Lecture 3
Microelectronic Devices

- Fermi statistics for the non-equilibrium conditions
- Electrostatics of semiconductors in equilibrium
  - effect of non-uniform doping
  - space-charge regions, built-in field and potential
  - electrostatics of pn junctions

Summary from L2: charge carriers in equilibrium

Fermi distribution (also known as Fermi-Dirac function):

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

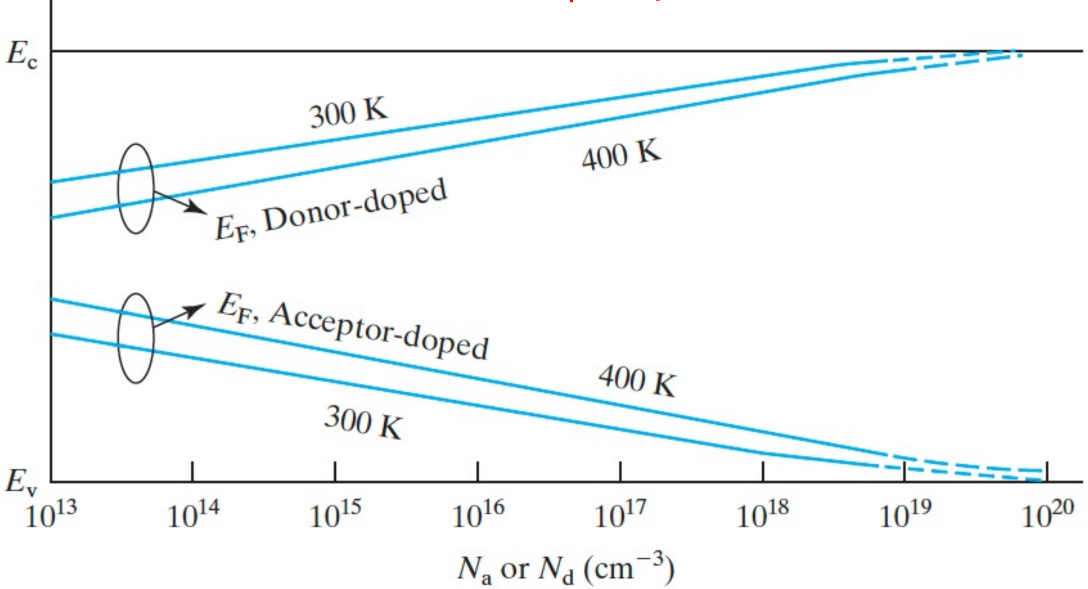
Boltzmann approximation, $E-E_F >> kT$

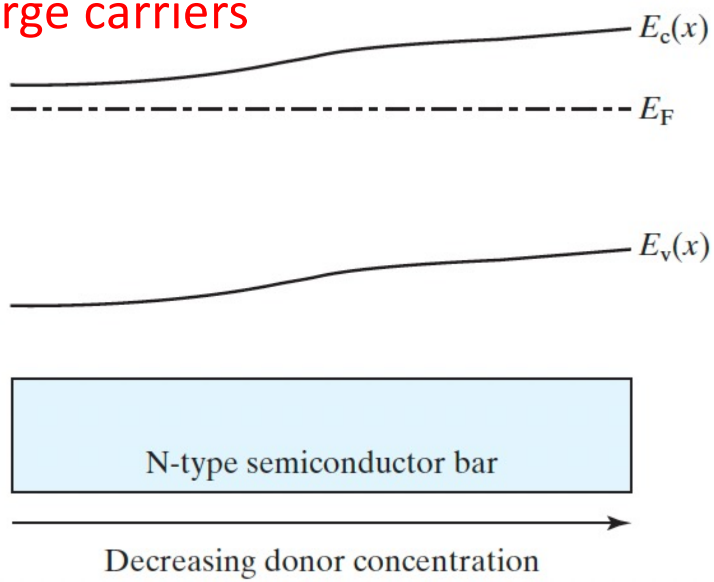
$$f(E) \approx e^{-(E-E_F)/kT}$$

$$n = N_c\, e^{-(E_c-E_F)/kT}$$

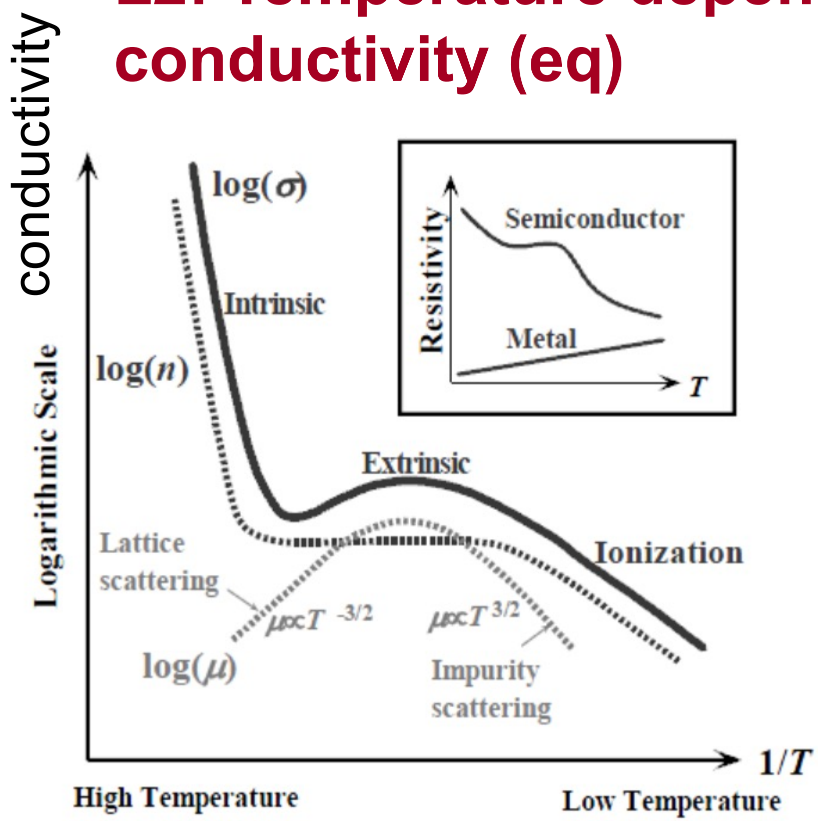
$$p = N_v\, e^{-(E_F-E_v)/kT}$$

Position of E_F depends on concentration of charge carriers





L2: Temperature dependence of semiconductor conductivity (eq)



Temperature dependence of electrical conductivity for a doped (*n*-type) semiconductor.

Calculate the intrinsic carrier density in germanium, silicon and gallium arsenide at 300, 400, 500 and 600 K.

Solution

The intrinsic carrier density in silicon at 300 K equals:

$$n_i(300K) = \sqrt{N_c N_v}\, exp\left(\frac{-E_g}{2\ KT}\right)$$

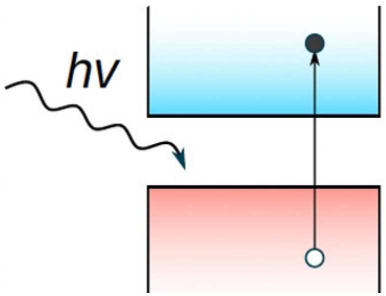
$$= \sqrt{2.81\ x\ 10^{19}\ x\ 1.83\ x\ 10^{19}}\ exp\left(\frac{-1.12}{2\ x\ 0.0258}\right)$$

$$= 8.72\ x\ 10^{9} m^{-3}$$

Try this for other materials and temperatures

	Germanium	Silicon	Gallium Arsenide
300 K	2.02×10^{13}	8.72×10^{9}	2.03×10^{6}
400 K	1.38×10^{15}	4.52×10^{12}	5.98×10^{9}
500 K	1.91×10^{16}	2.16×10^{14}	7.98×10^{11}
600 K	1.18×10^{17}	3.07×10^{15}	2.22×10^{13}

Fermi statistics for the non-equilibrium conditions



• Photogeneration of electron/hole pairs – the system is driven out of equilibrium

$$n = N_c e^{-(E_c - E_F)/kT}$$

$$p = N_v e^{-(E_F - E_v)/kT}$$

$$np \neq n_i^2$$

$$n = N_c e^{-(E_c - E_{Fn})/kT}$$

$$p = N_v e^{-(E_{Fp} - E_v)/kT}$$

• $\boldsymbol{E_{Fn}}$ and $\boldsymbol{E_{Fp}}$ are the electron and hole quasi-Fermi levels. When electrons and holes are at equilibrium, i.e., when $\boldsymbol{np = n_i^2}$, $\boldsymbol{E_{Fn}}$ and $\boldsymbol{E_{Fp}}$ coincide and this is known as $\boldsymbol{E_F}$. Otherwise, $\boldsymbol{E_{Fn} \neq E_{Fp}}$.

• The concept of quasi-Fermi levels is usable, because even when electrons and holes, as two groups, are not at equilibrium with each other, the electrons (and holes) can still be at equilibrium among themselves.

Fermi statistics for the non-equilibrium conditions: numerical example

Consider a Si sample with $N_d = 10^{17}\text{cm}^{-3}$. Find the location of E_F.
Find the location of E_{Fn} and E_{Fp} when excess carriers are introduced such that the excess carrier concentrations $p' = n' = 10^{15}\text{cm}^{-3}$.

$$n = N_d = 10^{17}\text{cm}^{-3} = N_c e^{-(E_c - E_F)/kT}$$

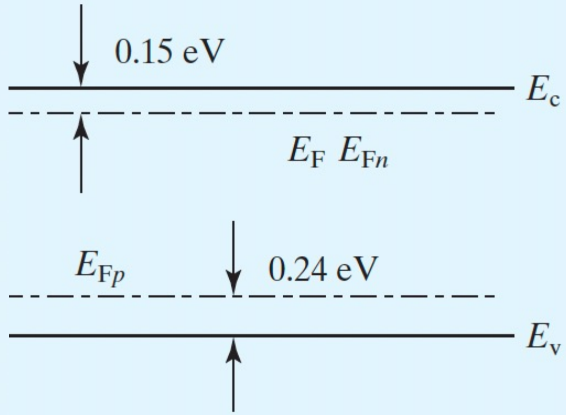
$$E_c - E_F = kT \ln \frac{N_c}{10^{17}\text{cm}^{-3}} = 26\text{ meV} \cdot \ln \frac{2.8 \times 10^{19}\text{cm}^{-3}}{10^{17}\text{cm}^{-3}} = 0.15\text{ eV}$$

- this is E_F in equilibrium, E_{Fn} remains nearly same because $10^{15} << 10^{17}$

$$p = p_0 + p' = \frac{n_i^2}{N_d} + p' = 10^3\text{cm}^{-3} + 10^{15}\text{cm}^{-3} = 10^{15}\text{cm}^{-3}$$

$$10^{15}\text{cm}^{-3} = N_v e^{-(E_{Fp} - E_v)/kT}$$

$$E_{Fp} - E_v = kT \ln \frac{N_v}{10^{15}\text{cm}^{-3}} = 26\text{ meV} \cdot \ln \frac{1.04 \times 10^{19}\text{cm}^{-3}}{10^{15}\text{cm}^{-3}} = 0.24\text{ eV}$$



Electrostatics of semiconductors:

- non-uniform doping

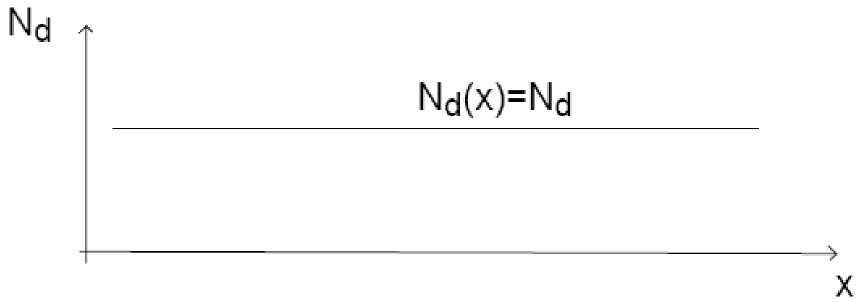
- thermal equilibrium

• Is it possible to have **an electric field inside a semiconductor in thermal equilibrium**?

• If there is a **doping gradient in a semiconductor, what is the resulting majority carrier concentration in thermal equilibrium**?

Uniformly doped semiconductor in thermal equilibrium

- Uniformely doped semiconductor, n-type



n-type → lots of electrons, few holes

$n_o = N_d$ independent of x

Volume charge density [C/cm^3]:

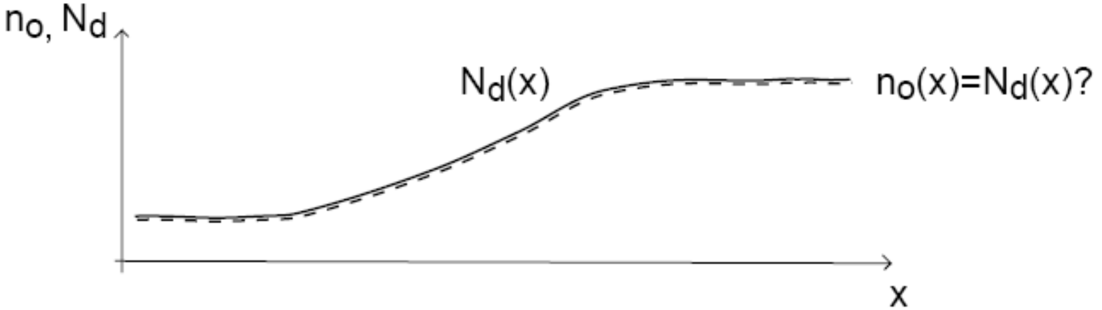
$$\rho = q(N_d - n_o) = 0$$

Non-uniformly doped semiconductor in thermal equilibrium

Consider a piece of n-type Si in thermal equilibrium
with non-uniform dopant distribution: ***what is the resulting electron concentration in thermal equilibrium?***

Option 1:

$$n_o(x) = N_d(x)$$



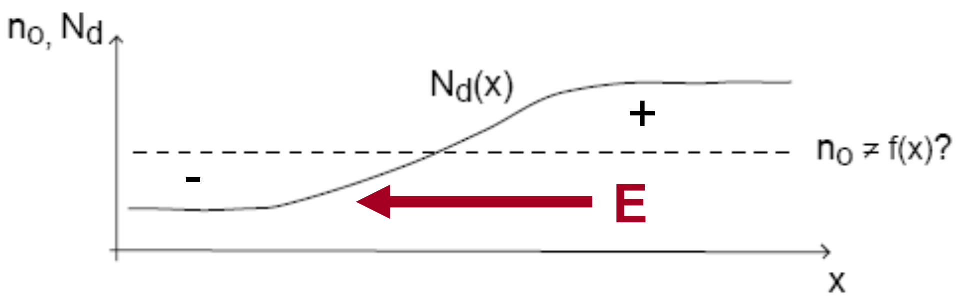
Gradient of electron concentration:

- net electron diffusion
- this is not thermal equilibrium! Not the good solution.

Non-uniformly doped semiconductor in thermal equilibrium

<u>Option 2:</u> Electron concentration uniform in space:

$$n_o = n_{ave} \neq f(x)$$



Space charge density: $\rho(x) = q[N_d(x) - n_o(x)]$

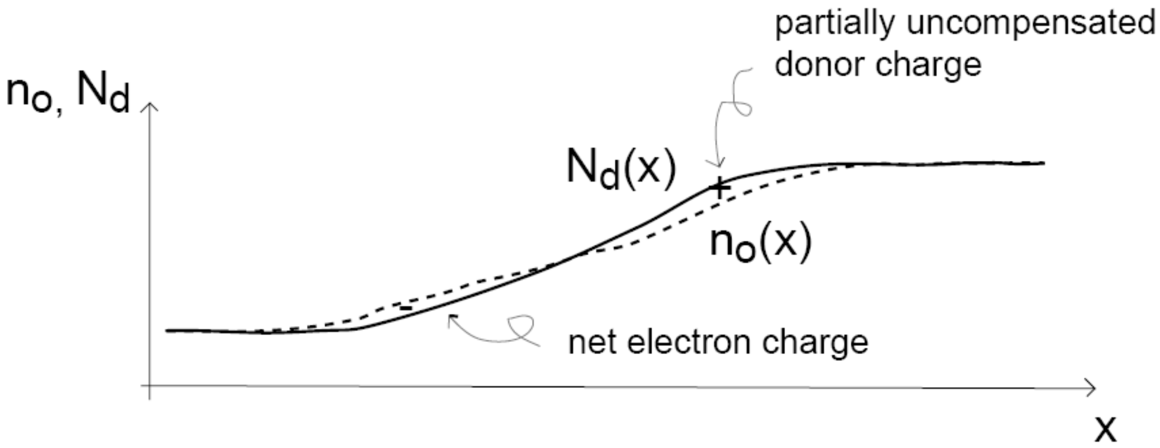
If $N_d(x) \neq n_o(x) \Rightarrow \rho(x) \neq 0$
$\Rightarrow$ electric field
$\Rightarrow$ <u>net electron drift</u>
$\Rightarrow$ not thermal equilibrium!

<u>Not the good solution</u>

Non-uniformly doped semiconductor in thermal equilibrium (3)

Option 3: Diffusions balances drift: $Jn=0$ and $Jp=0$ @ any point

$$J_n(x) = J_n^{drift}(x) + J_n^{diff}(x) = 0$$



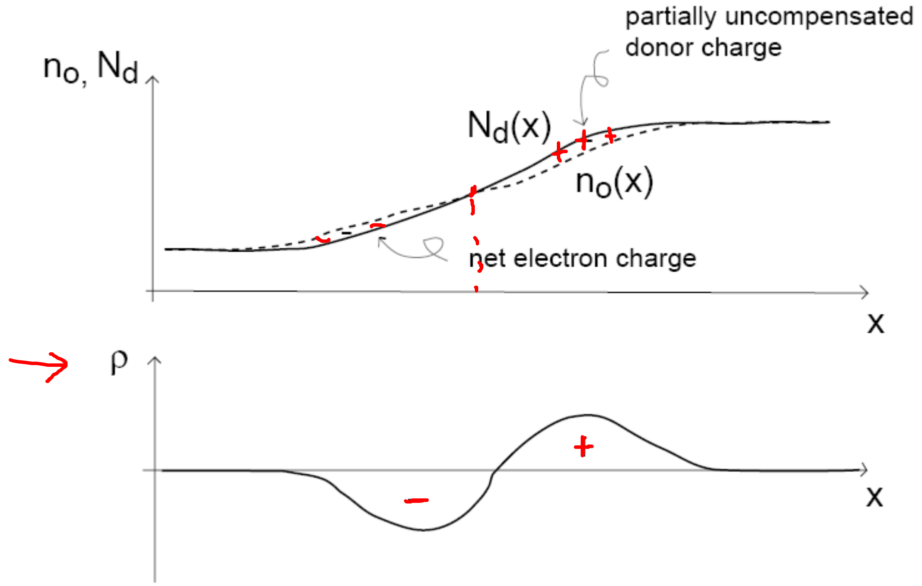
What n(x) profile meets this condition?

$$n_o(x) \neq N_d(x)$$

Non-uniformly doped semiconductor in thermal equilibrium

Resulting space charge density:

$$\rho(x) = q[N_d(x) - n_o(x)] \neq 0$$

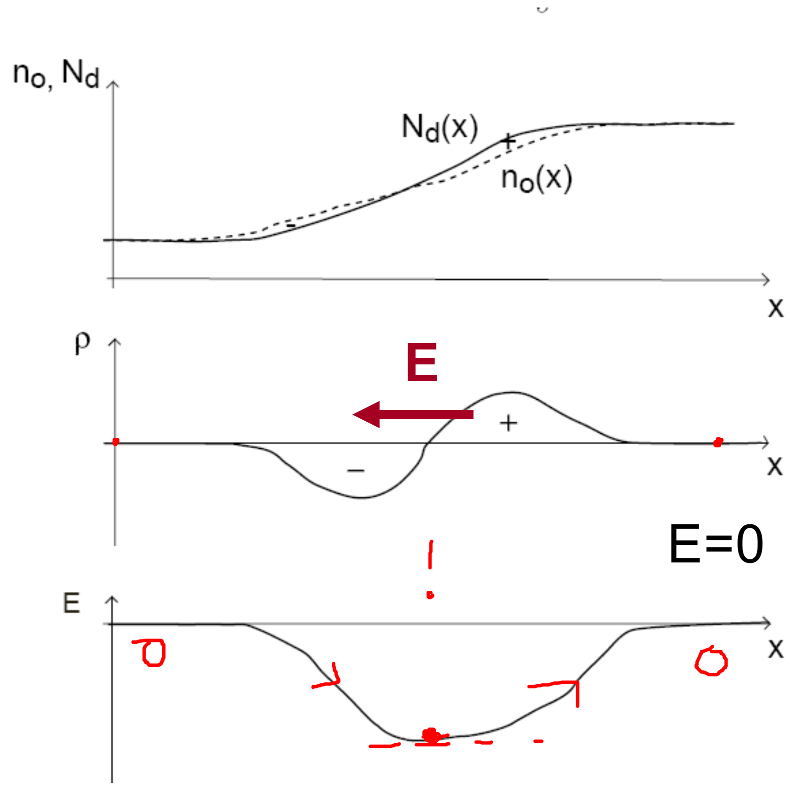


Non-uniformly doped semiconductor in thermal equilibrium (5)

Resulting electrical field, from Gauss equation:

$$\frac{dE}{dx} = \frac{\rho}{\epsilon_s}$$

$$\int \rho dx = \epsilon_s E$$



Integrate from 0 to x:

$$E(x) - E(0) = \frac{1}{\epsilon_s} \int_0^x \rho(x) dx$$

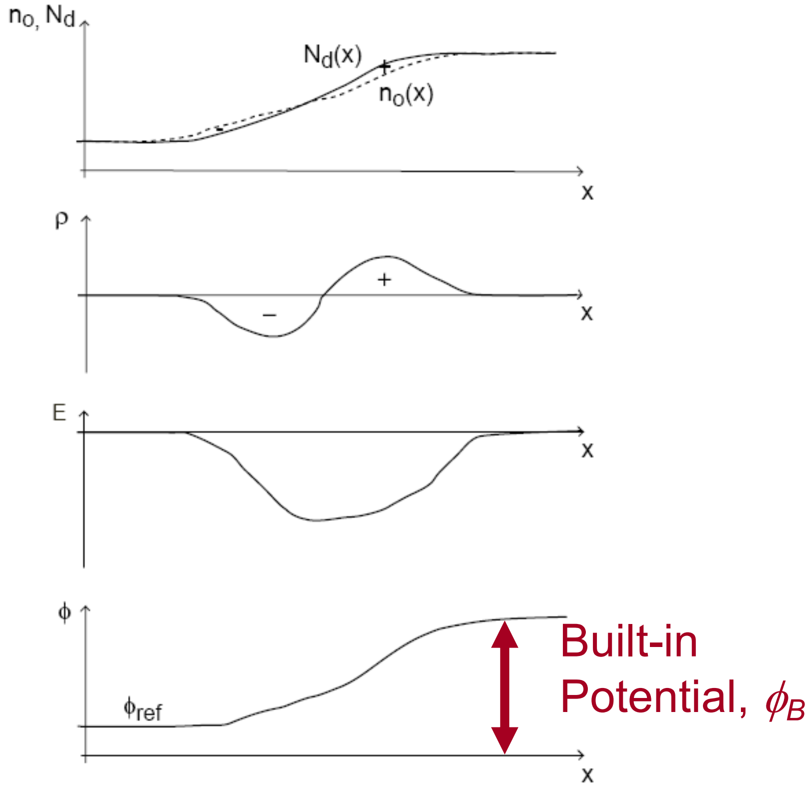
Non-uniformly doped semiconductor in thermal equilibrium

What about electrostatic potential?

$$\frac{d\phi}{dx} = -E$$

Integrate from 0 to x:

$$\phi(x) - \phi(0) = -\int_0^x E(x)dx$$



Equations for solving the non-uniformly doped semiconductor in thermal equilibrium

Question: Given $N_d(x)$, want to know $n_o(x)$, $\rho(x)$, $E(x)$, and $\phi(x)$.

In terms of electric field:

$$J_e = q\mu_n n_o E + qD_n \frac{dn_o}{dx} = 0$$

$$\frac{dE}{dx} = \frac{q}{\epsilon_s}(N_d - n_o)$$

In terms of potential:

$$-q\mu_n n_o \frac{d\phi}{dx} + qD_n \frac{dn_o}{dx} = 0$$

$$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s}(n_o - N_d)$$

One eq. with one unknown.
Given $N(x)$ solve for n_0.

$$\frac{d^2(\ln n_o)}{dx^2} = \frac{q^2}{\epsilon_s kT}(n_o - N_d)$$

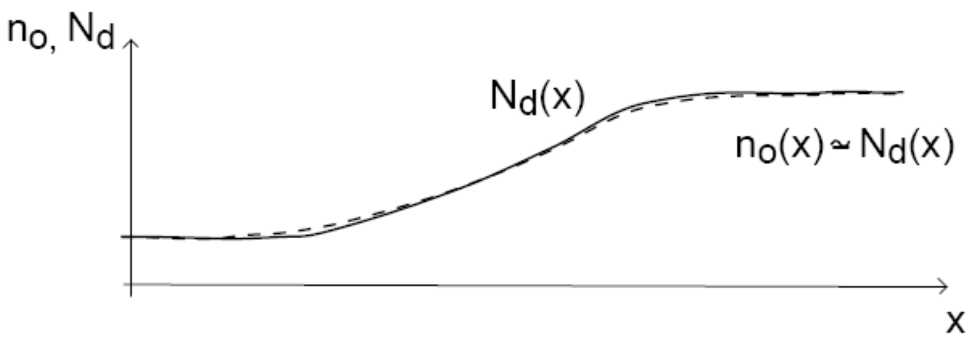
Quasi-neutral condition

From:

$$\frac{d^2(\ln n_o)}{dx^2} = \frac{q^2}{\epsilon_s kT}(n_o - N_d)$$

If $N_d(x)$ chages slowly with x: $n_o(x) \simeq N_d(x)$ (approximation)

• minimum space charge → semiconductor is quasi-neutral



Quasi—neutrality approximation, valid if:

$$\left|\frac{n_o - N_d}{n_o}\right| \ll 1 \quad \text{or} \quad \left|\frac{n_o - N_d}{N_d}\right| \ll 1$$

Boltzmann relation: relationship between n_0 and ϕ

From the drift/diffusion balance:

$$-q\mu_n n_o \frac{d\phi}{dx} + qD_n \frac{dn_o}{dx} = 0$$

We established:

$$\frac{\mu_n}{D_n}\frac{d\phi}{dx} = \frac{1}{n_o}\frac{dn_o}{dx}$$

→ using the Einstein relation:

$$\frac{q}{kT}\frac{d\phi}{dx} = \frac{d(\ln n_o)}{dx}$$

Integrating:

$$\frac{q}{kT}(\phi - \phi_{ref}) = \ln n_o - \ln n_o(ref) = \ln \frac{n_o}{n_o(ref)}$$

$$n_o = n_o(ref) e^{q(\phi - \phi_{ref})/kT}$$

Boundary conditions: $\phi_{ref} = 0 \text{ at } n_o(ref) = n_i.$

→ $$n_o = n_i e^{q\phi/kT}$$

→ $$p_o = n_i e^{-q\phi/kT}$$

Boltzmann relation between n_0 and ϕ

$$\phi = \frac{kT}{q} \ln \frac{n_o}{n_i}$$

$$\phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}$$

The 60mV rule for Si, @ room temperature:

$$\phi = (25\ mV) \ln \frac{n_o}{n_i}$$

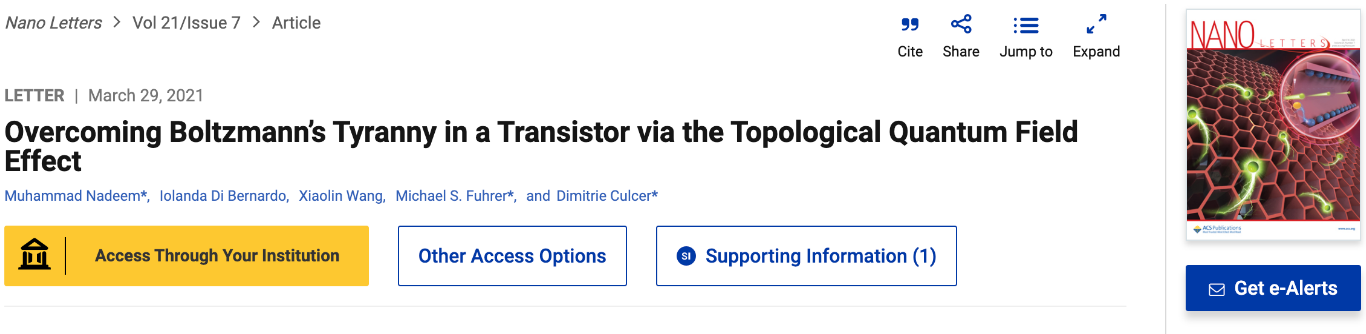
$$= (25\ mV) \ln(10) \log \frac{n_o}{n_i}$$

$$\phi \simeq (60\ mV) \log \frac{n_o}{10^{10}}$$

For every decade of increase in n_0, ϕ increases by 60 mV at 300K.

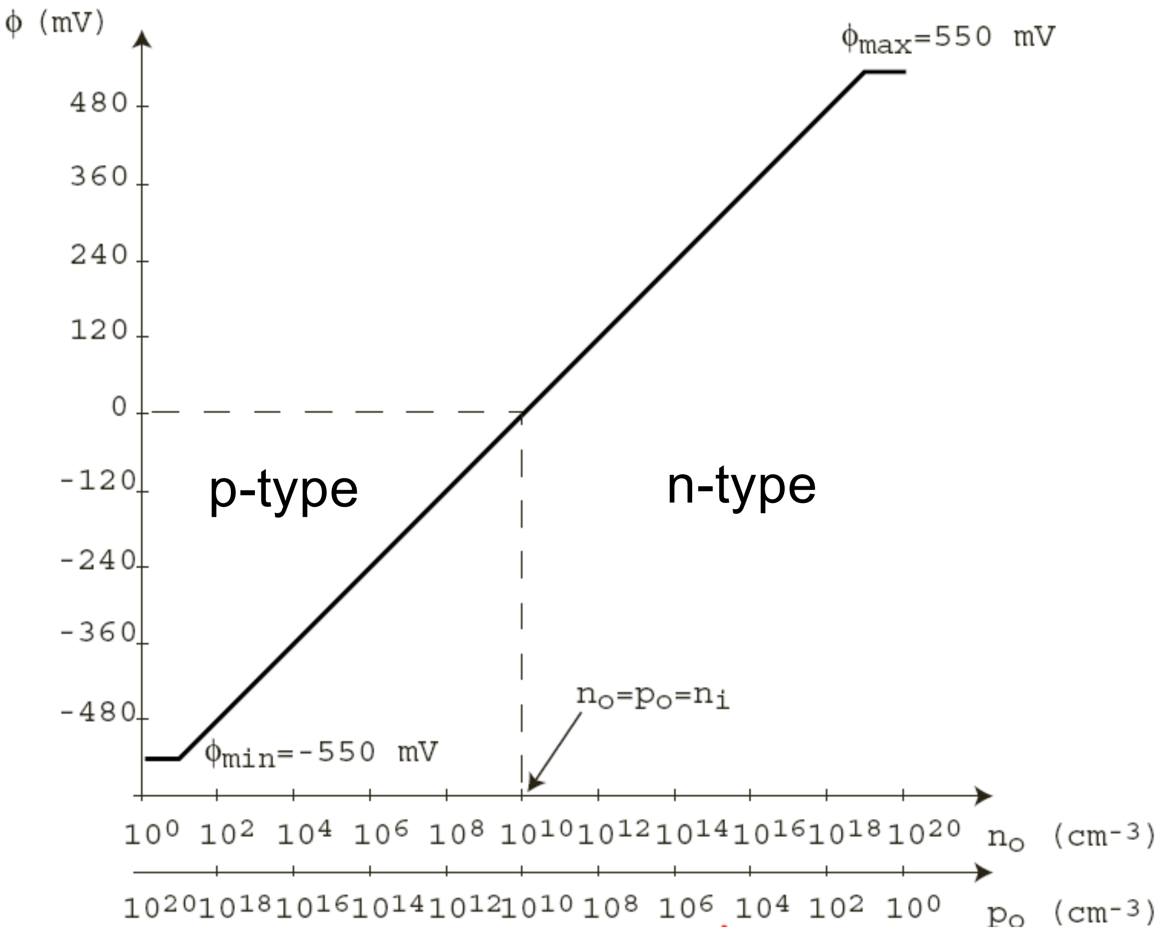
"Boltzmann Tyranny" a fundamental limitation of silicon technology

60 mV rule (at 300K)



For every decade of increase in n_0, ϕ increases by 60 mV at 300K.

ϕ limits in Silicon



Potential difference between n-type and p-type Si

- **EXAMPLE:** Compute potential difference in thermal equilibrium between region where $n_o = 10^{17}\ cm^{-3}$ and region where $p_o = 10^{15}\ cm^{-3}$:

$$\phi(n_o = 10^{17}\ cm^{-3}) = 60 \times 7 = 420\ mV$$

$$\phi(p_o = 10^{15}\ cm^{-3}) = -60 \times 5 = -300\ mV$$

$$\phi(n_o = 10^{17}\ cm^{-3}) - \phi(p_o = 10^{15}\ cm^{-3}) = 720\ mV$$

Potential difference between n-type and p-type Si

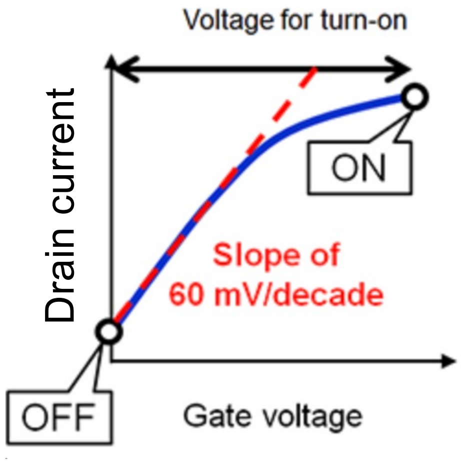
- **EXAMPLE:** Compute potential difference in thermal equilibrium between region where $n_o = 10^{20}\ cm^{-3}$ and region where $p_o = 10^{16}\ cm^{-3}$:

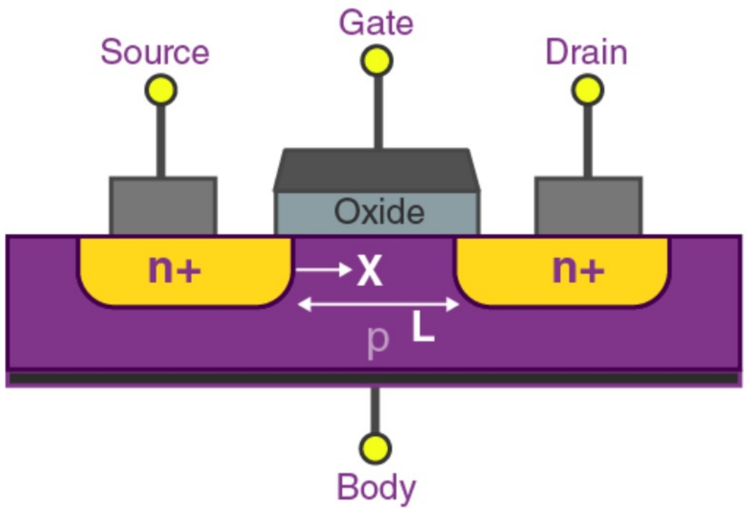
$$\phi(n_o = 10^{20}\ cm^{-3}) = \phi_{max} = 550\ mV$$

$$\phi(p_o = 10^{16}\ cm^{-3}) = -60 \times 6 = -360\ mV$$

$$\phi(n_o = 10^{20}\ cm^{-3}) - \phi(p_o = 10^{16}\ cm^{-3}) = 910\ mV$$

The 60 mV/dec limitation is real, it limits the transfer characteristics of MOSFETs





Transfer characteristics of MOSFET (Drain current vs gate voltage dependence) is mainly determined by carrier concentration in the channel. The carrier concentration is controlled by the gate voltage, the rule of 60mV/decade (Si at RT) applies.

Key conclusions

• It is possible to have an electric field inside a semiconductor in thermal equilibrium (non-uniform doping distribution).

• In a slowly varying doping profile, majority carrier concentration tracks well doping concentration.

• In thermal equilibrium, there are fundamental relationships between ϕ(x) and the equilibrium carrier concentrations called Boltzmann relations (or ”60 mV Rule”).

$$\phi = \frac{kT}{q} \ln \frac{n_o}{n_i} \qquad \phi = -\frac{kT}{q} \ln \frac{p_o}{n_i} \qquad \phi \simeq (60\ mV) \log \frac{n_o}{10^{10}}$$

25 meV

Microelectronic Devices: pn junction

- 1. Introduction to pn junction
- 2. Electrostatics of pn junction in thermal equilibrium
- 3. The depletion approximation
- 4. Contact potentials

Key questions

- What happens if the doping distribution in a semiconductor abruptly changes from n-type to p-type?
- Is there a simple description of the electrostatics of a pn junction?

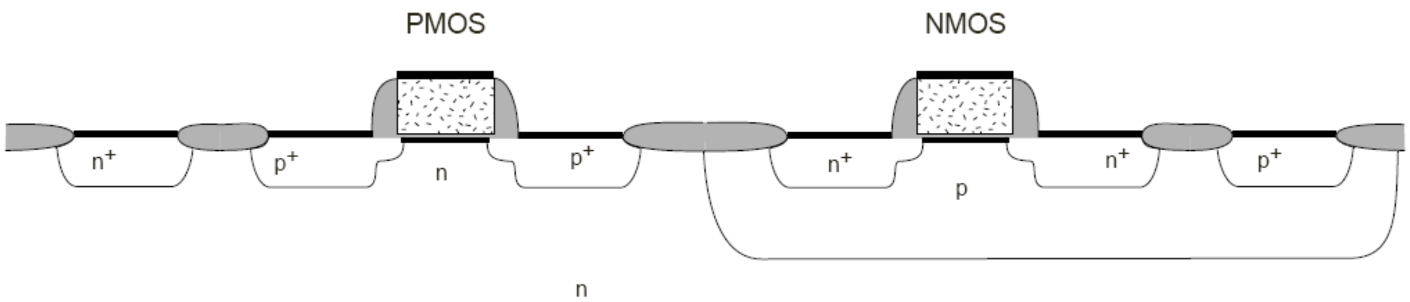
Introduction to pn junction

• pn junction: p-region and n-region in intimate contact

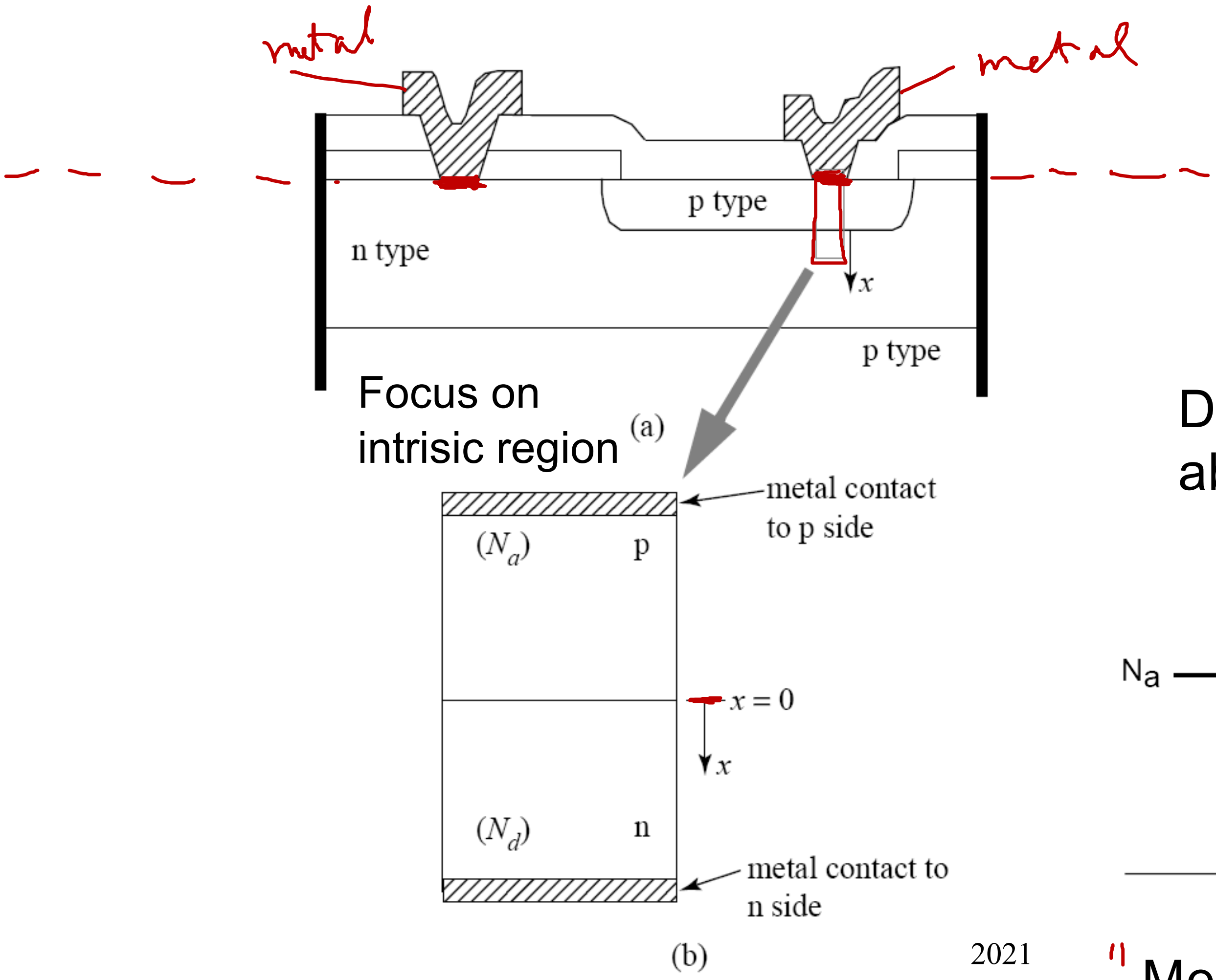
• Why is the p-n junction worth studying?

It is present in virtually every semiconductor device! Including the MOSFET switch, which is the core device for binary logic circuits (microprocessors).

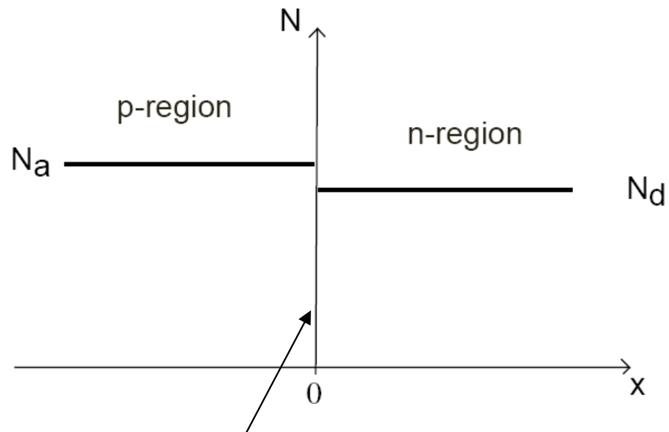
Understanding pn junction is essential to understanding transistor operation.



Electrostatics of p-n junction in equilibrium (1)



Doping distribution in abrupt pn junction

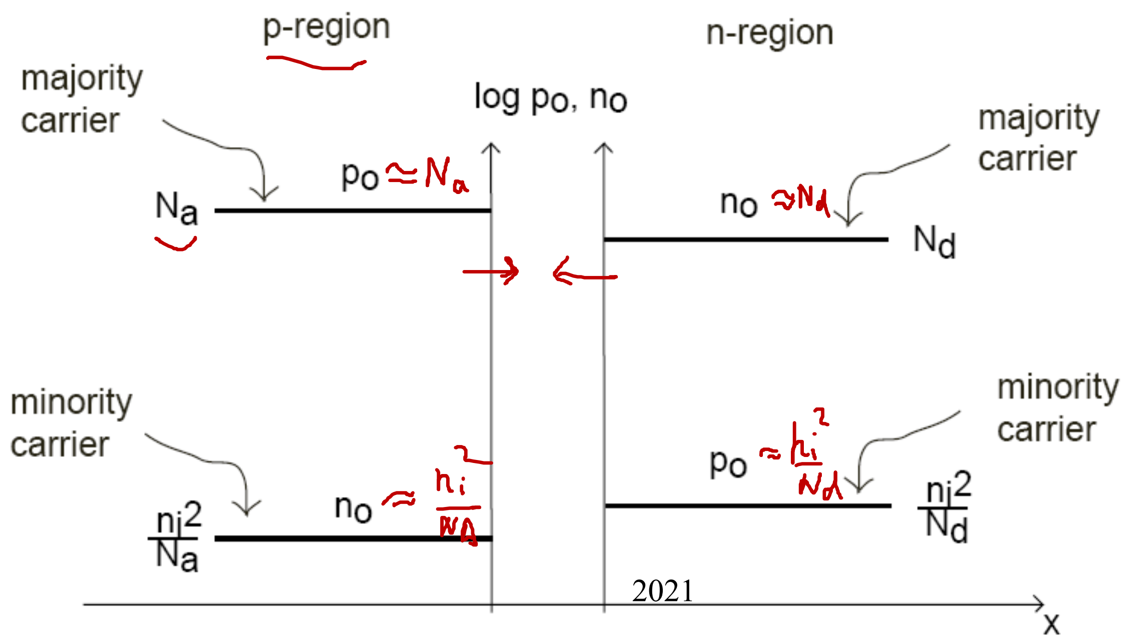


"Metallurgical" junction

Electrostatics of p-n junction in equilibrium (2)

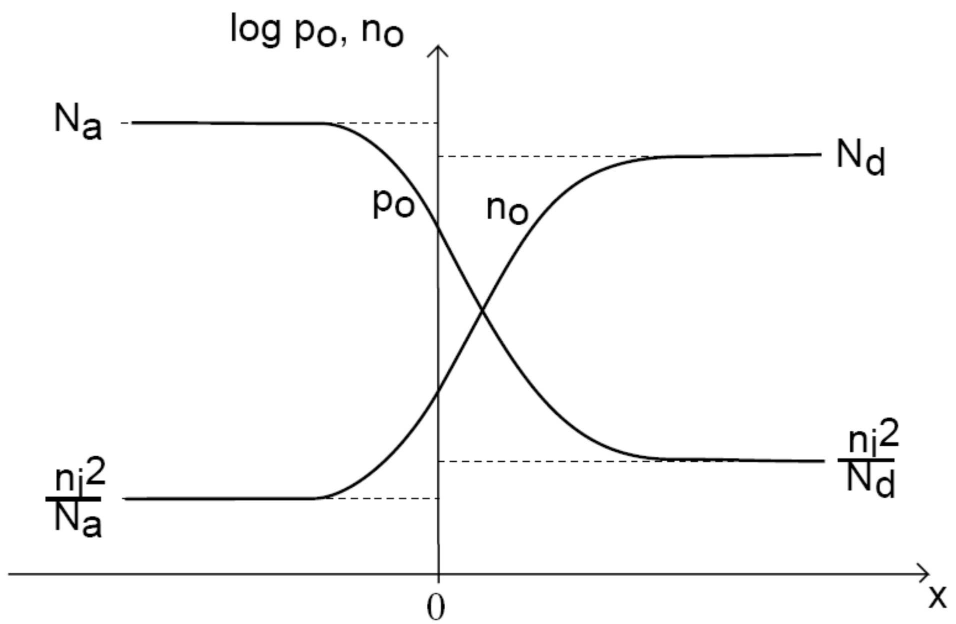
- What is the carrier concentration distribution in thermal equilibrium?

Bring together the two sides. *Effect*: diffusion of electrons and holes from majority carrier side to minority carrier side until drift balances diffusion.



Electrostatics of p-n junction in equilibrium (3)

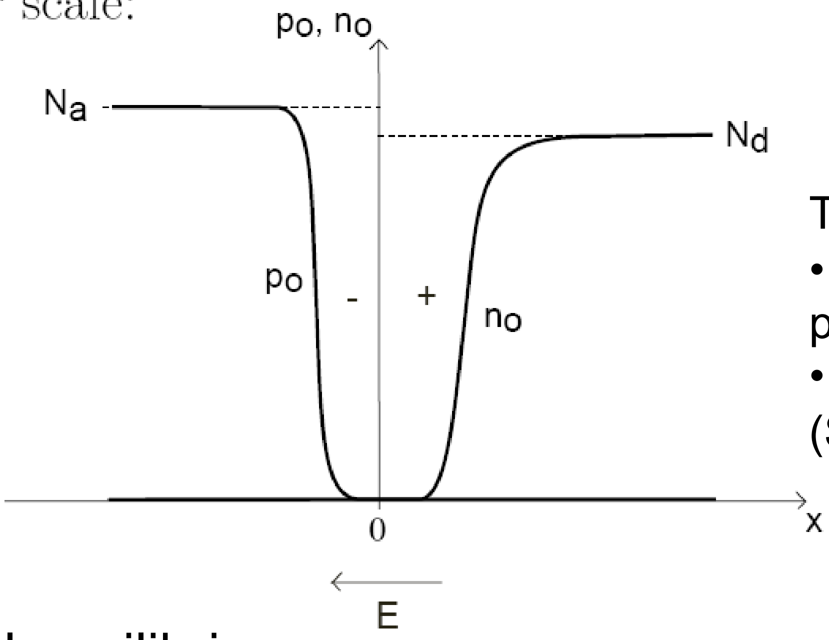
Resulting carrier profile in thermal equilibrium:



- Far away from metallurgical junction: nothing happens
  - – two quasi-neutral regions
- Around metallurgical junction: carrier drift must cancel diffusion
  - – space-charge region formation

Electrostatics of p-n junction in equilibrium (4)

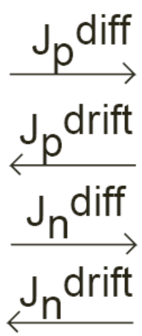
In a linear scale:



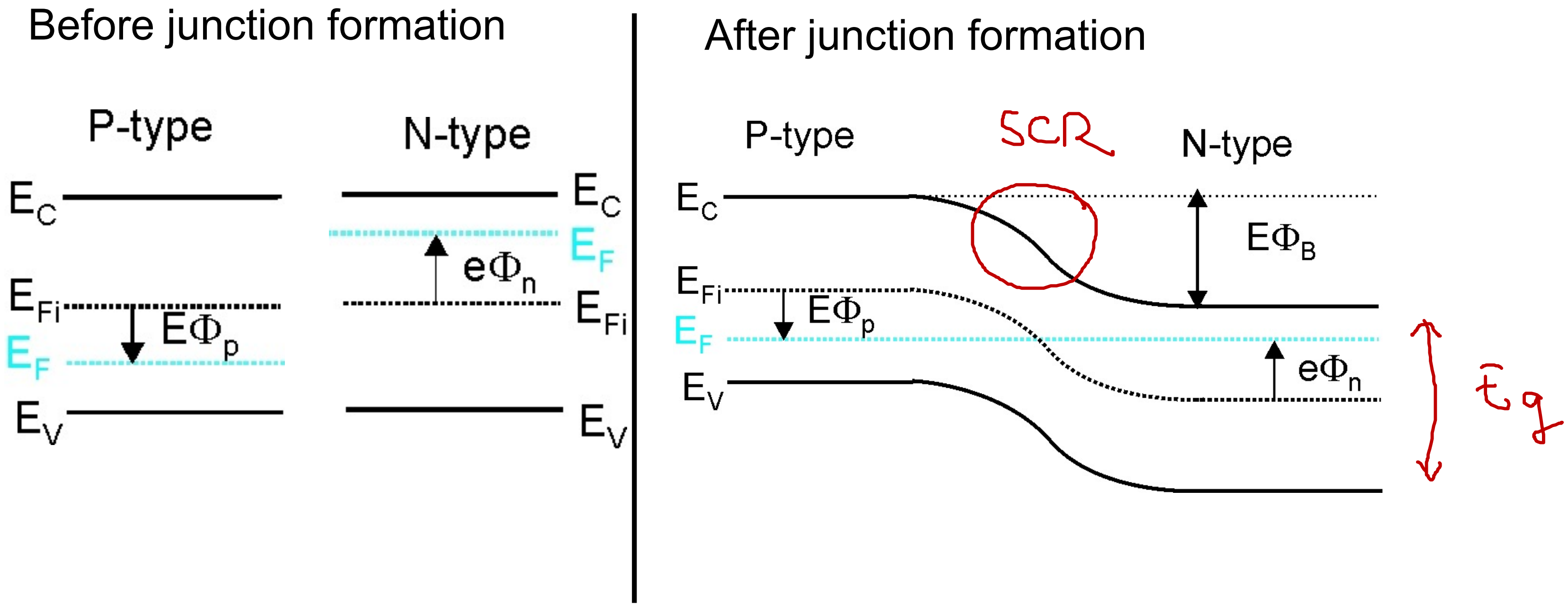
Three major regions:

- two quasi-neutral n- and p-regions (QNR's)
- one space charge region (SCR)

Thermal equilibrium: balance between drift and diffusion



Energy-band diagram



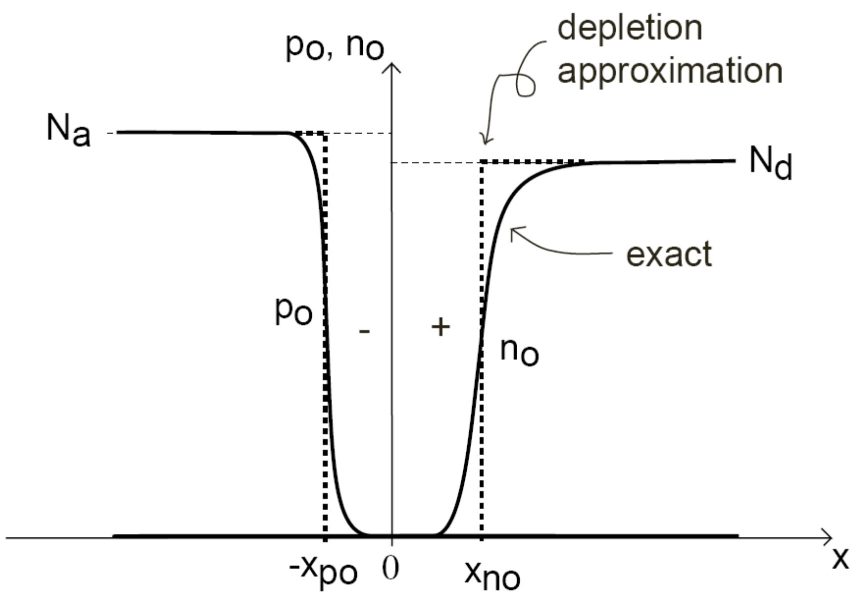
- Thermal equilibrium
  - The Fermi level is constant throughout the junction.
  - band-bending in the space charge region.

The depletion approximation

Goal: $n_o(x)$, $p_o(x)$, $\rho(x)$, $E(x)$, and $\phi(x)$.

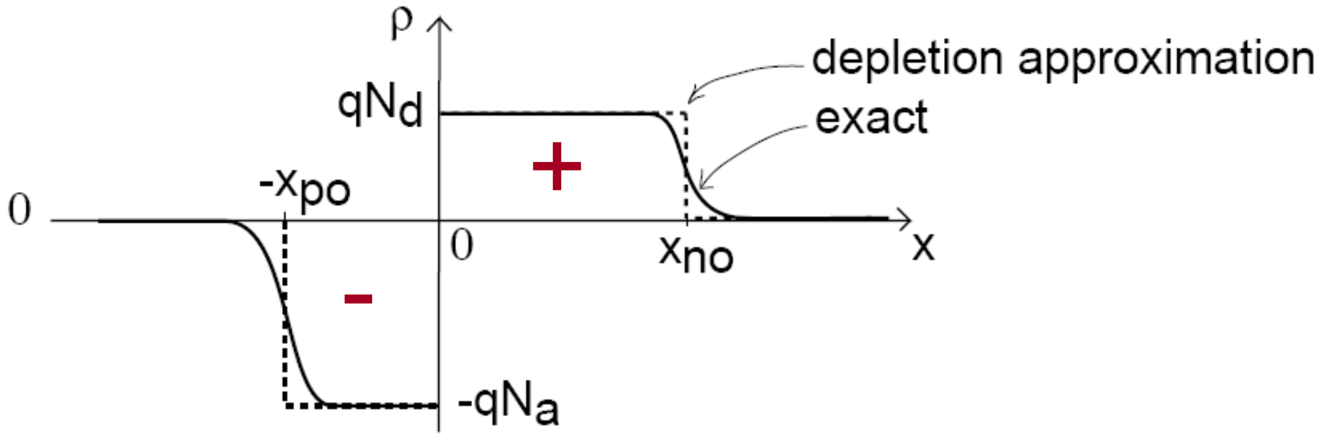
- Assume QNR's perfectly charge neutral
- assume SCR depleted of carriers (depletion region)
- transition between SCR and QNR's sharp

(must calculate where to place $-x_{p0}$ and x_{n0})



- $x < -x_{po}$ $\quad p_o(x) = N_a,\ n_o(x) = \frac{n_i^2}{N_a}$
- $-x_{po} < x < 0$ $\quad p_o(x),\ n_o(x) \ll N_a$
- $0 < x < x_{no}$ $\quad n_o(x),\ p_o(x) \ll N_d$
- $x_{no} < x$ $\quad n_o(x) = N_d,\ p_o(x) = \frac{n_i^2}{N_d}$

Space charge density

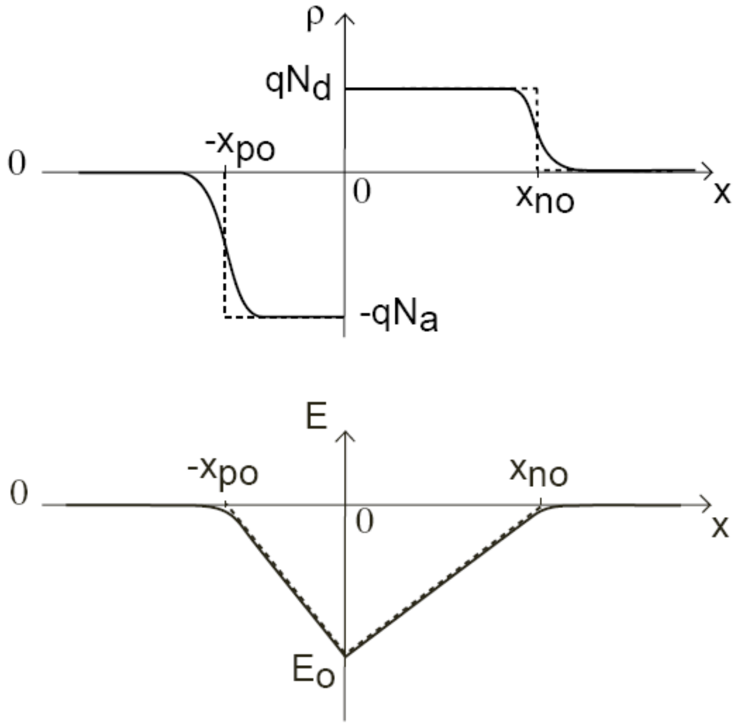


$$
\begin{aligned}
\rho(x) &= 0 & x < -x_{po} \\
&= -qN_a & -x_{po} < x < 0 \\
&= qN_d & 0 < x < x_{no} \\
&= 0 & x_{no} < x
\end{aligned}
$$

Electric field

Integrate Gauss' equation:

$$E(x_2) - E(x_1) = \frac{1}{\epsilon_s} \int_{x_1}^{x_2} \rho(x)dx$$



- $x < -x_{po}$ $\quad E(x) = 0$
- $-x_{po} < x < 0$ $\quad E(x) - E(-x_{po}) = \frac{1}{\epsilon_s} \int_{-x_{po}}^{x} -qN_a dx$
  $= \frac{-qN_a}{\epsilon_s} x|_{-x_{po}}^{x} = \frac{-qN_a}{\epsilon_s}(x + x_{po})$
- $0 < x < x_{no}$ $\quad E(x) = \frac{qN_d}{\epsilon_s}(x - x_{no})$
- $x_{no} < x$ $\quad E(x) = 0$

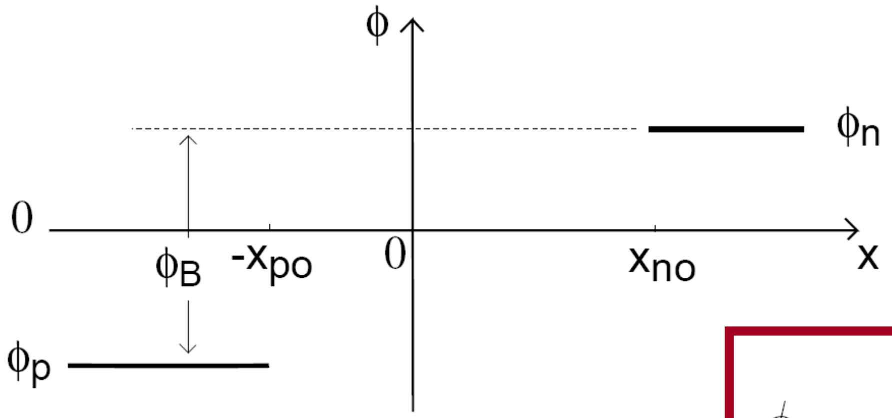
Electrostatic potential (1)

With $\phi = 0$ @ $n_0 = p_0 = n_i$): $\phi = \frac{kT}{q} \ln \frac{n_o}{n_i}$ $\phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}$

In QNR's, n_o, p_o known $\Rightarrow$ can determine ϕ:

in p-QNR: $p_o = N_a \Rightarrow \phi_p = -\frac{kT}{q} \ln \frac{N_a}{n_i}$

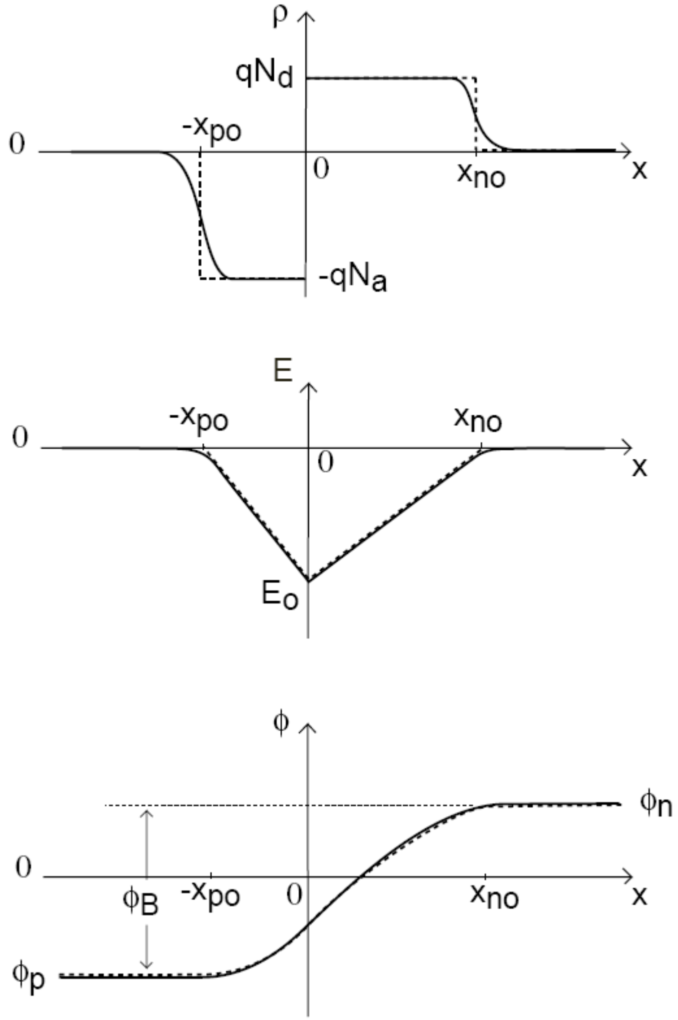
in n-QNR: $n_o = N_d \Rightarrow \phi_n = \frac{kT}{q} \ln \frac{N_d}{n_i}$



Built-in potential:

$$\phi_B = \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

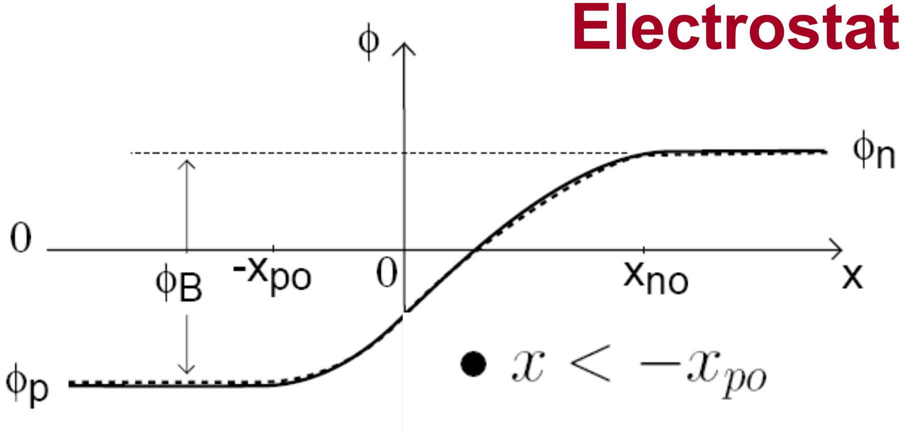
Electrostatic potential (2)



Integrate *E(x)*:

$$\phi(x_2) - \phi(x_1) = -\int_{x_1}^{x_2} E(x)dx$$

Electrostatic potential (3)



- $x < -x_{po}$ $\qquad \phi(x) = \phi_p$

- $-x_{po} < x < 0$ $\qquad$
$$\phi(x) - \phi(-x_{po}) = -\int_{-x_{po}}^{x} -\frac{qN_a}{\epsilon_s}(x + x_{po})dx = \frac{qN_a}{2\epsilon_s}(x + x_{po})^2$$
$$\phi(x) = \phi_p + \frac{qN_a}{2\epsilon_s}(x + x_{po})^2$$

- $0 < x < x_{no}$ $\qquad \phi(x) = \phi_n - \frac{qN_d}{2\epsilon_s}(x - x_{no})^2$

- $x_{no} < x$ $\qquad \phi(x) = \phi_n$

Electrostatic potential (4)

Problem: don't know x_{n0} and x_{p0} → need two more equations

1. Require overall charge neutrality:

$$qN_a x_{po} = qN_d x_{no}$$

2. Require $\phi(x)$ continuous at $x = 0$:

$$\phi_p + \frac{qN_a}{2\epsilon_s} x_{po}^2 = \phi_n - \frac{qN_d}{2\epsilon_s} x_{no}^2$$

- 2 equations
- 2 unknowns

Solution:

$$x_{no} = \sqrt{\frac{2\epsilon_s \phi_B N_a}{q(N_a + N_d) N_d}} \qquad x_{po} = \sqrt{\frac{2\epsilon_s \phi_B N_d}{q(N_a + N_d) N_a}}$$

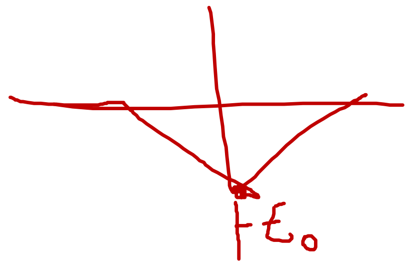
Complementary results

- Total width of space charge region:

$$x_{do} = x_{no} + x_{po} = \sqrt{\frac{2\epsilon_s \phi_B (N_a + N_d)}{q N_a N_d}}$$

- Field at metallurgical junction:

$$|E_o| = \sqrt{\frac{2q\phi_B N_a N_d}{\epsilon_s (N_a + N_d)}}$$



Three junction cases (1)

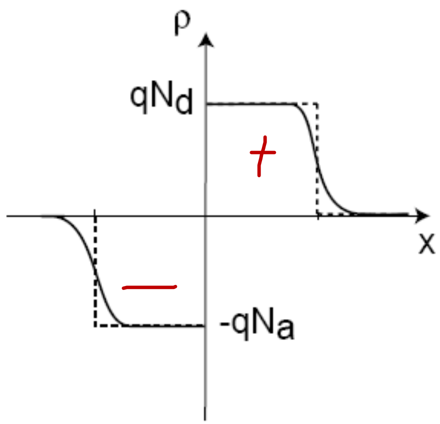
- Symmetric junction: $N_a = N_d \;\Rightarrow\; x_{po} = x_{no}$
- Asymmetric junction: $N_a > N_d \;\Rightarrow\; x_{po} < x_{no}$
- Strongly asymmetric junction:
  i.e. $\mathrm{p^+n}$ junction: $N_a \gg N_d$

$$x_{po} \ll x_{no} \simeq x_{do} \simeq \sqrt{\frac{2\epsilon_s \phi_B}{qN_d}} \propto \frac{1}{\sqrt{N_d}}$$

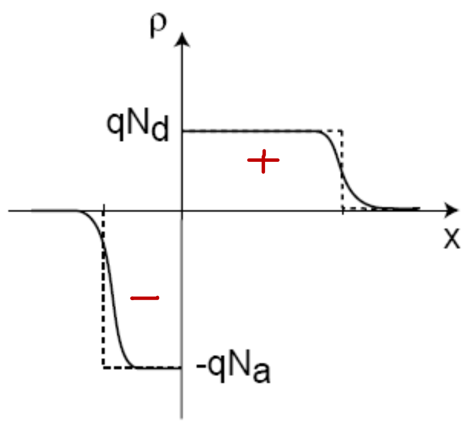
$$|E_o| \simeq \sqrt{\frac{2q\phi_B N_d}{\epsilon_s}} \propto \sqrt{N_d}$$

Three junction cases (2)

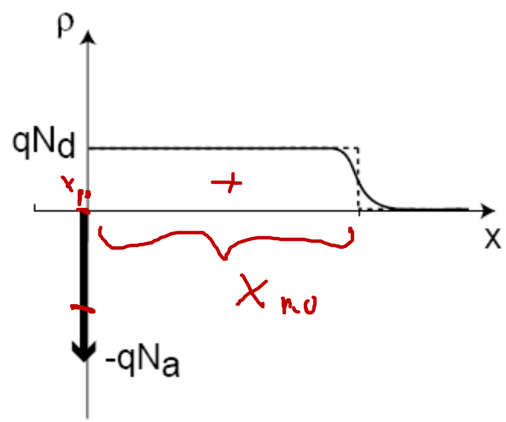
The lowly-doped side controls the electrostatics of the pn junction.



Symmetric junction



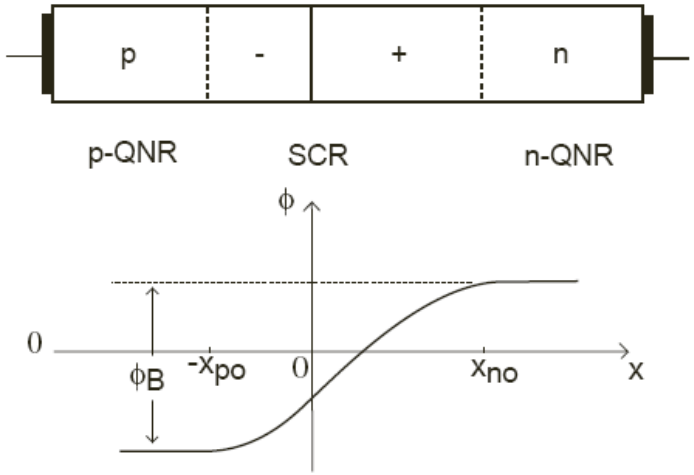
Asymmetric junction



Strongly asymmetric junction

Contact potentials

Potential distribution in thermal equilibrium so far:



Question 1: If I apply a voltmeter across diode, do I measure ϕ_B?

☐ yes ☐ no ☐ it depends

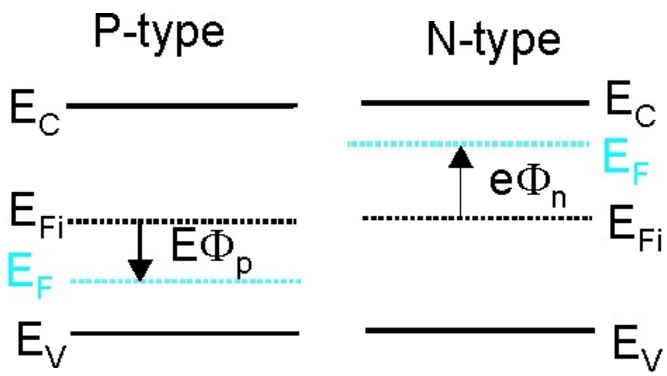
Question 2: If I short diode terminals, does current flow on outside circuit?

☐ yes ☐ no ☐ sometimes

Energy-band diagram and potential profile of pn junction

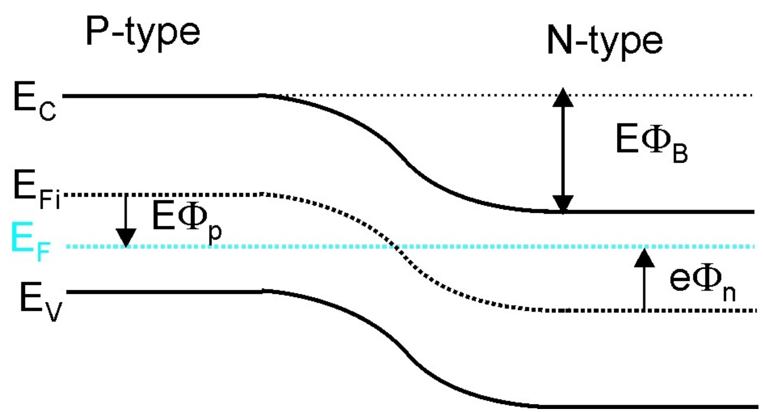
Band diagram

Before junction formation

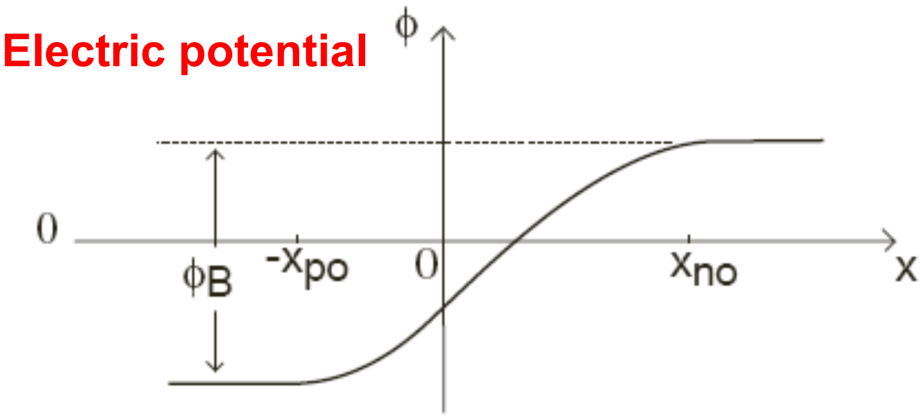


Band diagram

After junction formation



Electric potential



- Note that the diagrams of energy bands and electrostatic potential show opposite behavior (increase of one = decrease of the other)
- the reason is that the band diagram is developed for electrons (negative charges)

Key conclusions

- Electrostatics of pn junction in equilibrium:
  - a space-charge region
  - surrounded by two quasi-neutral regions

  → **built-in potential across p-n junction**

- The carrier concentrations in space-charge region are considered much smaller than doping level

  → **depletion approximation**

- Contact potential at metal-semiconductor junctions:

→ from contact to contact, there is no potential buildup across pn junction