \right)$$

**Compare with:**

$$I_C = -J_{nB}A_E = qA_E \frac{D_n}{W_B} n_{pB0} \exp \frac{qV_{BE}}{kT}$$

For  $V_{BE} \gg \frac{kT}{q}$ :

$$I_B \simeq \frac{I_C}{\beta_F}$$

$$I_B = \frac{I_S}{\beta_F} \left( \exp \frac{qV_{BE}}{kT} - 1 \right)$$



# What are the values of $\beta_F$ in a 'very good' transistor....?

Collector current is nearly identical to the (magnitude) of the emitter current ... define

$$I_C = \alpha_F I_E \quad \alpha_F = .999$$

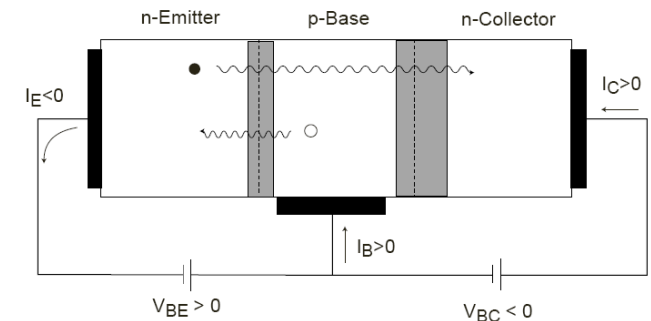
Kirchhoff:

$$I_E = I_C + I_B$$

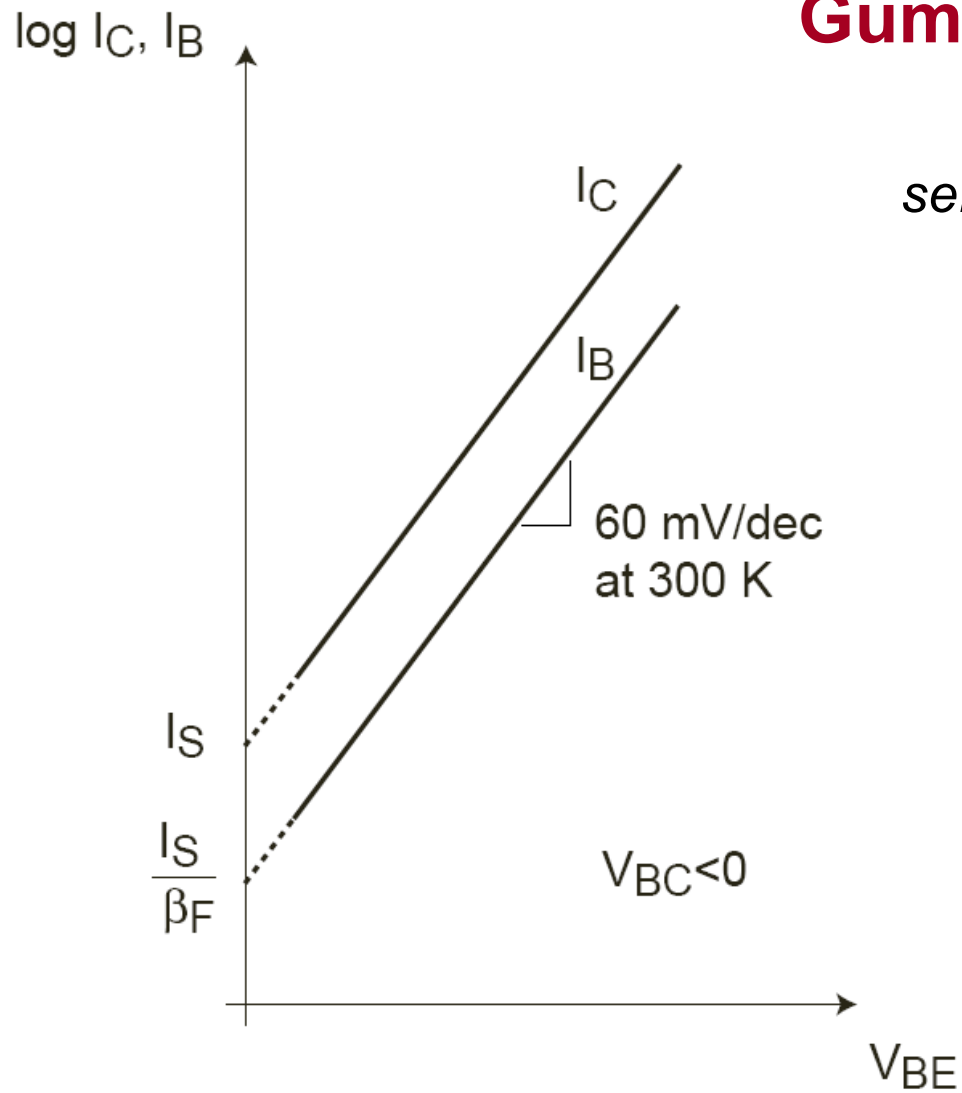
DC Current Gain:

$$I_C = \alpha_F I_E = \alpha_F (I_B + I_C)$$

$$I_C = \frac{\alpha_F}{1 - \alpha_F} I_B = \beta_F I_B \quad \beta_F = \frac{\alpha_F}{1 - \alpha_F} = \frac{.999}{.001} = 999$$



## Gummel plot



*semilog plot of  $I_C$  and  $I_B$  vs.  $V_{BE}$ :*

State-of-the-art IC BJT's :

$I_C \sim 0.1\text{--}1\text{ mA}$

$\beta_F \sim 50 - 300$

Current density  $10\text{--}100\text{A/cm}^2$

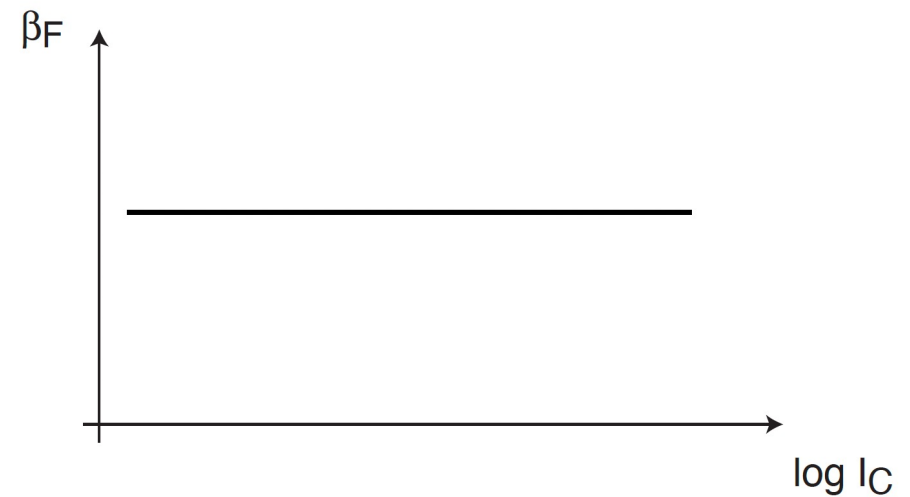
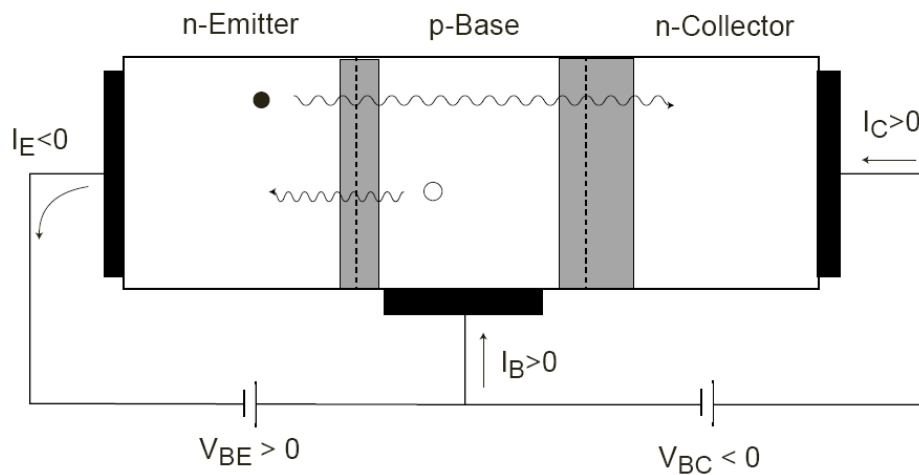
High-speed BJT: up to  $10^5\text{A/cm}^2$

# Key conclusions

- npn bipolar transistor operation in forward mode (régime direct):  
 $V_{BE} > 0$ ,  $V_{BC} < 0$  ( $\rightarrow V_{CE} > 0$ )
- Emitter "injects" electrons into Base, Collector "collects" electrons from Base  $\Rightarrow I_C$  controlled by  $V_{BE}$ , independent of  $V_{BC}$  (transistor effect)

$$I_C \propto \exp \frac{qV_{BE}}{kT}$$

$$I_C = \beta_F \times I_B$$



## Key conclusions

Current gain:

$$\beta_F = \frac{I_C}{I_B} = \frac{n_{pBo} \frac{D_n}{W_B}}{p_{nEo} \frac{D_p}{W_E}} = \frac{N_{dE} D_n W_E}{N_{aB} D_p W_B}$$

To maximize  $\beta_F$ :

- $N_{dE} \gg N_{aB}$
- $W_E \gg W_B$

For best current gain:

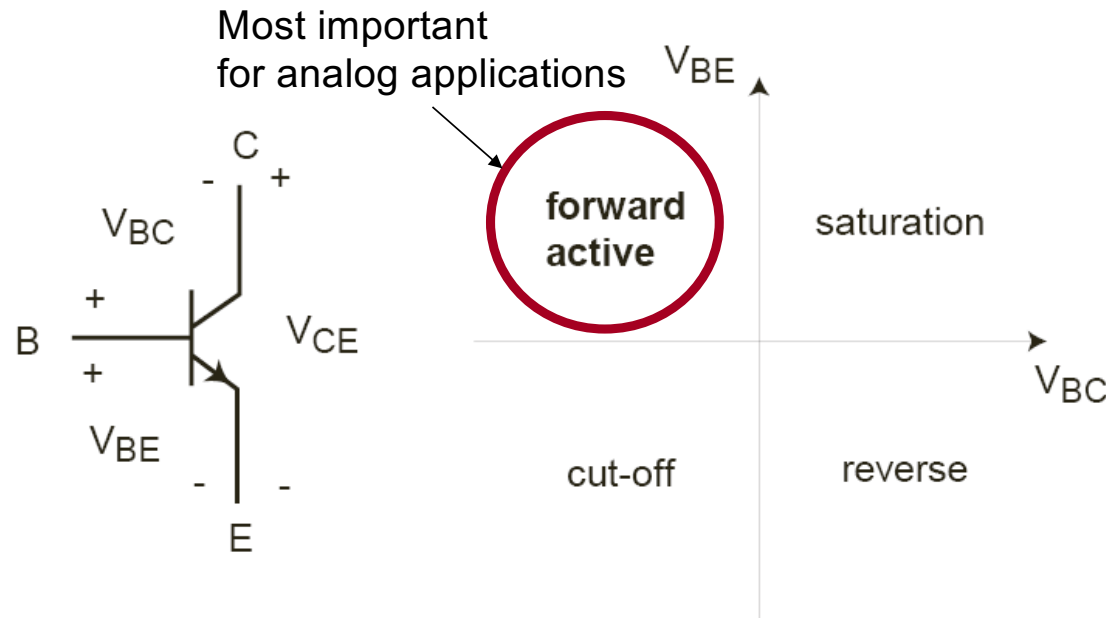
Narrow base, wide, highly doped emitter

prefer npn rather than pnp because  $D_n > D_p$

## Other key questions about BJT

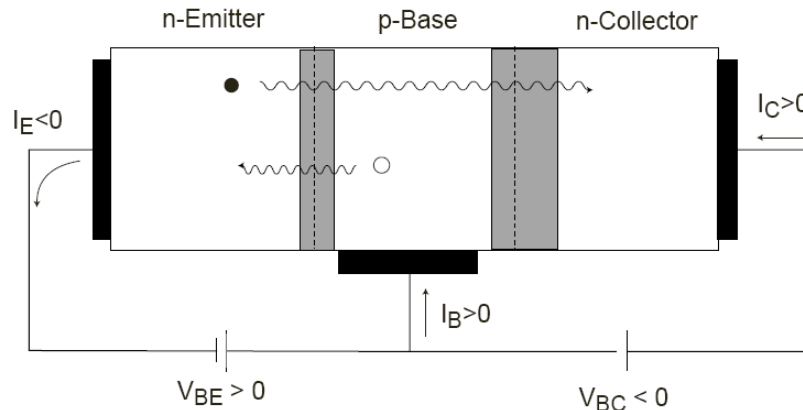
- What other regimes of operation are there for the BJT?
- What is important about each regime?
- How do equivalent circuit models for the BJT look like?

# Regimes of BJT operation



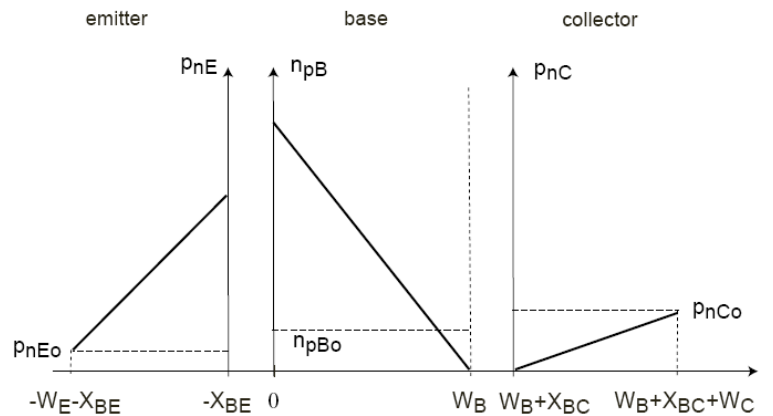
- **forward active**: device has good isolation and high gain; **most useful regime**;
- **saturation**: device has no isolation and is flooded with minority carriers  $\Rightarrow$  takes time to get out of saturation; **avoid it** for the majority of cases!
- **reverse**: poor gain; **not useful**;
- **cut-off**: negligible current: nearly an open circuit; **useful**.

## Forward-active regime: $V_{BE} > 0$ , $V_{BC} < 0$



- Emitter injects electrons into base, collector collects electrons from base
- Base injects holes into emitter, recombine at emitter contact

### Minority carrier profiles

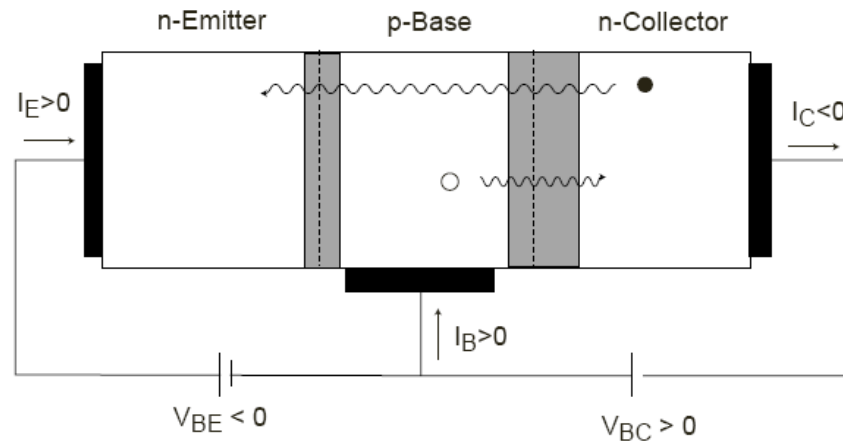


$$I_C = I_S \exp \frac{qV_{BE}}{kT}$$

$$I_B = \frac{I_S}{\beta_F} \left( \exp \frac{qV_{BE}}{kT} - 1 \right)$$

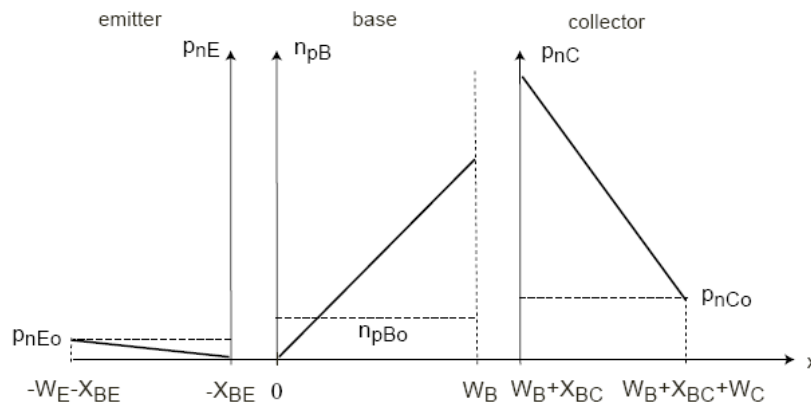
$$V_{BE} \sim 0.6 - 0.7V$$

## Reverse regime: $V_{BE} < 0$ , $V_{BC} > 0$



- Similar with the direct regime with the difference that  $\beta_F$  is replaced by  $\beta_R$  (the npn transistor is not symmetrical, the transistor effect is less efficient in reverse mode!)
- $I_E \rightarrow I_C$ ,  $V_{BE} \rightarrow V_{BC}$

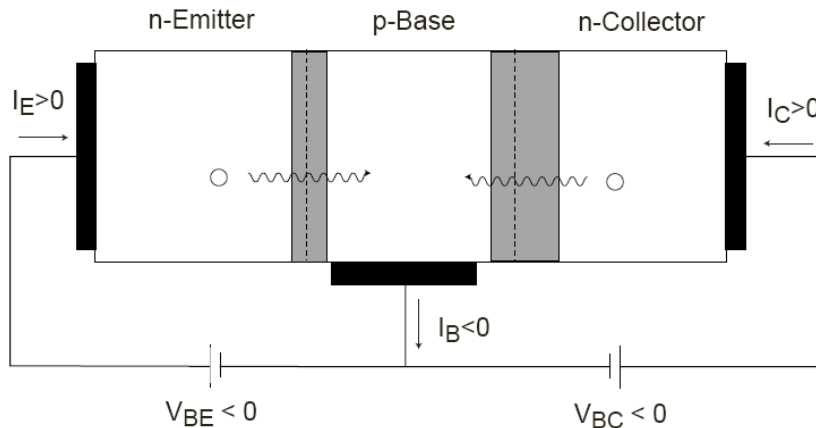
### Minority carrier profiles



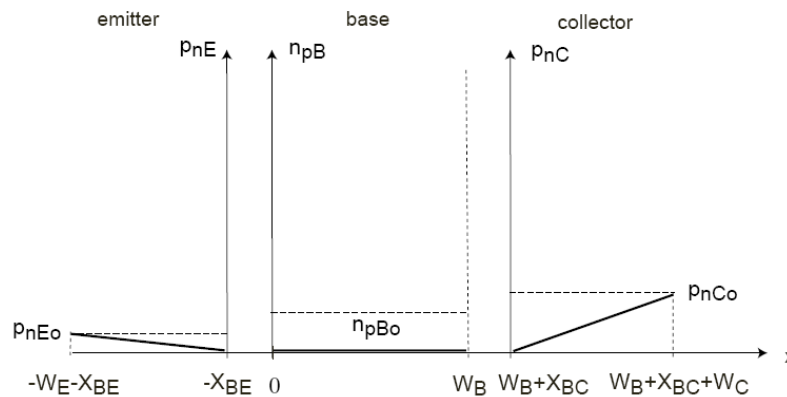
Typically,  $\beta_R \simeq 0.1 - 5 \ll \beta_F$ .



## Cut-off regime: $V_{BE} < 0$ , $V_{BC} < 0$



### Minority carrier profiles



- Base extracts holes from emitter:

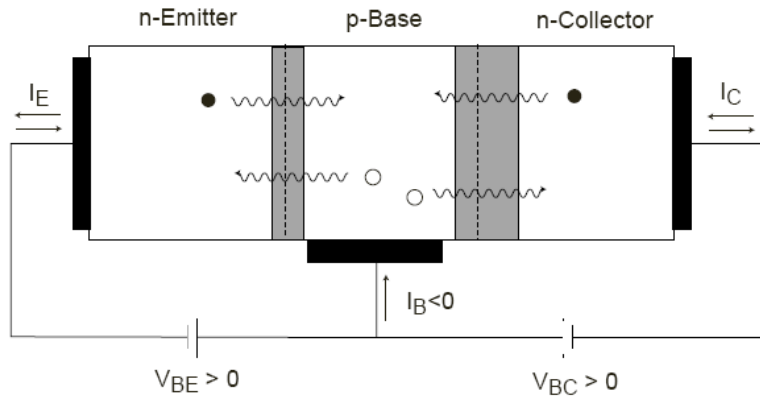
$$I_{B1} = -\frac{I_S}{\beta_F} = -I_E$$

- Base extracts holes from collector:

$$I_{B2} = -\frac{I_S}{\beta_R} = -I_C$$

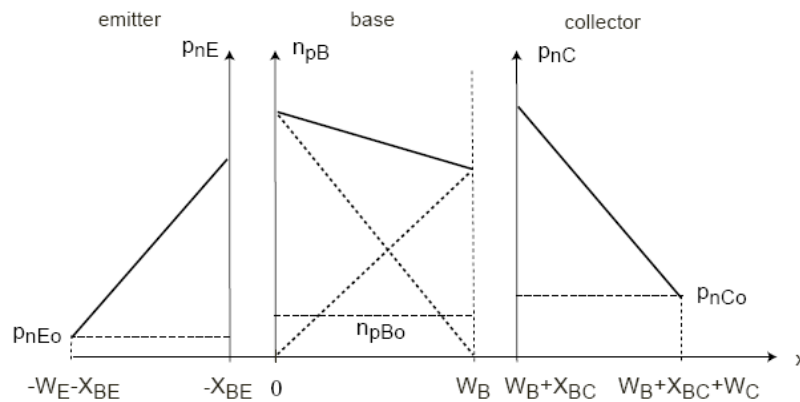
- junction leakage currents ( $\sim 10^{-12}$  A) – junctions in reverse bias

**Saturation:  $V_{BE} > 0$ ,  $V_{BC} > 0$  ( $V_{CE} \sim 0V$ !)**



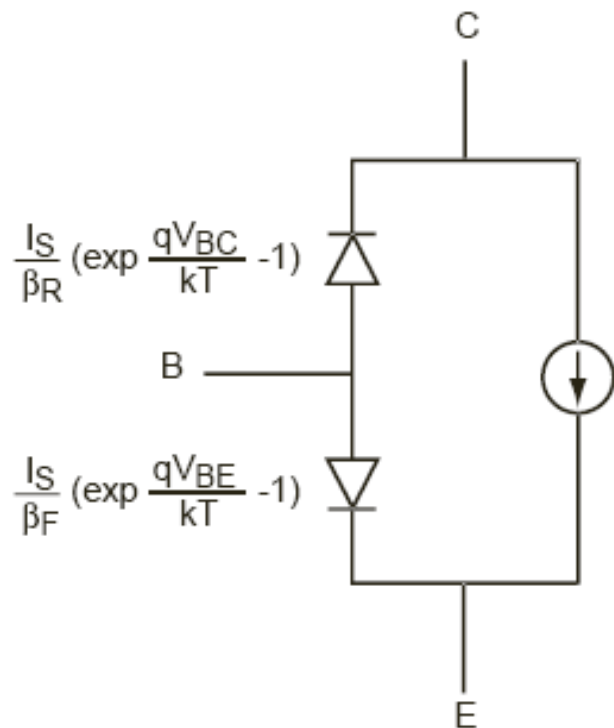
- Saturation is superposition of forward active + reverse
- $I_C$  and  $I_E$  can have either sign, depending on relative magnitude of  $V_{BE}$  and  $V_{BC}$ , and  $\beta_F$  and  $\beta_R$ .
- In saturation, collector and base flooded with excess minority carriers  $\Rightarrow$  takes lots of time to get transistor out of saturation.

### Minority carrier profiles



# Large-signal equivalent circuit model

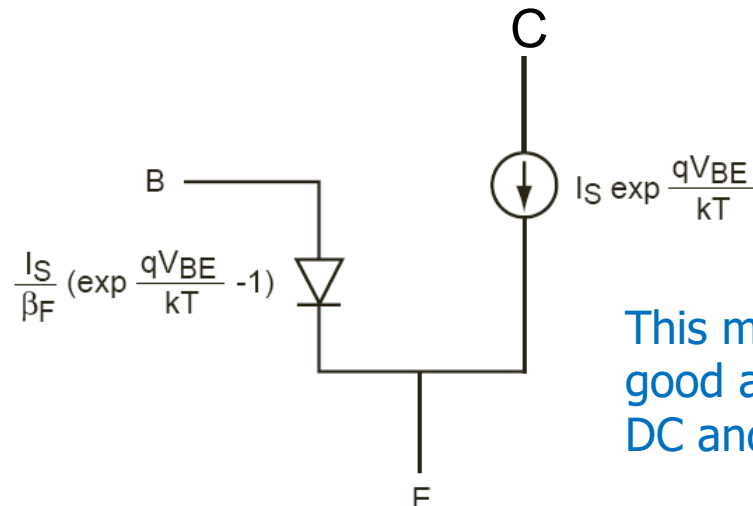
## Equivalent circuit model



Only three parameters in this model:

$I_S$ ,  $\beta_F$ , and  $\beta_R$ .

Example: forward-active regime,  $V_{BE} > 0$ ,  $V_{BC} < 0$



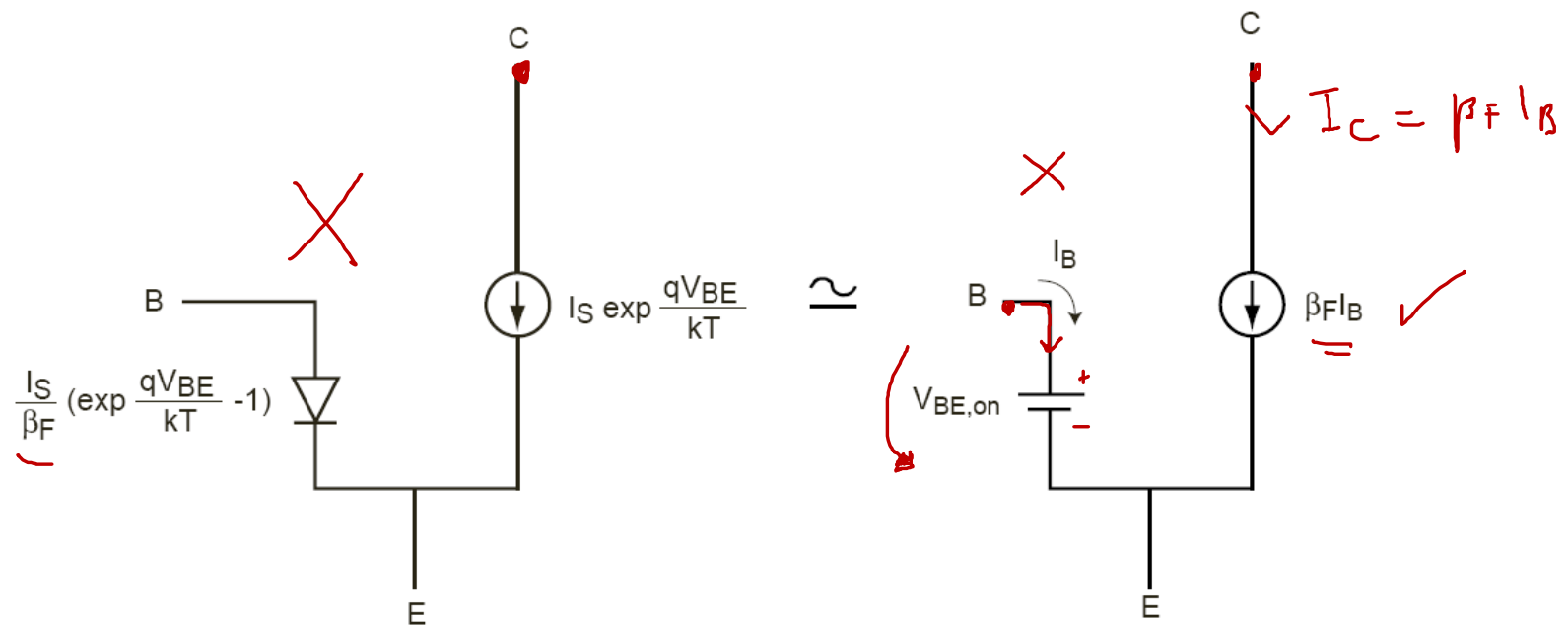
## Ebers-Moll model

A functional model of the BJT utilizes ideal diodes to model the base-emitter and base-collector junctions and a current source.

This model is sufficient to achieve good analysis results with a variety of DC and low frequency circuits

## Simplified equivalent circuit (1)

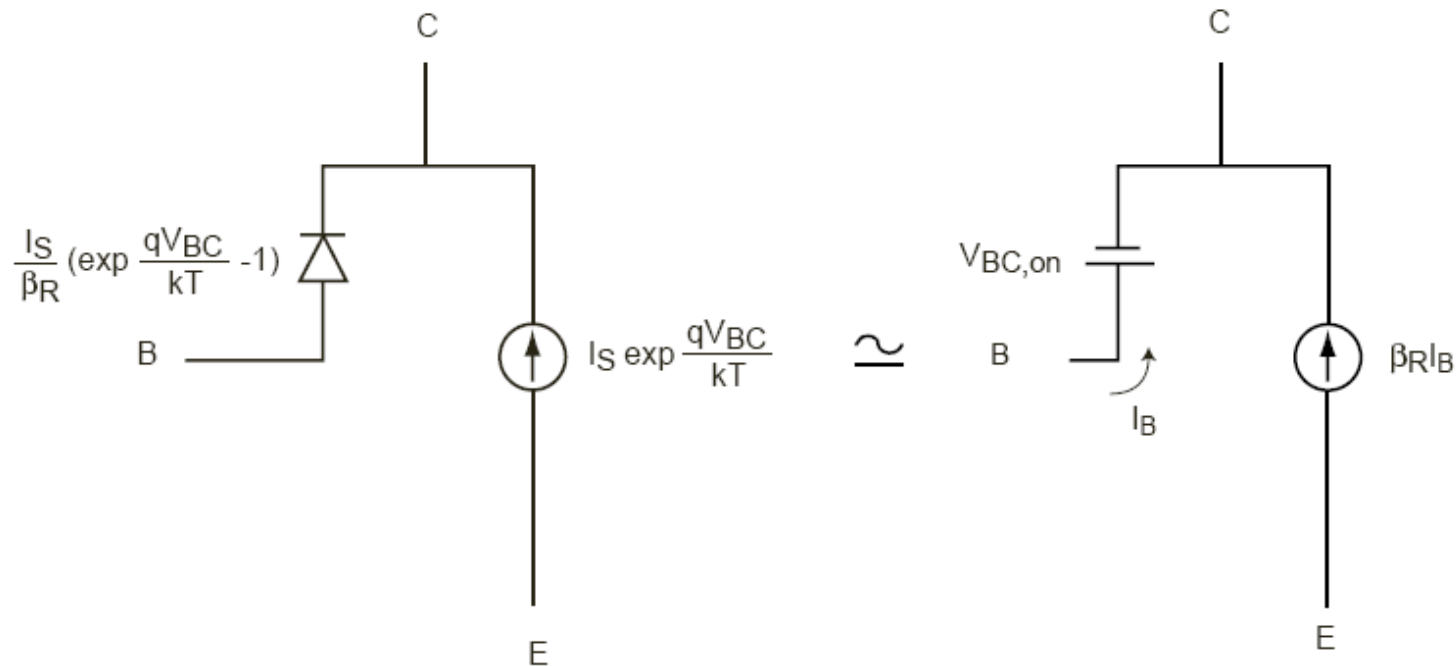
- Forward-active regime:  $V_{BE} > 0$ ,  $V_{BC} < 0$



For today's technology:  $V_{BE,on} \approx 0.7 \text{ V}$ .

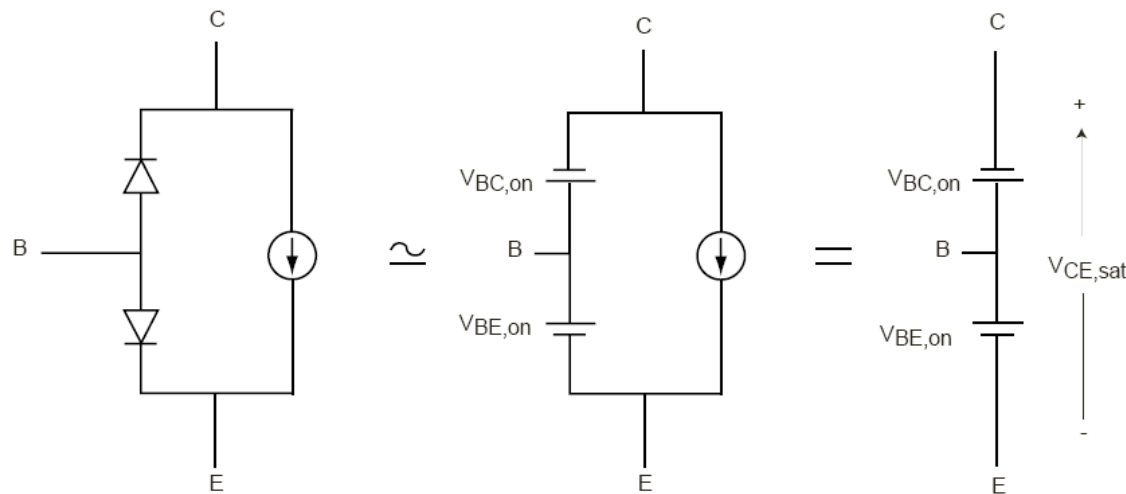
## Simplified equivalent circuit (2)

- Reverse:  $V_{BE} < 0$ ,  $V_{BC} > 0$



## Simplified equivalent circuit (3)

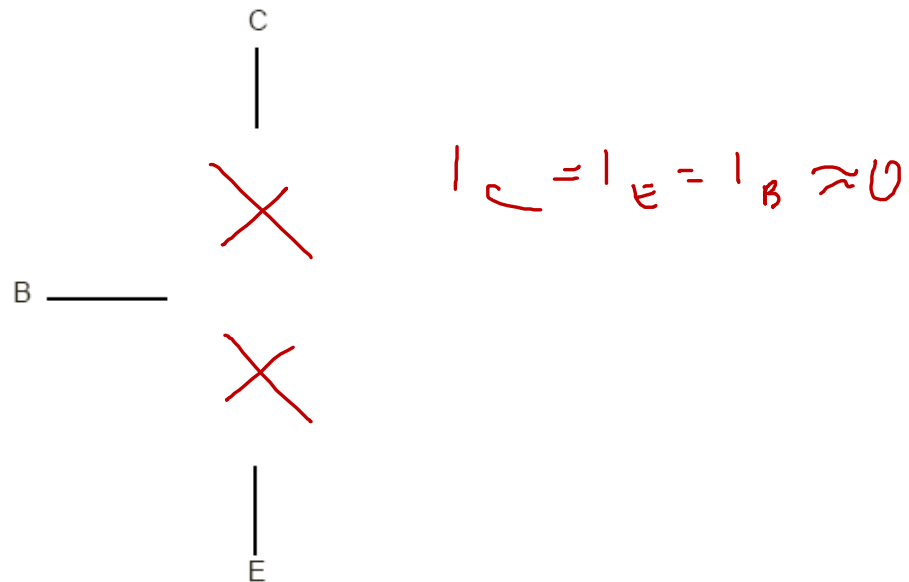
- Saturation:  $V_{BE} > 0$ ,  $V_{BC} > 0$



Today's technology:  $V_{CE,sat} = V_{BE,on} - V_{BC,on} \simeq 0.2 \text{ V}$ .  
 $I_B$  and  $I_C$  depend on outside circuit.

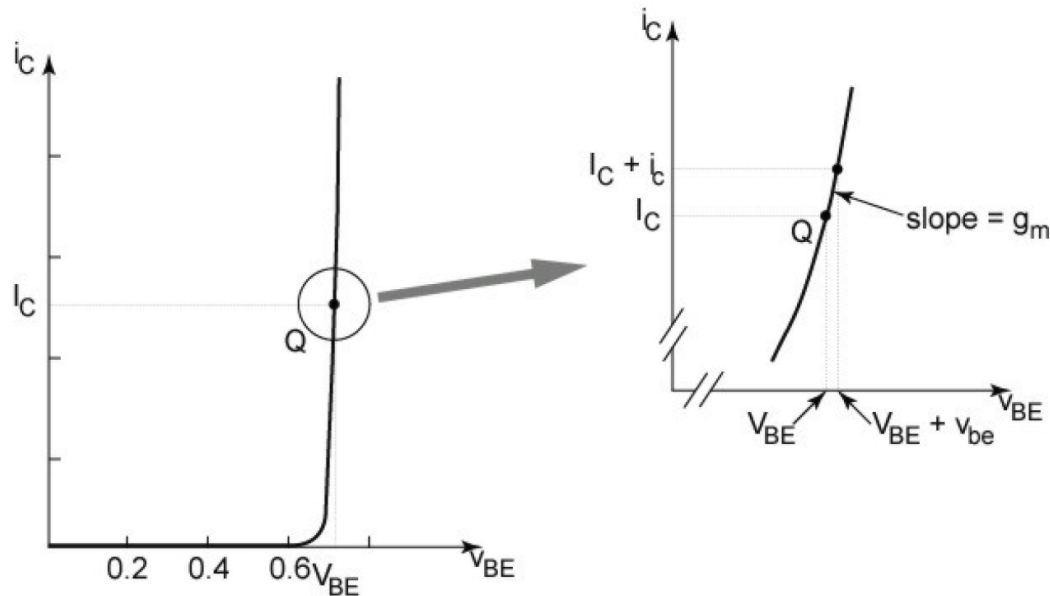
## Simplified equivalent circuit (4)

- Cut-off:  $V_{BE} < 0$ ,  $V_{BC} < 0$



Only negligible leakage currents.

# Transfer characteristics, $I_C(V_{BE})$ : BJT transconductance



Differentiating and evaluating at  $Q = (V_{BE}, V_{CE})$

$$\left. \frac{\partial i_C}{\partial v_{BE}} \right|_Q = \frac{q}{kT} I_S e^{qV_{BE}/kT} (1 + V_{CE}/V_A)$$

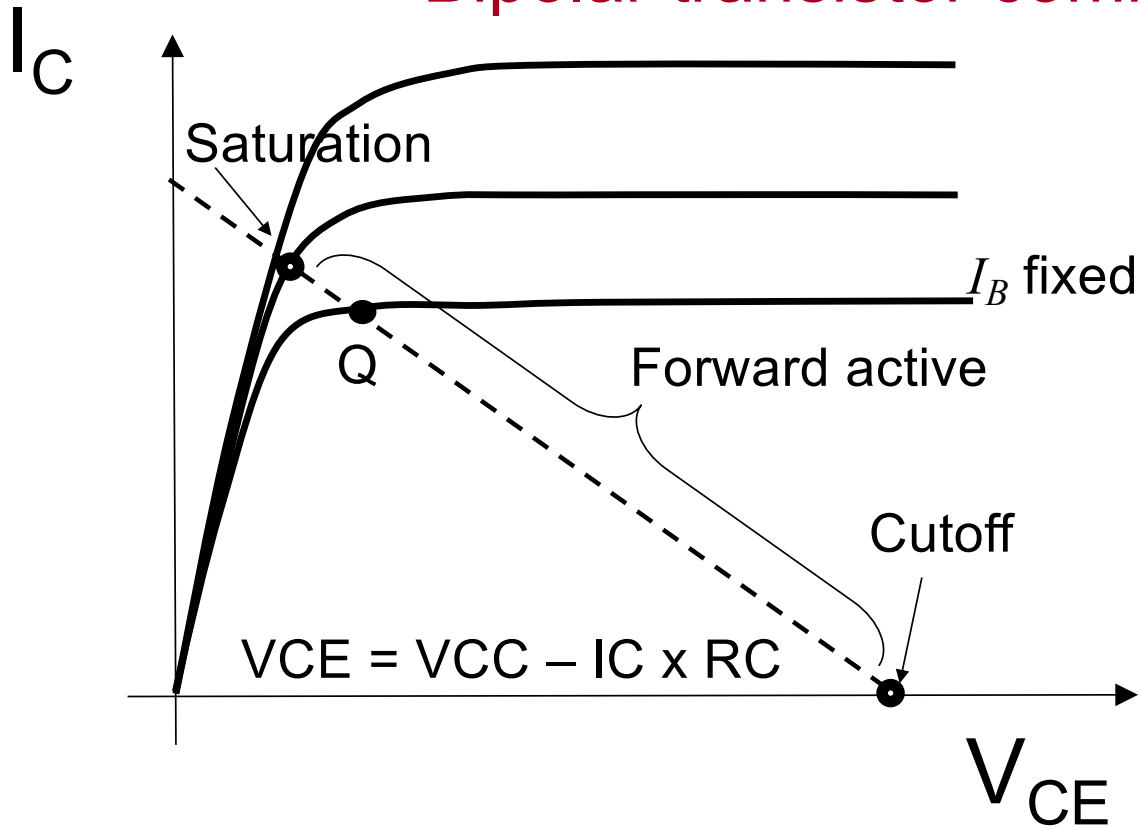
$$g_m = \left. \frac{\partial i_C}{\partial v_{BE}} \right|_Q = \frac{qI_C}{kT}$$

- The transconductance is analogous to diode conductance

**Q-point** - “Quiescent” means “at rest”—so the Q-point represents the steady-state values of collector current



## Bipolar transistor common-emitter I-V with load line



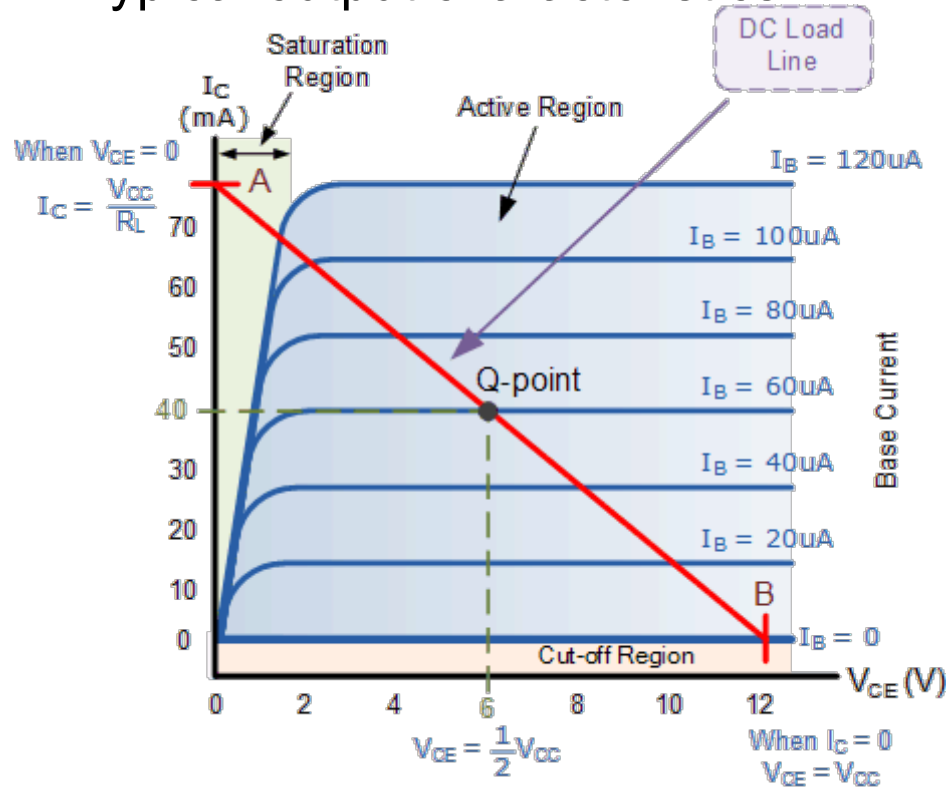
- The Q-point of a BJT is the point on its output characteristics that defines the transistor's DC operating condition—specifically, when no input signal is present
- Q-point represents the steady-state  $I_c$  and  $V_{ce}$
- Q-point is the intersection point of the transistor's DC load line and the output characteristic curve **for a given base current**
- The Q-point determines how the transistor will behave when an AC signal is applied (i.e., how it will amplify signals)

### • Why Q-point Matters:

- A properly chosen Q-point ensures linear amplification and avoids distortion.
- If the Q-point is too close to cutoff (where the transistor is off) or saturation (where it's fully on), the output signal may get clipped.

# Bipolar transistor common-emitter I-V with load line

## Typical output characteristics



- This BJT output characteristics helps to avoid the saturation regime

Pick a Q-point in the middle third of the load line:

This gives room for the signal to swing up and down without pushing the transistor into saturation (left) or cutoff (right)

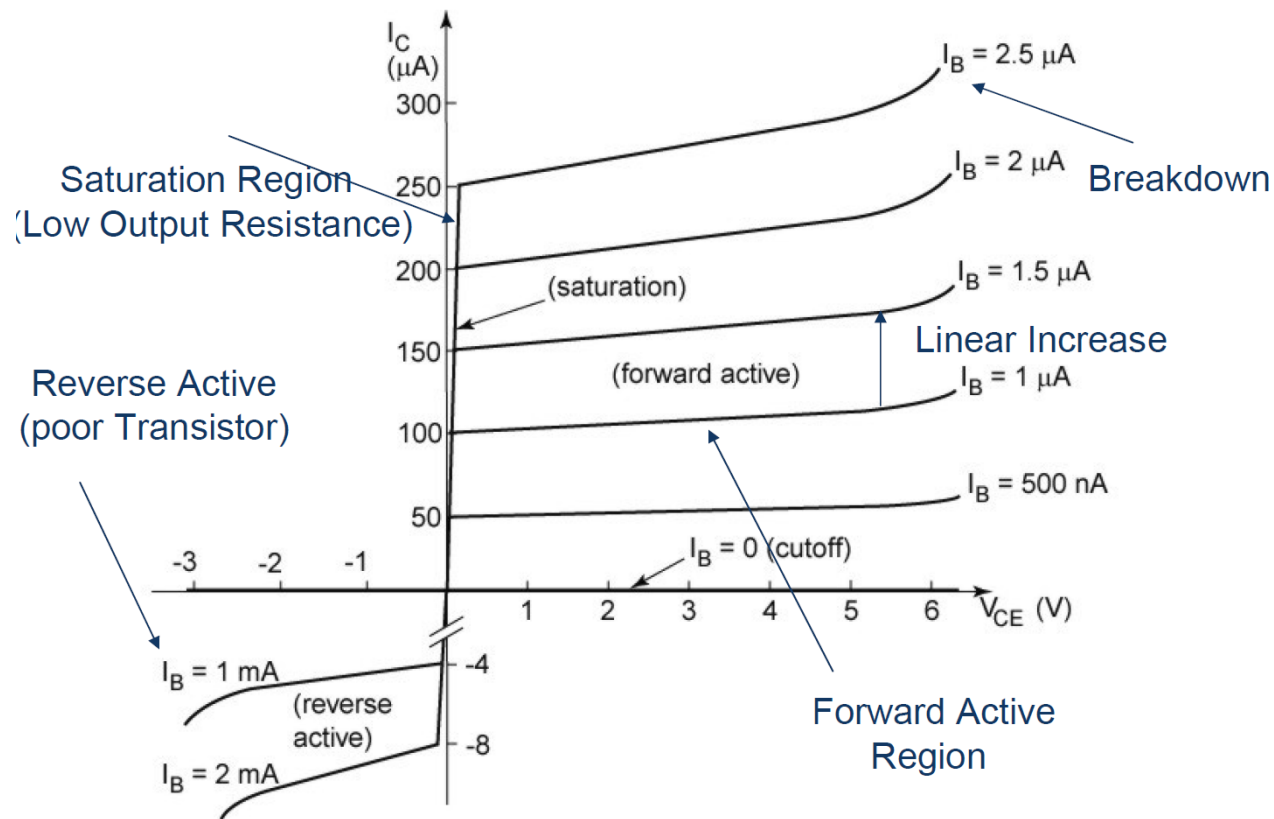
- If your Q-point (red dot) is too close to the bottom-left corner of the graph low  $V_{CE}$ , it's entering saturation. Then moved the Q-point to the right by reducing  $I_B$

### • Why Q-point Matters:

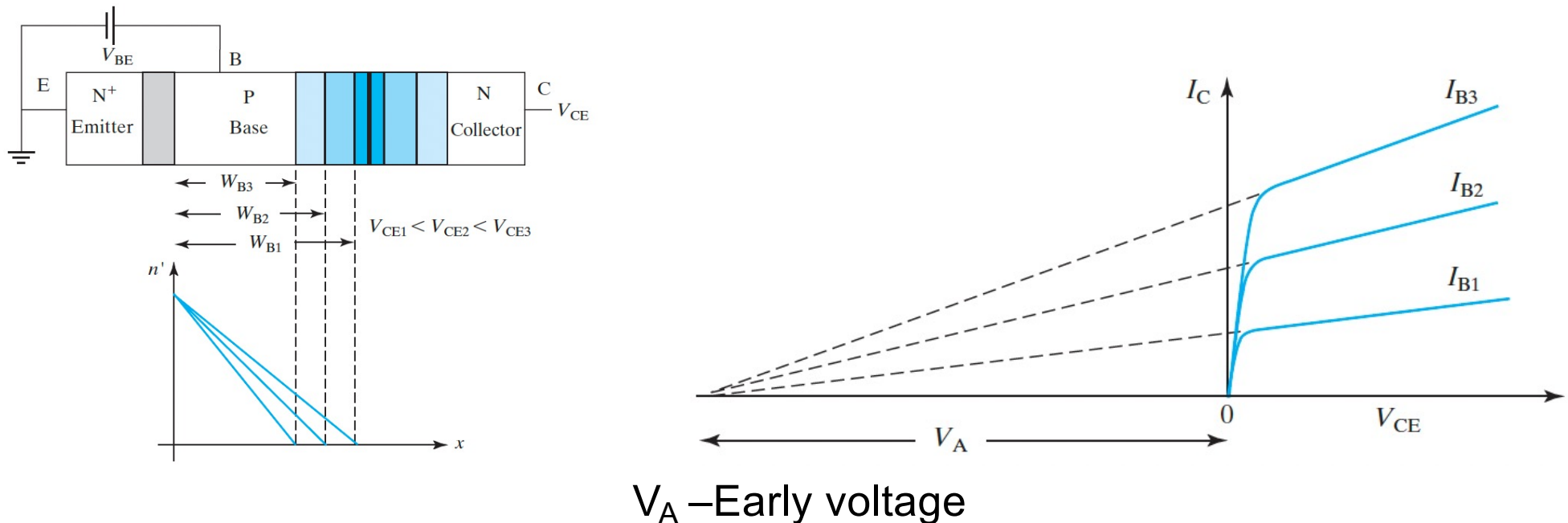
- A properly chosen Q-point ensures linear amplification and avoids distortion.
- If the Q-point is too close to cutoff (where the transistor is off) or saturation (where it's fully on), the output signal may get clipped.

# Output characteristics: in real transistors $I_C$ tends to increase with $V_{CE}$

Why?



# Output characteristics: varying width of the base

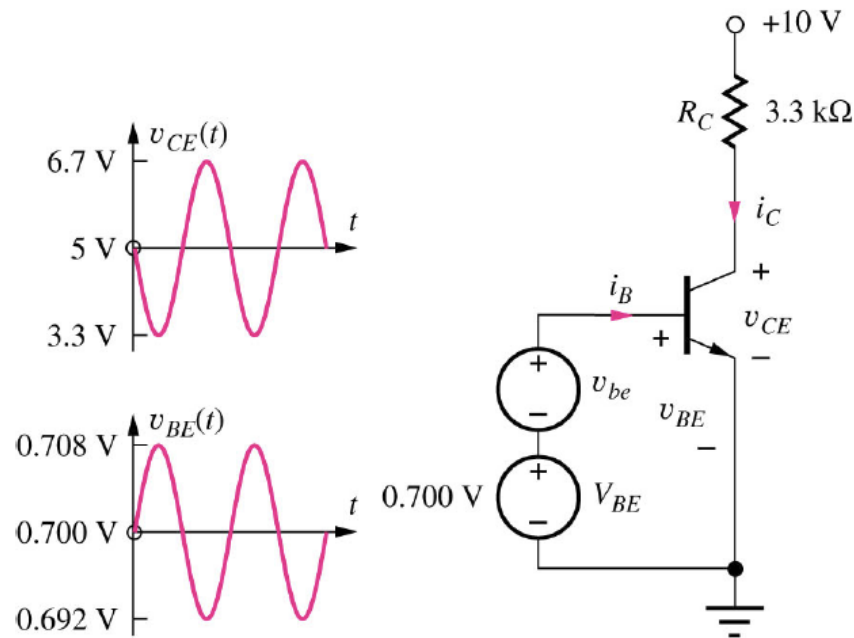


As  $V_C$  increases, the BC depletion layer width increases and  $W_B$  decreases causing  $dn/dx$  and  $I_C$  to increase.

The effect is described using the Early voltage approximation  
(avoids complex maths)

# BJT amplifier concept (1)

*Common emitter configuration – the most used one*



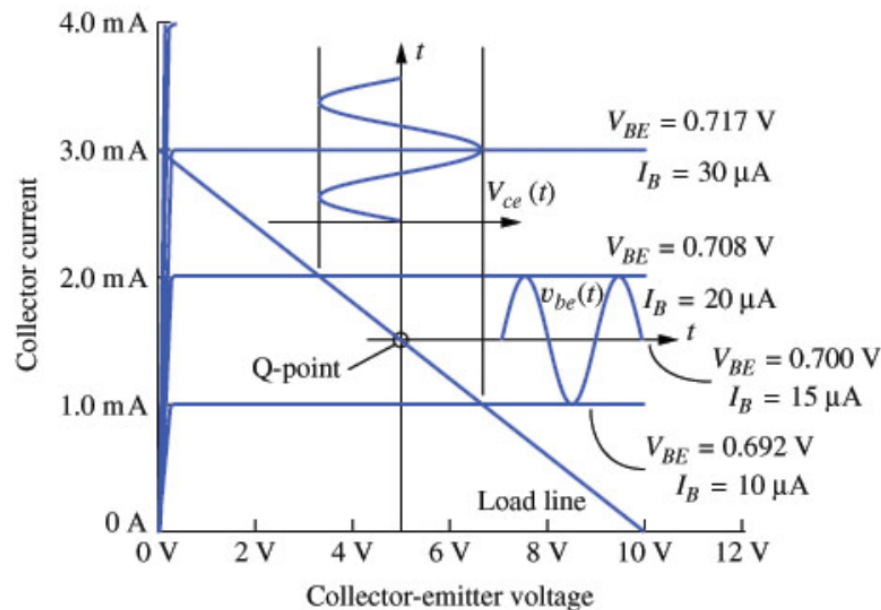
- Q-point is set at  $(I_C, V_{CE}) = (1.5\text{ mA}, 5\text{ V})$  with  $I_B = 15\text{ }\mu\text{A}$  ( $\beta_F = 100$ )
- Total base-emitter voltage is:  $V_{BE} = V_{BE\text{ const}} + v_{be}$
- Collector-emitter voltage is:

$$v_{CE} = V_{CC} - i_C R_C$$

This is the load line equation.

(10V / 3300 Ohm = 3mA; 1.5mA is a good Q-point)

# BJT amplifier concept (2)



**8 mV peak change in  $v_{BE}$  gives  $5 \mu A$  change in  $i_B$  and 0.5 mA change in  $i_C$ .**

**0.5 mA change in  $i_C$  produces a 1.65 V change in  $v_{CE}$ .**

For 8mV ac signal added to  $V_{BE}$

$$\Delta I_C / \Delta V_{BE} = I_C q / kT \text{ (low-signal conduction)}$$

A small voltage change at the base causes a large voltage change at collector. Voltage gain is given by:

$$A_v = \frac{V_{ce}}{V_{be}} = \frac{1.65 \angle 180^\circ}{0.008 \angle 0^\circ} = 206 \angle 180^\circ = -206$$

**Minus sign indicates 180° phase shift between the input and output signals.**

# BJT - applications

- TTL logic (Transistor-Transistor Logic) - digital logic design built using BJTs. It's used to implement logic operations like AND, OR, NOT - basic building blocks of digital circuits
  - Legacy design - Introduced in the 1960s, was popular in the 70s and 80s, now replaced by CMOS
- Amplifiers: BJTs are still widely used in audio, RF, and instrumentation amplifiers due to their high gain and linearity.
- RF (Radio Frequency) Circuits: BJTs perform well at high frequencies and are used in oscillators, mixers, and RF amplifiers

## Advantages (compared to MOSFET):

- Lower output impedance (great for analog)
- Better performance in certain linear applications
- Less sensitive to static discharge (unlike MOSFETs)

## Disadvantages of BJT :

- large chip area is needed for fabrication
- Power loss is high
- very temperature sensitive
- more sensitive to radiations than MOSFET