Lecture 2
Microelectronic Devices

Electrons and holes in semiconductors

Carrier transport : drift and diffusion

Statistics of electrons and holes

Important takeaway from lecture 1

For electrons and holes in semiconductors in equilibrium $\boldsymbol{n_0 p_0 = n_i^2 (T)}$

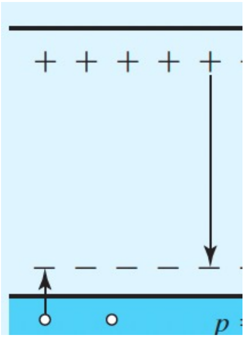
(valid for intrinsic and extrinsic semiconductors)

Compensational doping: a p-type semiconductor is converted to n-type by adding $N_d > N_a$

More donors $N_d - N_a >> n_i$, $n = N_d - N_a$, $p = n_i^2 / (N_d - N_a)$

More acceptors $N_a - N_d >> n_i$, $p = N_a - N_d$, $n = n_i^2 / (N_a - N_d)$

Compensation doping: equilibrium concentration of electrons and holes is maintained: excessive carriers recombine



Important takeaway from lecture 1

- Carrier transport : drift under external electric field (classic Drude theory), controlled by mobility

$$J^{drift} = J_n^{drift} + J_p^{drift} = q(n\mu_n + p\mu_p)E$$

Two key parameters for conduction (or resistivity) of a semiconductor: carrier concentration (cm^{-3}) and mobility (cm^2/(Vs)

$$J_n^{drift} = -qnv_{dn} = qn\mu_n E$$

$$\mu_{n,p} = \frac{q\tau_c}{2m_{n,p}} \equiv mobility\,[cm^2/V \cdot s]$$

$$v_{dn} = -\mu_n E$$
$$v_{dp} = \mu_p E$$

Electrical conduction: Drude model and Ohm law

$$J^{drift} = J_n^{drift} + J_p^{drift} = q(n\mu_n + p\mu_p)E$$

Ohm's law:

$$J = \sigma E = \frac{E}{\rho}$$

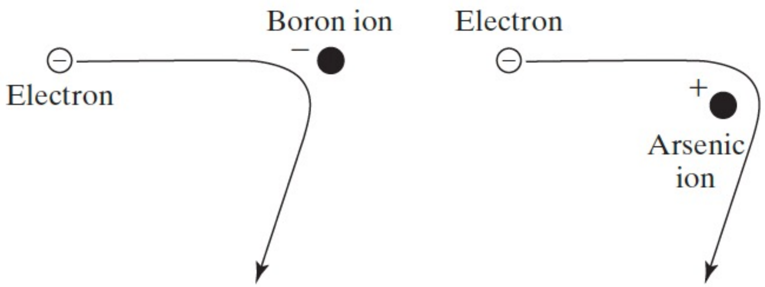
$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

Mobility is measure of *ease* of carrier drift

- Increases with longer time between collisions
- Decreases with the higher effective mass

$\mu_{phonon} \propto \tau_{ph} \propto T^{-3/2}$ decreases with temperature

$\mu_{impurity} \propto T^{3/2}$ Increases with temperature



For Si at 300K

$\tau_c \simeq 10^{-14} \sim 10^{-13}\ s \ll 1ps$

$v_{th} \simeq 10^7\ cm/s$

$\Rightarrow \lambda \simeq 1 \sim 10\ nm$

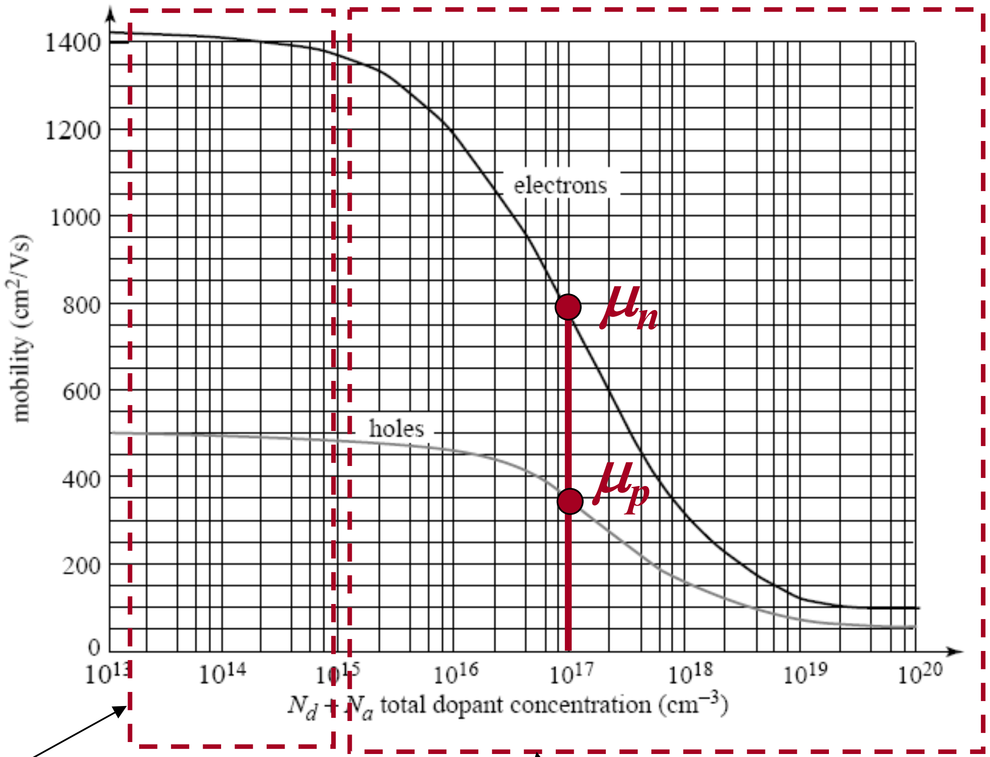
- apply $|E| = 1\ kV/cm$

$|v_{dn}| \simeq 10^6\ cm/s \ll v_{th}$

Time to drift through L = 100nm: 10ps !

Carrier mobility in Si vs doping concentration

- Carrier mobility depends on doping – see below dependence for Si @ 300K



Remark: electrons lighter than holes (better mobility at same doping):

$$\mu_n > \mu_p$$

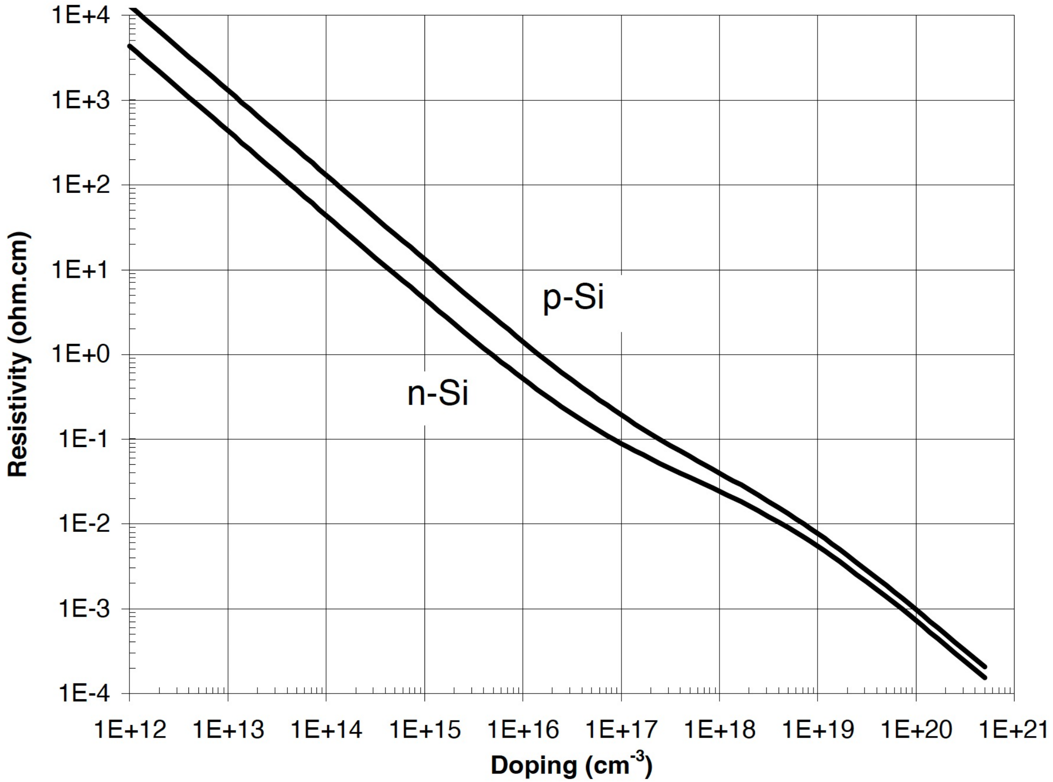
Mobility limited by scattering with Si lattice (phonons)

Mobility limited by scattering with ionized impurities

Resistivity and doping level, some numbers

For n-type and p-type semiconductors,

$$\rho_n \simeq \frac{1}{qN_d\mu_n} \qquad \rho_p \simeq \frac{1}{qN_a\mu_p}$$



Resistivity is commonly used to characterise the doping level

- Si with $N_d = 3 \times 10^{16}\ cm^{-3}$ at 300 K

$$\mu_n \simeq 1000\ cm^2/V \cdot s$$

$$\rho_n \simeq 0.21\ \Omega \cdot cm$$

- apply $|E| = 1\ kV/cm$

$$|v_{dn}| \simeq 10^6\ cm/s \ll v_{th}$$

$$|J_n^{drift}| \simeq 4.8 \times 10^3\ A/cm^2$$

- time to drift through $L = 0.1\ \mu m$:

$$t_d = \frac{L}{v_{dn}} = 10\ ps$$

Carrier transport in semiconductors: Drift and Diffusion

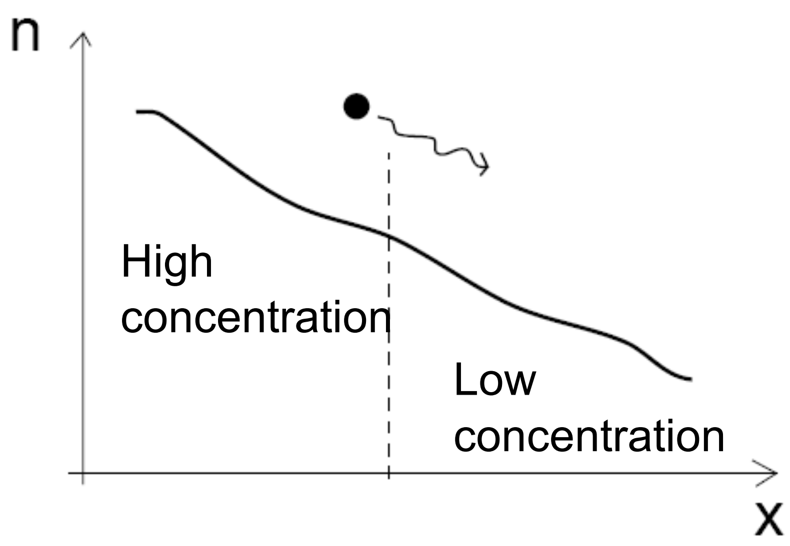
- What are two principle driving forces responsible for current flow in semiconductors
- How do electrons and holes in a semiconductor behave in an electric field? Carrier drift
- How do electrons and holes in a semiconductor behave if their concentration is non-uniform in space ? Carrier diffusion
- How do drift and diffusion counterbalance each other?

Carrier diffusion (1)

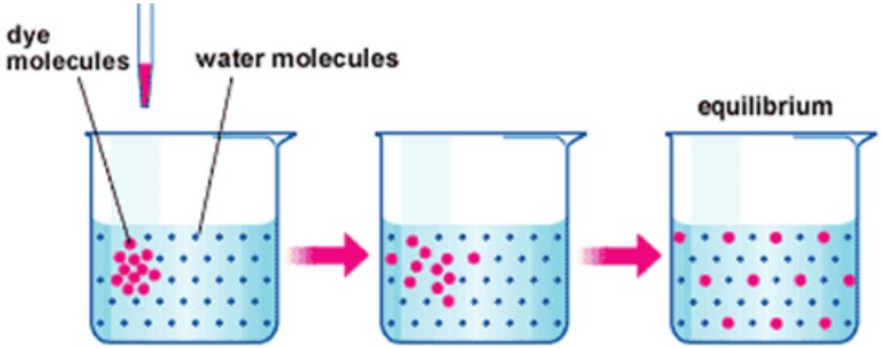
Diffusion:
Particle movement in response to concentration gradient.

Origin of diffusion current:

- a medium (Si crystal)
- a gradient of concentration of particles (electrons and holes) inside the medium
- collisions between particles and medium send particles off in random directions: overall, there is a particle movement down the gradient



diffusion of pigment in water: same principle



Carrier diffusion (2)

- Key diffusion relationship - Fick's law:

Diffusion flux ~ - concentration gradient

Definition:
Flux = number of particles crossing unit area per unit time $[cm^{-2} \cdot s^{-1}]$

electrons

$$F_n = -D_n \frac{dn}{dx}$$

holes

$$F_p = -D_p \frac{dp}{dx}$$

$D_{n,\,p}$ is electron/hole diffusion coefficient $[cm^2/s]$

- D measures the ease of carrier diffusion in response to a concentration gradient
- D limited by phonons and ionized dopants

Diffusion current

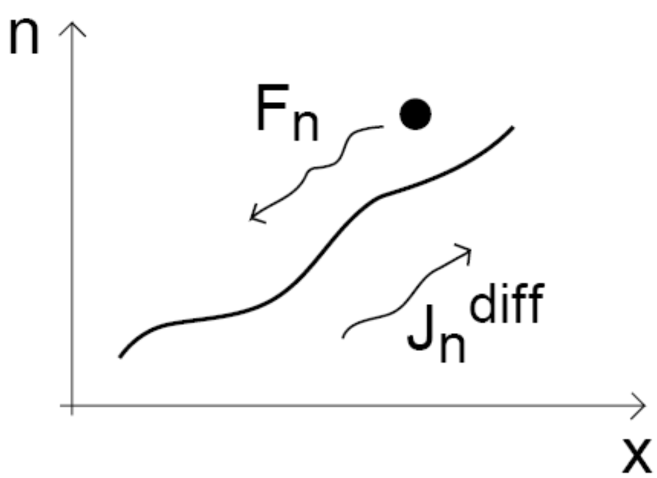
Diffusion current density = charge × carrier flux

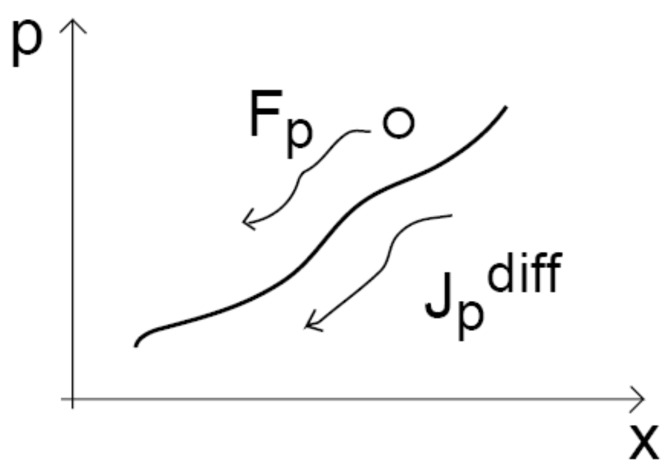
electrons

$$J_n^{diff} = qD_n \frac{dn}{dx}$$

holes

$$J_p^{diff} = -qD_p \frac{dp}{dx}$$





Einstein relation between *D* and *μ*

• Physically drift and diffusion are both determined by collisions among particles and medium atoms: there should be a relationship (Einstein) between D and μ

$$\frac{D}{\mu} = \frac{kT}{q}$$

$$D_n = \frac{kT}{q}\mu_n \qquad D_p = \frac{kT}{q}\mu_p$$

In a semiconductor: $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$

@ T=300K

$$\frac{kT}{q} \simeq 25\ mV$$

For example: for $N_d = 3 \times 10^{16}\ cm^{-3}$:

$$\mu_n \simeq 1000\ cm^2/V \cdot s \rightarrow D_n \simeq 25\ cm^2/s$$
$$\mu_p \simeq 400\ cm^2/V \cdot s \rightarrow D_p \simeq 10\ cm^2/s$$

Total current in a semiconductor: drift + diffusion

$$J_{total} = J_n + J_p$$

Electrons: $J_n = J_n^{drift} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$

Drift term

Diffusion term

Holes: $J_p = J_p^{drift} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$

Summary: drift and diffusion

• Electrons and holes in semiconductors are 2 types of mobile charge carriers, which support electrical current!

• Drift current: produced by electric field

$$J^{drift} \propto E$$

• Diffusion current: produced by concentration gradient

$$J^{diff} \propto \frac{dn}{dx}, \frac{dp}{dx}$$

• Carriers move fast in response to fields and gradients, and diffusion and drift currents can be quite strong in modern devices

In the equilibrium situation, the electron drift and diffusion currents will perfectly cancel each other out for an arbitrary doping profile (same holds true for holes)

Practical aspects: how to determine important characteristics of a semiconductor:

-sign/concentration of mobile charge carriers,

-mobility

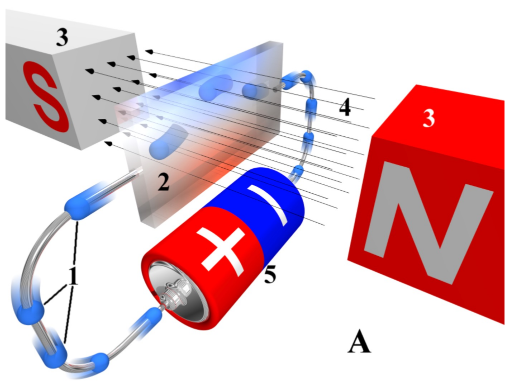
-Magnetotransport measurements (Hall effect) is a well-established method to find out if the current is dominated by holes of electrons

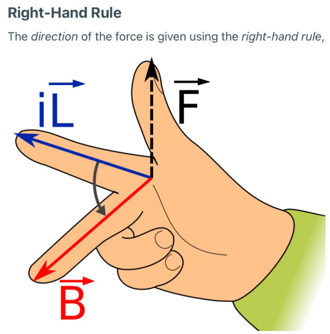
The Hall effect measurements also permit to measure the concentration

The resistance measurements performed on the same sample together with the Hall data permit calculation of mobility

$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

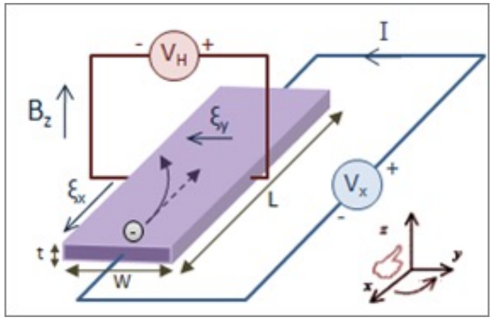
How to measure the sign, concentration, mobility of carriers?



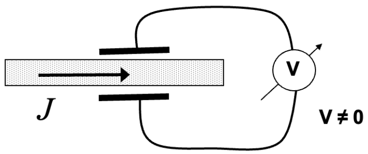


- magnetotransport of charge carriers:

Lorentz force results in transverse Hall voltage



$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$



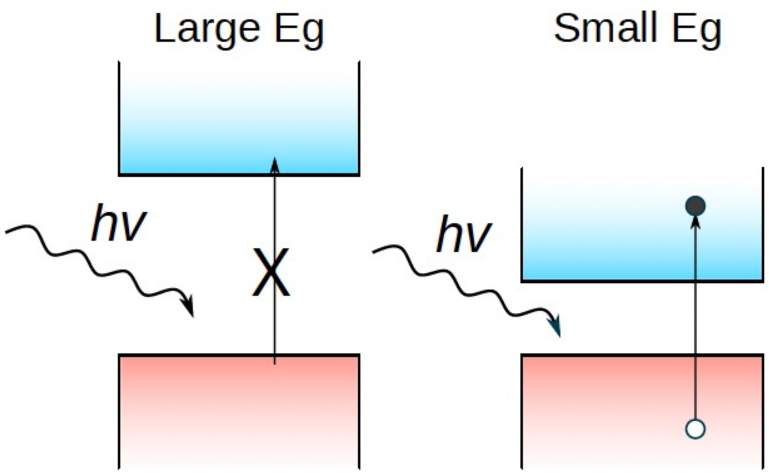
$V_H=(I_xB_z)/(qnt)$

- Sign of V_H – sign of carriers
- Magnitude of V_H – concentration of carriers

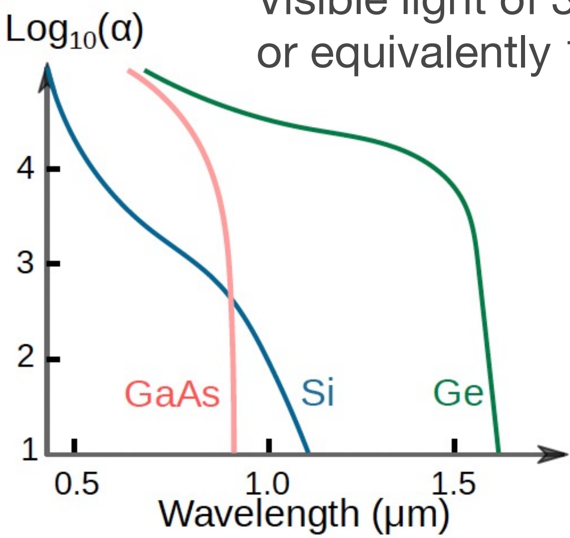
• Hall effect measurements – a standard technique to determine the sigh and concentration of charge carriers

Semiconductors: how to measure the band gap

How to prove that semiconductors have band gap? – Light absorption experiments



Visible light of 380 nm ~ 780 nm, or equivalently 1.60 eV ~ 3.26 eV



- Light absorption in a semiconductor: 0nly the light with wavelength $h\nu$ larger than the bandgap E_{g}, can be absorbed by the semiconductor. When a photon is absorbed by the semiconductor, a pair of electron-hole is generated.

- Thermal equilibrium is disturbed by the photogeneration of carriers

Statistics of charge carriers in semiconductors

- Fermi distribution
- Boltzmann approximation
- Carrier concentration in semiconductors vs temperature
- Density of states, effective mass and other parameters influencing the electronic properties

- Practical aspects – how to calculate the concentration of electrons, holes, ionized/non-ionized impurities

Boltzmann and Fermi distributions

What is the probability of finding a particle with the energy E?

Classic systems of particles is described by Boltzmann probability function:
This assumes that any number of particles may have the same energy E

$$f(E) = \frac{1}{Ae^{E/kT}}$$

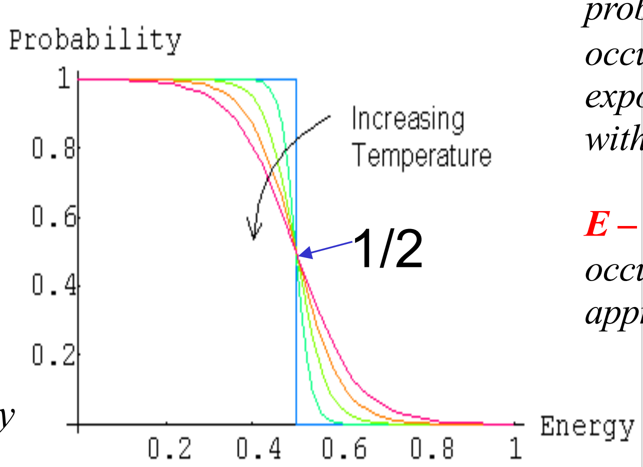
Statistics of electrons takes into account the Pauli Exclusion principle (one electron in a particular quantum state) Fermi distribution applies

At absolute zero temperature (*T* = 0 K), the energy levels are all filled up to a maximum energy, which we call the Fermi level. No states above the Fermi level are filled.

At higher T, the transition between completely filled states and completely empty states is gradual rather than abrupt.

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

- describes the probability of finding an electron at a certain energy



$E - E_F \gg kT$ - *the probability of a state being occupied decreases exponentially with increasing E*

$E - E_F \ll -kT$ - *the occupation probability approaches 1*

Electrons are Fermions: the Fermi function provides the probability that an energy level at energy, *E*, in thermal equilibrium is occupied by an electron. The system is characterized by its temperature, *T*, and its Fermi energy, E_F.

Fermi distribution and Boltzmann approximation

Fermi distribution (also known as Fermi-Dirac function):

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

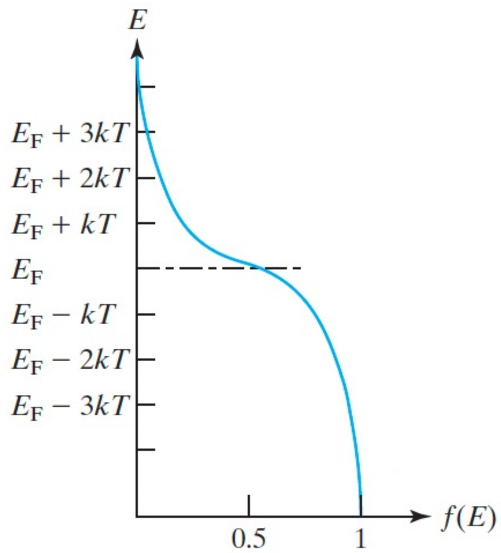
For $E - E_F >> kT)$ - the probability of a state being occupied decreases exponentially with increasing E

The resulting distribution is known as Boltzmann approximation (important for semiconductors)

$$f(E) \approx e^{-(E - E_F)/kT}$$

$$f(E) \approx e^{-(E - E_F)/kT}$$

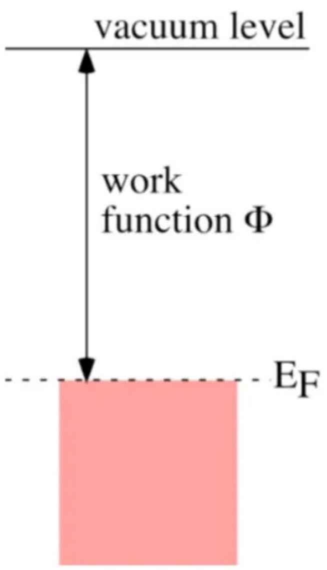
$$f(E) \approx 1 - e^{-(E_F - E)/kT}$$



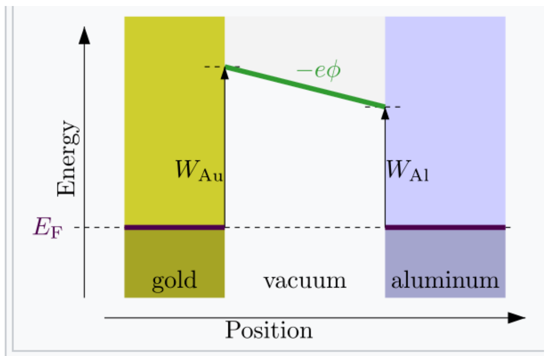
Significance of Fermi energy

What does the Fermi energy formally represent?

- The energy where the probability of occupancy is ½ (near zero K)
- the Fermi level in a metal at absolute zero is the energy of the highest occupied single particle state
- The relation to electrochemical potential of the system: The Fermi level is the electrochemical potential of electrons: the change in free energy when one electron is added to the system



The work function Φ refers to removal of an electron to a position that is far enough from the surface (many nm) that the force between the electron and its image charge in the surface can be neglected (vacuum level)

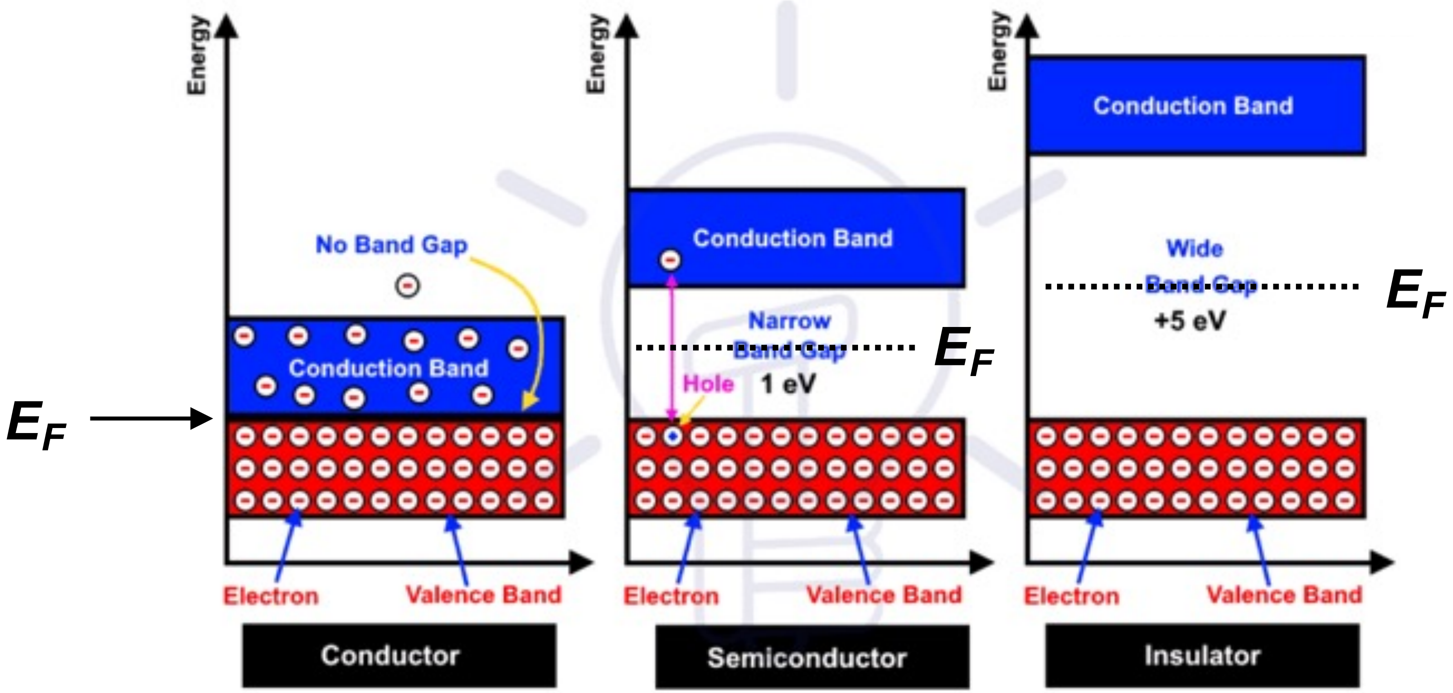


Here is a system with two different metals (Au and Al), with work functions of 5.5eV and 4.1eV
In equilibrium the position of Fermi level with be the same, but there will be a potential difference of 1.4V in the gap

Fermi level and carrier concentration in semiconductors

Using the Fermi-Dirac distribution one can find concentration of free carriers of charge in semiconductors (insulators)

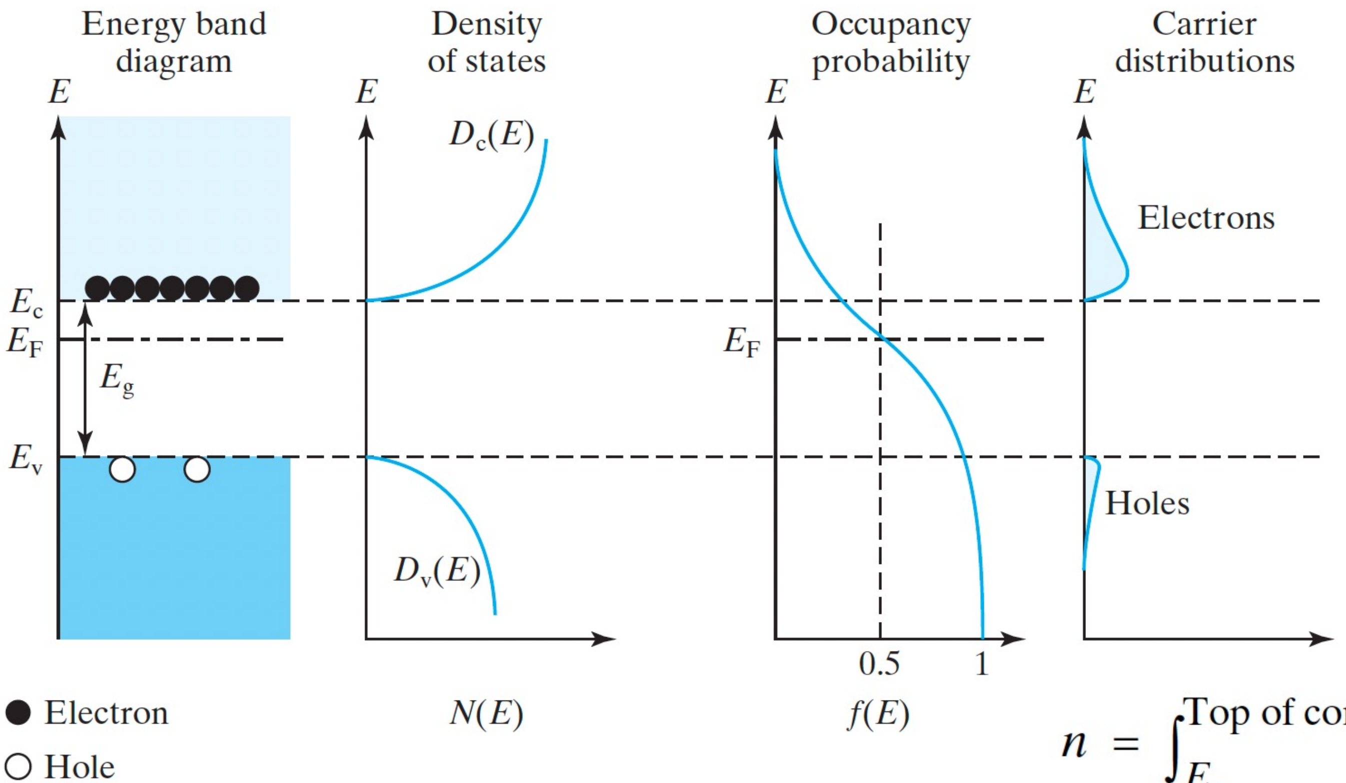
Where is the Fermi level in semiconductors ?



In semiconductors and insulators the Fermi level is situated within the band gap (forbidden band)

Carrier concentration in semiconductors

In addition to the probability distribution function one needs the density of states Density of states *Dc* (conduction band) and *Dv* (valence band) per cm^{-3} tells you how many energy states per energy interval (in a unit volume) are available



$$n = \int_{E_c}^{\text{Top of conduction band}} f(E) D_c(E)\, dE$$

Carrier concentration in intrinsic semiconductors

Typically, no need for calculation of this integral $n = \int_{E_c}^{\text{Top of conduction band}} f(E) D_c(E)\, dE$

Some simplifications are applicable:
- Boltzmann approximation (for electrons and holes)
- Effective density of states (for electrons and holes)

Fermi

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

$$p = N_v\, e^{-(E_F - E_v)/kT}$$

$$N_c \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

$$N_v \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2}$$

Effective density of states for some important semiconductors (300K), per cm^{-3}

	Ge	Si	GaAs
N_c (cm^{-3})	1.04×10^{19}	2.8×10^{19}	4.7×10^{17}
N_v (cm^{-3})	6.0×10^{18}	1.04×10^{19}	7.0×10^{18}

Carrier concentration in intrinsic semiconductors

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

$$p = N_v\, e^{-(E_F - E_v)/kT}$$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$np = n_i^2$$

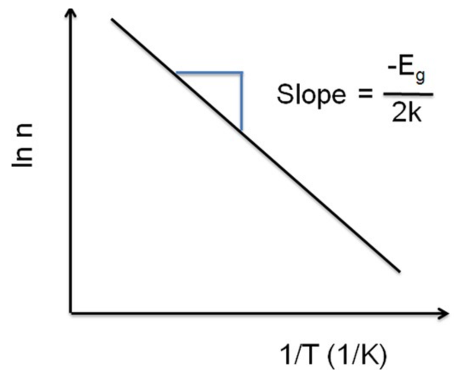
$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

Where is the intrinsic Fermi level F_i?

$$\ln n_i = \ln \sqrt{N_c N_v} - E_g/2kT$$

$$n_i = N_c e^{-(E_c - E_i)/kT}$$

$$E_i = E_c - kT \ln \frac{N_c}{n_i} = E_c + kT \ln n_i - kT \ln N_c = E_c - \frac{E_g}{2} - kT \ln \sqrt{\frac{N_c}{N_v}}$$



In intrinsic semiconductors E_i is very close to the middle of the gap;
It is situated exactly in the middle if $Nc = N_v$

Temperature dependence of n_i

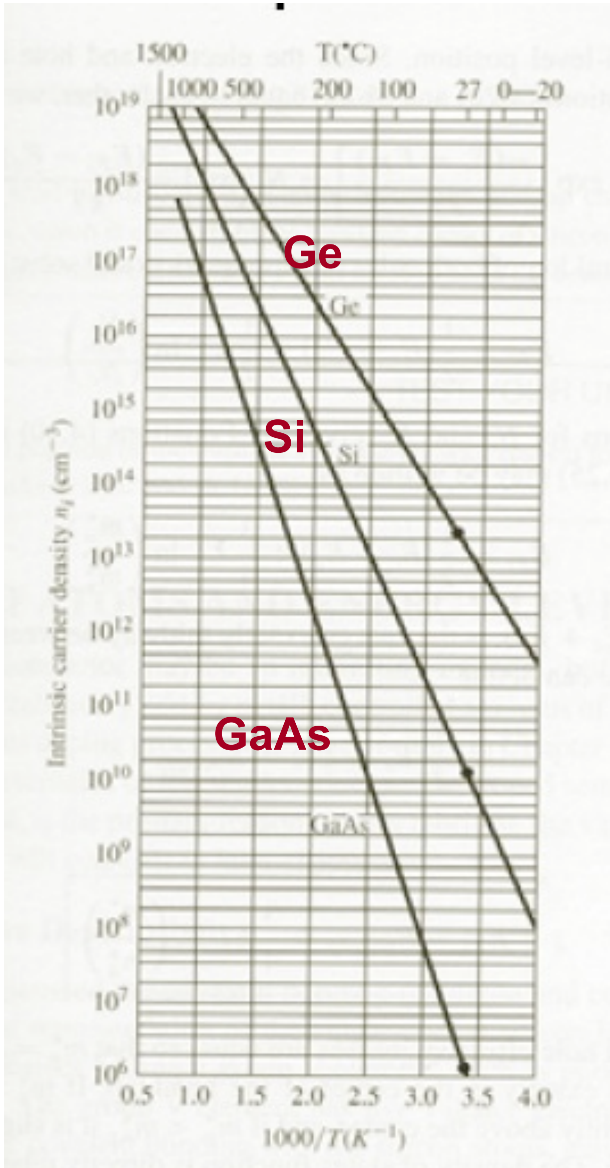
Two important factors for n_i:

• Temperature, T - for a given temperature T there are $n_i(T)$
Carriers - we call this the “intrinsic” carrier concentration.
• Energy gap, E_g - The value of $n_i(T)$ depends on E_g:

$$n_i(T)^2 \sim exp(-E_g/kT)$$

• the larger Eg the smaller n_i
• the higher the temperature the larger n_i

Semiconductor	InSb	Ge	Si	GaAs	GaP	ZnSe	Diamond
E_g (eV)	0.18	0.67	1.12	1.42	2.25	2.7	6.0



Donors and acceptors in a semiconductor

Energy diagrams of donors

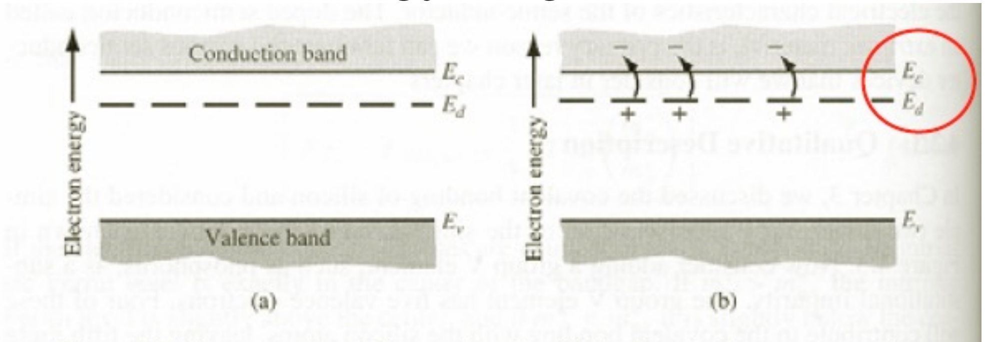
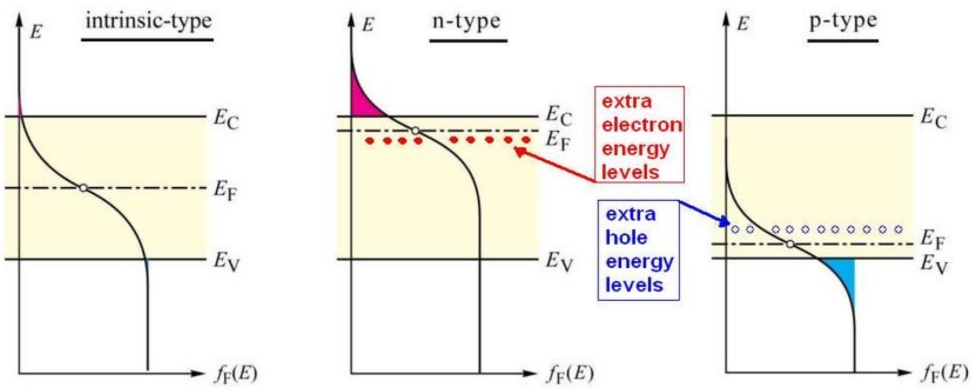


Figure 4.5 | The energy-band diagram showing (a) the discrete donor energy state and (b) the effect of a donor state being ionized.



Energy diagrams of acceptors

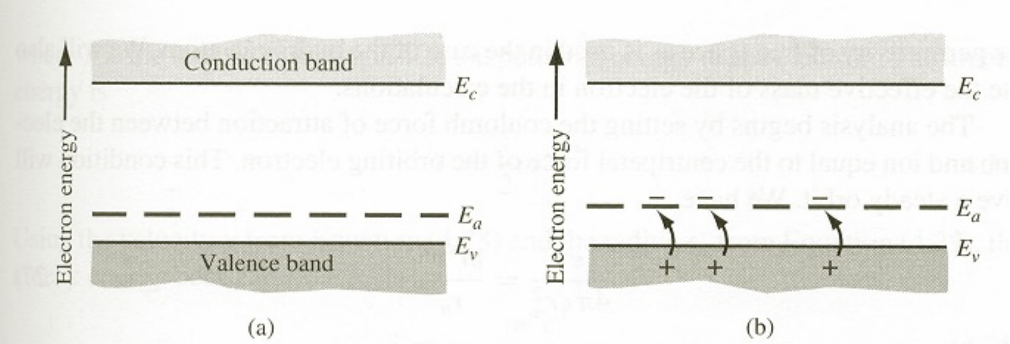


Figure 4.7 | The energy-band diagram showing (a) the discrete acceptor energy state and (b) the effect of an acceptor state being ionized.

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

$$p = N_v\, e^{-(E_F - E_v)/kT}$$

- n-type semiconductor: E_F is closer to the E_C
- p-type semiconductor: E_F is closer to the E_V

Donors and acceptors in a semiconductor, where is the Fermi level?

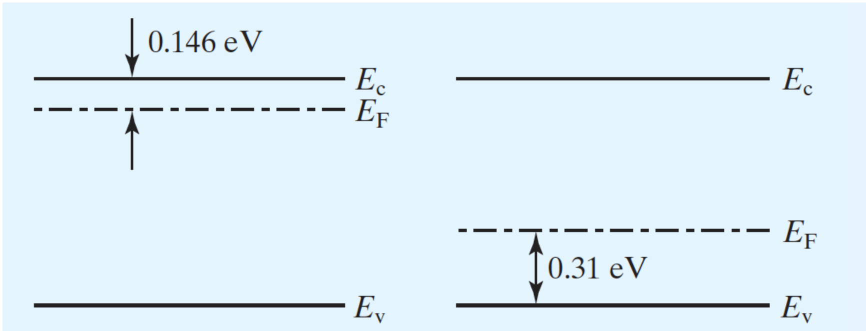
$$n = N_c \, e^{-(E_c - E_F)/kT}$$

The position of Fermi level can be determined for given concentration of electrons/holes

Numerical example: where is E_F located in the energy band of silicon, at 300K with $n = 10^{17} cm^{-3}$?

$$\begin{aligned} E_c - E_F &= kT \cdot \ln(N_c/n) \\ &= 0.026 \ln(2.8 \times 10^{19}/10^{17}) \\ &= 0.146 \text{ eV} \end{aligned}$$

The higher concentration of electrons/holes, the closer the fermi level approaches to the convuction/valence band edge



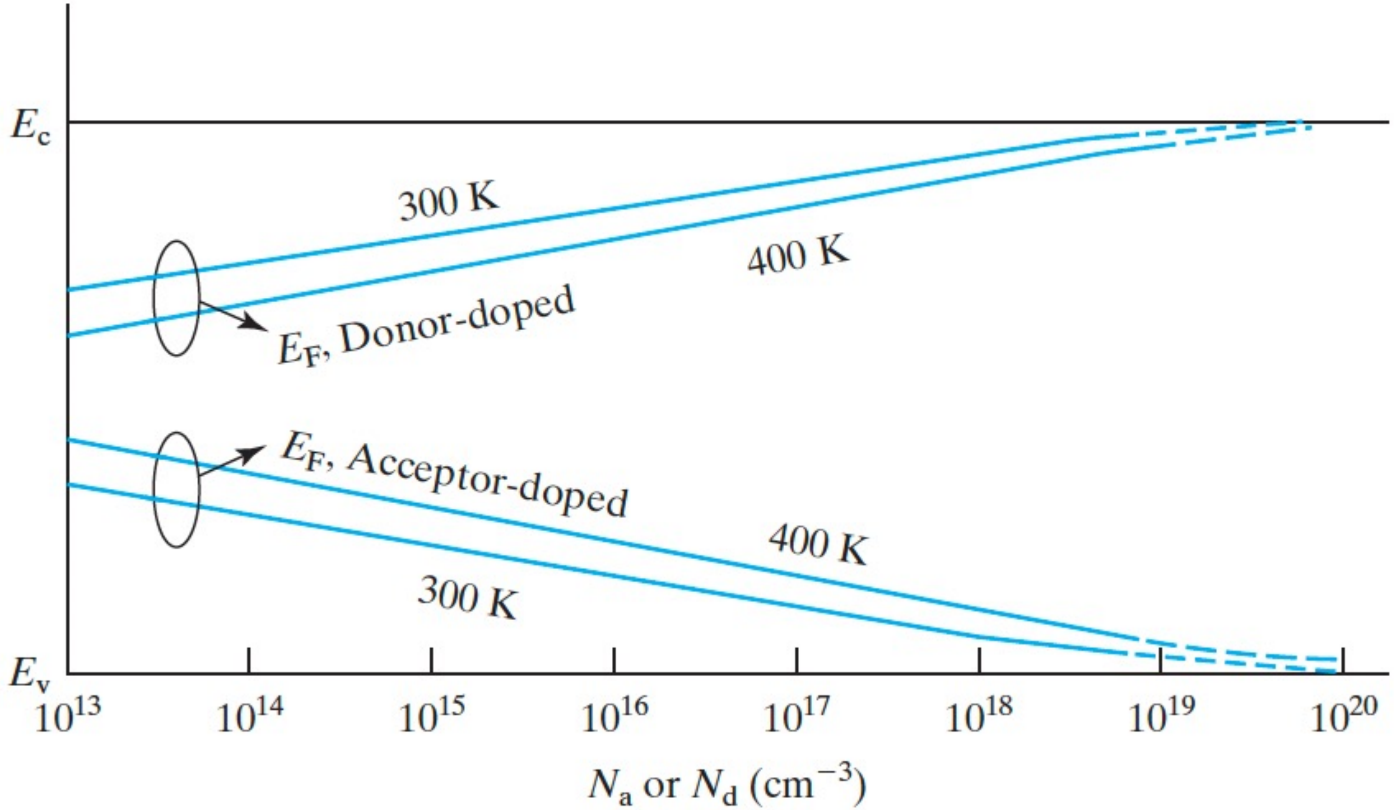
Now, let us the same for $p = 10^{14} cm^{-3}$

$$p = N_v \, e^{-(E_F - E_v)/kT}$$

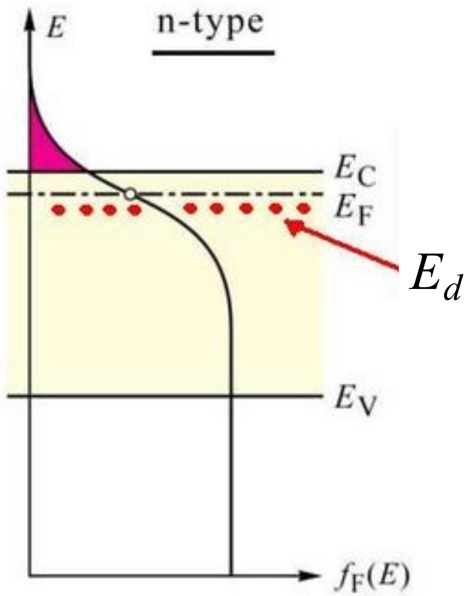
$$\begin{aligned} E_F - E_v &= kT \cdot \ln(N_v/p) \\ &= 0.026 \ln(1.04 \times 10^{19}/10^{14}) \\ &= 0.31 \text{ eV} \end{aligned}$$

Donors and acceptors in a semiconductor, E_F position vs doping level

Location of Fermi level vs. dopant concentration in Si at 300 and 400 K



Donors and acceptors in a semiconductor



For donors, the difference (E_d-E_F) determines the degree of ionization

The probability that a donor is not ionized, i.e., the probability that it is occupied by the “extra” electron, is expressed as follows:

$$n_d = \frac{N_d}{1+\frac{1}{g}exp\left(\frac{E_d-E_F}{kT}\right)}$$

The factor g=2 stems from the complication that a donor atom can hold an electron with upspin or downspin. This increases the probability that a donor state is occupied by an electron

Same way, for acceptors, the probability of occupation (non-ionization):

$$p_a = \frac{N_a}{1+\frac{1}{g}exp\left(\frac{E_F-E_a}{kT}\right)}$$

To calculate the concentration of non-ionized donors:

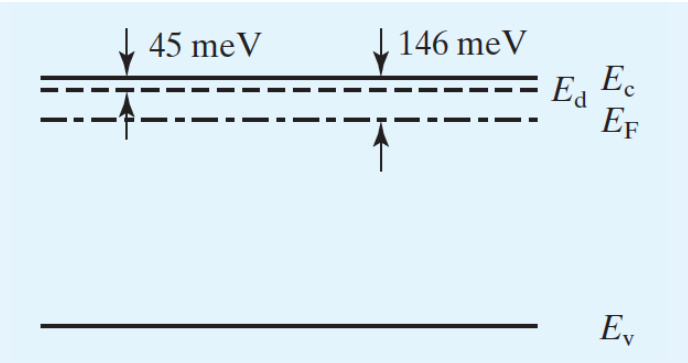
$$n_d \approx \frac{N_d}{1+\frac{1}{2}exp\left(\frac{E_d-E_F}{kT}\right)} = 2N_d exp\left[\frac{-(E_d-E_F)}{kT}\right]$$

Boltzmann approximation

Donors and acceptors in a semiconductor: numerical example

In a silicon sample doped with $10^{17} cm^{-3}$ of phosphorus atoms, what fraction of the donors are not ionized (T=300K)?
The donor level E_d is located at 45 meV below Ec for phosphorus

- Note that the concentration of electrons determines the E_F position as shown in previous example



kT= 26 meV – 300K

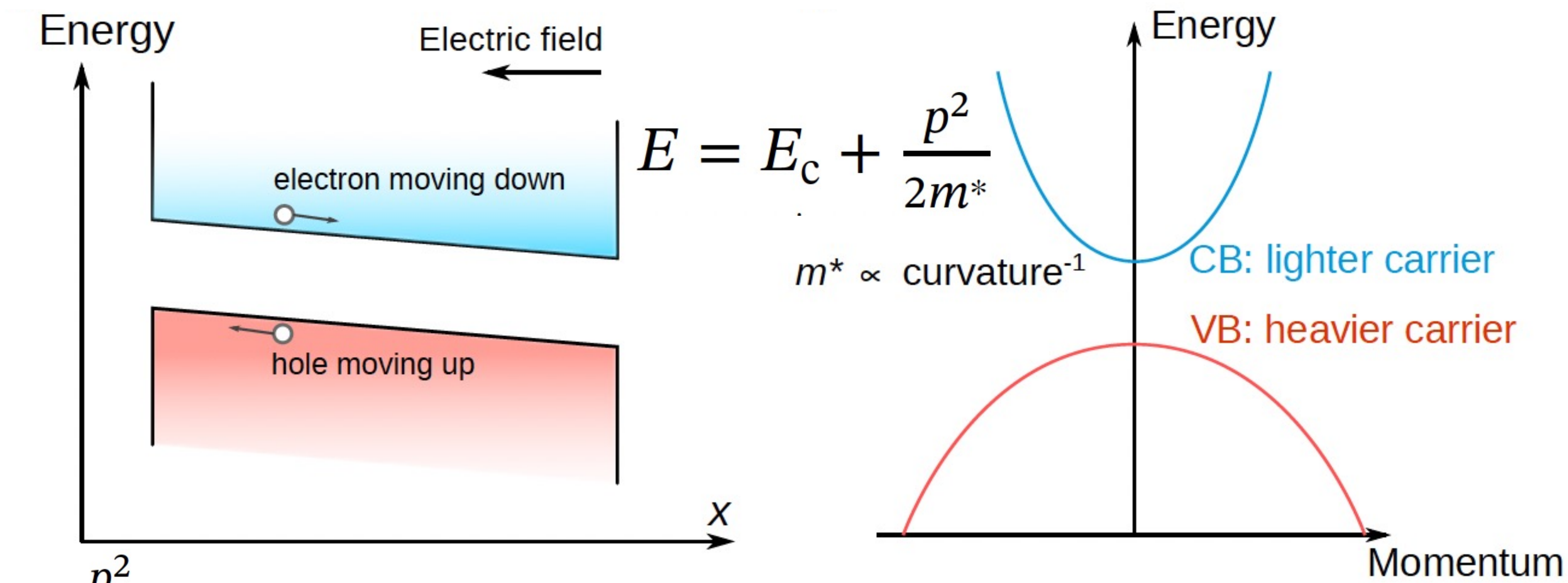
g-factor

$$n_d = \frac{N_d}{1 + \frac{1}{2} exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$\frac{1}{1 + \frac{1}{2} e^{((146-45)meV)/26meV}} = 3.9\%$$

The assumption about nearly complete ionization of donors is valid
(this may change when the temperature goes down!)

Effective mass for electrons and holes



$$E = E_c + \frac{p^2}{2m^*}$$

where Ec the energy of the conduction band edge, we can calculate m^*_n. Similarly, we can get m^*_p.

For silicon, m^*_n = 0.26 m_0 and m^*_p = 0.39 m_0, where m_0 is the mass of electron in vacuum. Thus, in silicon, the carriers travel faster than in vacuum and electrons travel faster than holes.

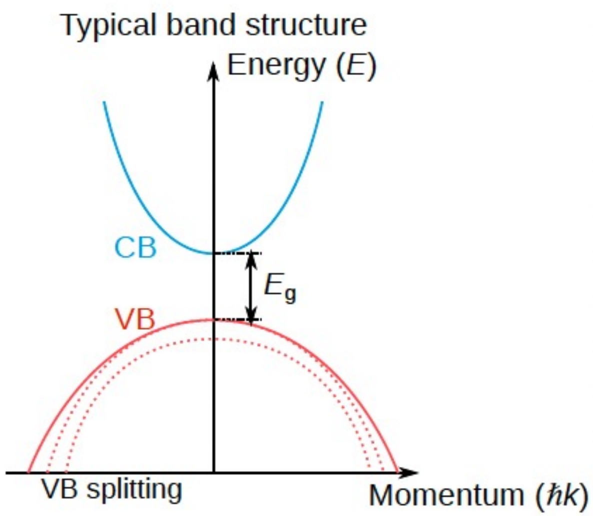
Degeneracy factor – to be taken into account for calculations of *n, p*

- **The degeneracy factor represents the number of ways a specific energy state can be occupied by particles.**

- conduction band: the degeneracy factor accounts for the number of equivalent energy valleys or minima where electrons can reside.
- valence band, the degeneracy factor accounts for the multiple energy maxima where holes (missing electrons) can exist
- in practrice, for calculations a factor is added before the effective density of states
- the degeneracy factor is **not necessary *equal to 2***

Example from your today's exercise
(degeneracy factor β=4 for B doping in Si):

$$N_A - N_A^- = \frac{N_A}{1 + \frac{1}{\beta_a} \cdot e^{\frac{E_f - E_A}{kT}}} \approx \beta_a N_A \cdot e^{-\frac{E_f - E_A}{kT}}$$

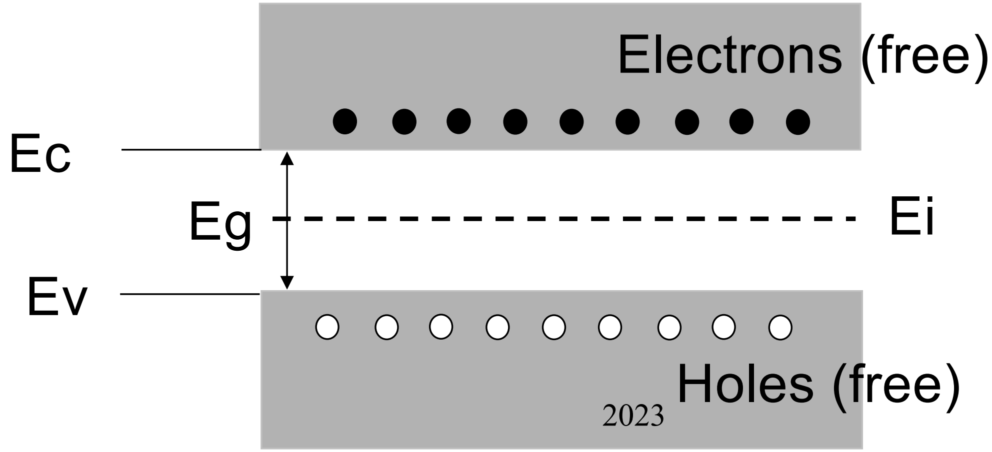


The relation $pn = n_i^2$ holds for either intrinsic or doped semiconductors (assuming that the system is in thermal equilibrium)

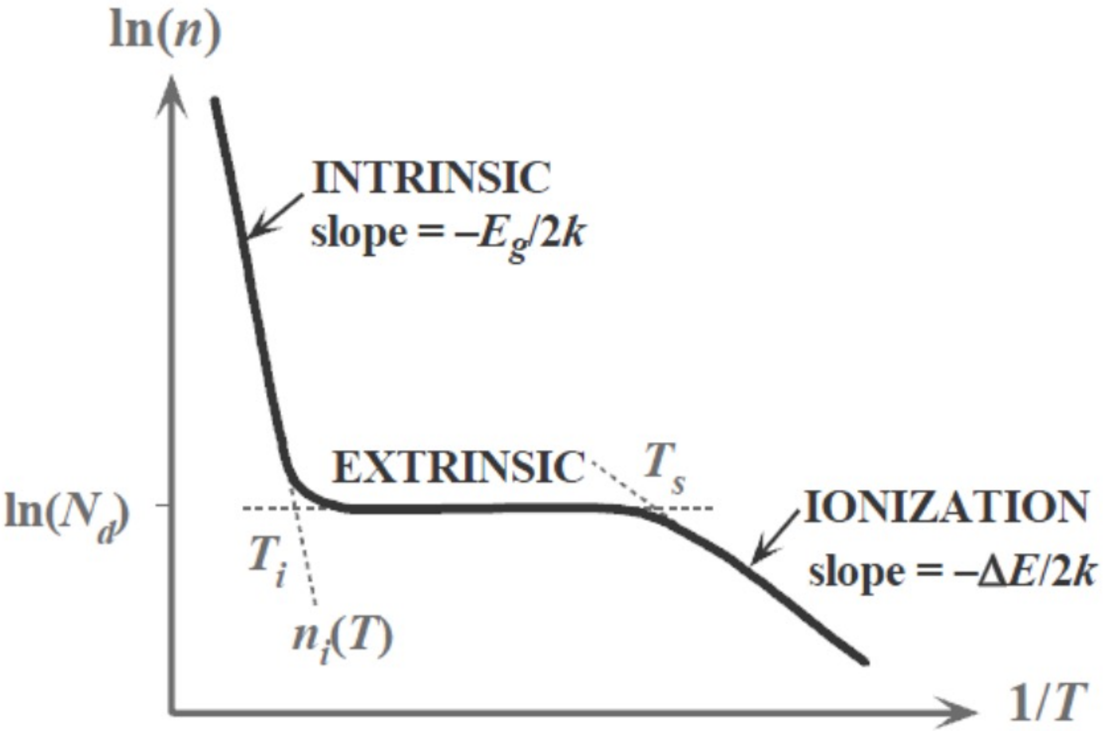
$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

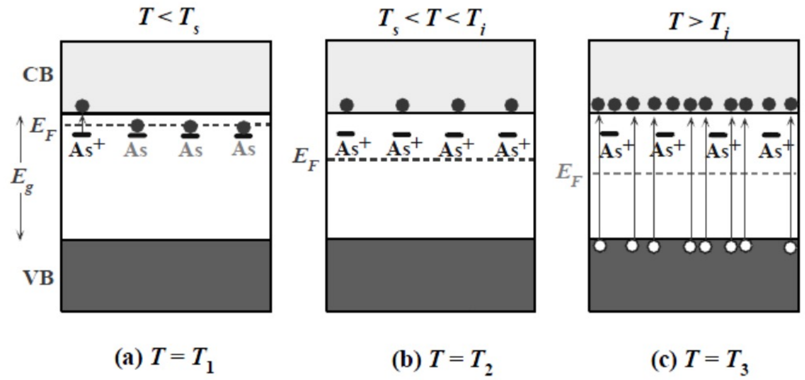
$$np = \sqrt{N_c N_v} e^{-(E_c - E_v)/kT} = n_i^2$$



Temperature dependence of carrier concentration

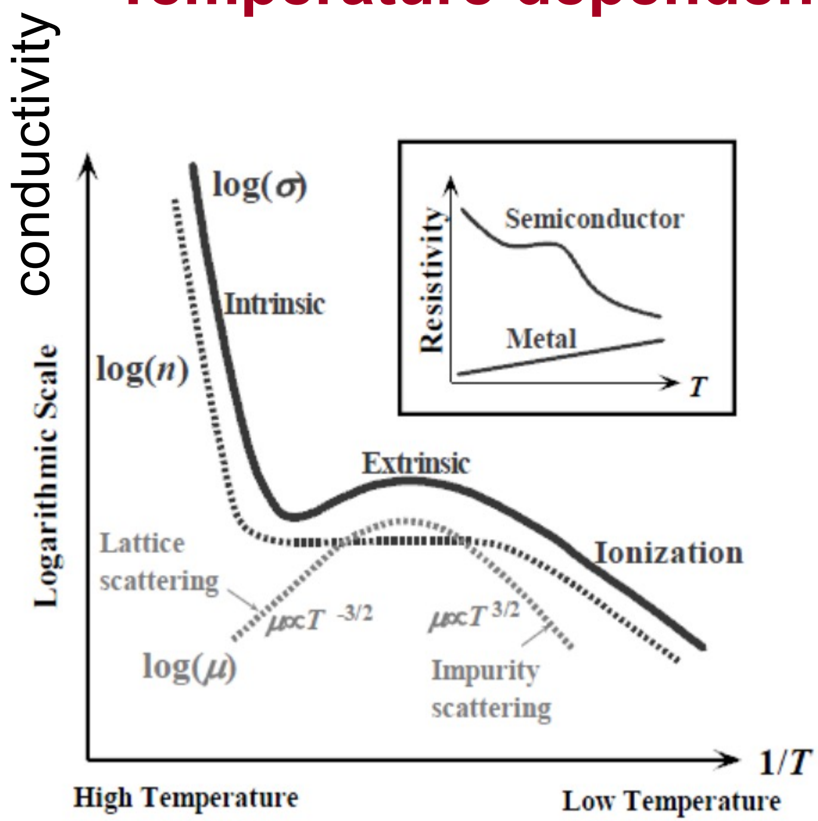


Ionization of Dopants



(a) Below T_s, the electron concentration is controlled by the ionization of the donors. (b) Between T_s and T_i, the electron concentration is equal to the concentration of donors since they would all have ionized. (c) At high temperatures, thermally generated electrons from the VB exceed the number of electrons from ionized donors and the semiconductor behaves as if intrinsic.

Temperature dependence of semiconductor conductivity



Temperature dependence of electrical conductivity for a doped (*n*-type) semiconductor.

From *Principles of Electronic Materials and Devices, Second Edition*, S.O. Kasap (© McGraw-Hill, 2002)
http://Materials.Usask.Ca

Calculate the intrinsic carrier density in germanium, silicon and gallium arsenide at 300, 400, 500 and 600 K.

Solution

The intrinsic carrier density in silicon at 300 K equals:

$$n_i(300K) = \sqrt{N_c N_v}\, exp\left(\frac{-E_g}{2\,KT}\right)$$

$$= \sqrt{2.81\ x\ 10^{19}\ x\ 1.83\ x\ 10^{19}}\ exp\left(\frac{-1.12}{2\ x\ 0.0258}\right)$$

$$= 8.72\ x\ 10^{9} m^{-3}$$

Try this for other materials and temperatures

	Germanium	Silicon	Gallium Arsenide
300 K	2.02 x 10^{13}	8.72 x 10^{9}	2.03 x 10^{6}
400 K	1.38 x 10^{15}	4.52 x 10^{12}	5.98 x 10^{9}
500 K	1.91 x 10^{16}	2.16 x 10^{14}	7.98 x 10^{11}
600 K	1.18 x 10^{17}	3.07 x 10^{15}	2.22 x 10^{13}

Summary: concentration of charge carriers in equilibrium

Fermi distribution (also known as Fermi-Dirac function):

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

Bolzmann approximation (valid for $E\text{-}E_F >> kT$

$$f(E) \approx e^{-(E - E_F)/kT}$$

Electron and hole concentration can be calculated (for different temperatures)

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

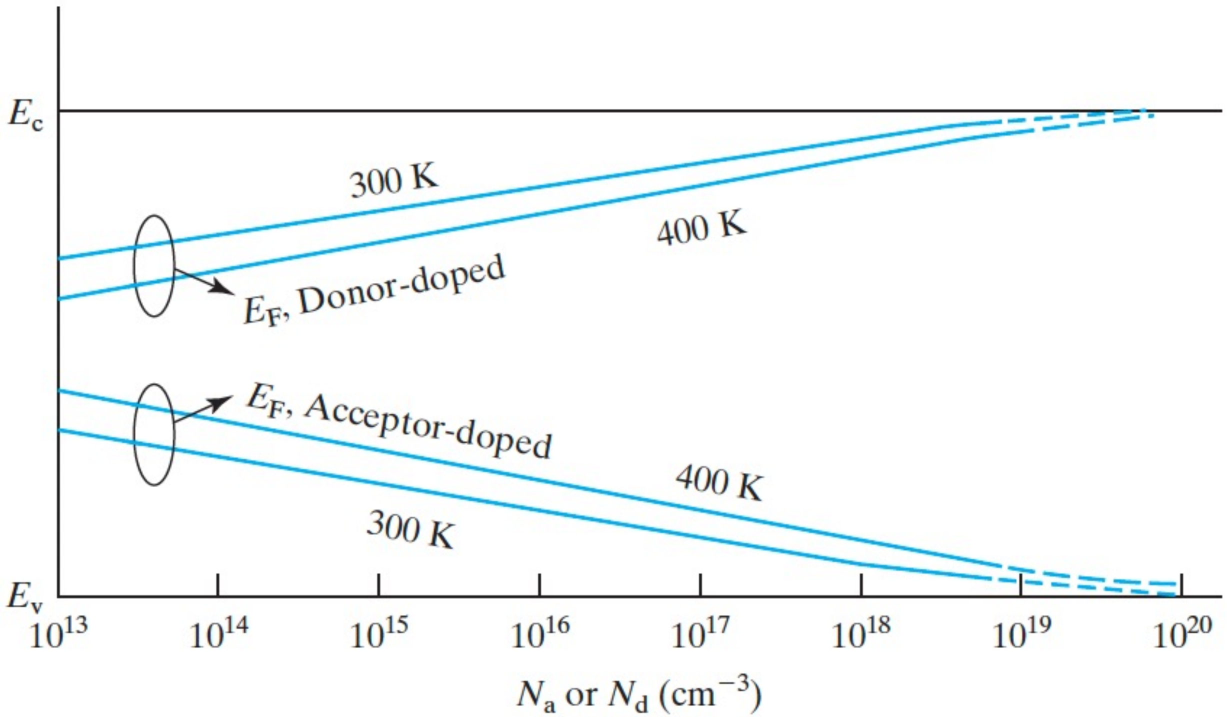
$$p = N_v\, e^{-(E_F - E_v)/kT}$$

To calculate the concentration of non-ionized donors (acceptors):

$$n_d \approx \frac{N_d}{1+\frac{1}{2}exp\left(\frac{E_d-E_F}{kT}\right)} = 2N_d exp\left[\frac{-(E_d-E_F)}{kT}\right]$$

$$p_a = \frac{N_a}{1+\frac{1}{g}exp\left(\frac{E_F-E_a}{kT}\right)}$$

Summary: behavior of Fermi level



- Donors are nearly fully ionized is their energy levels are situated between E_F and Ec

- Acceptors are nearly fully ionized is their energy levels are situated between E_F and E_V

For the non-equilibrium situation, Fermi statistics can applied applied with some modifications (quasi-Fermi levels), will be examined in further lectures