

# Dispositifs microélectroniques

## Microelectronic devices

### Lectures:

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### Exercices & support:

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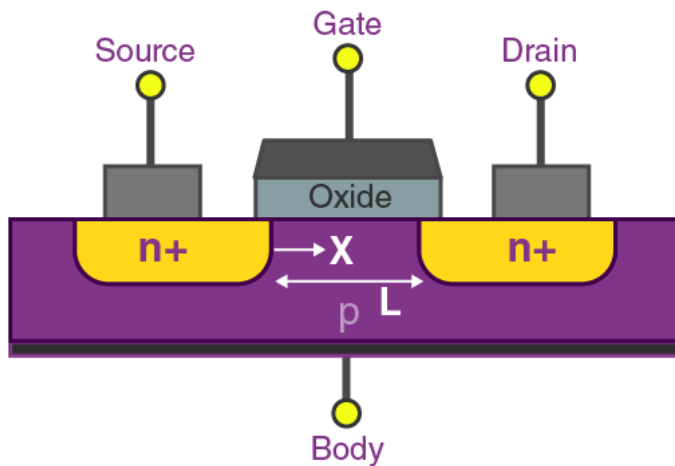
*niccolo.martinolli@epfl.ch*

*edoardo.tenna@epfl.ch*



- Zettabyte Era (officially since 2016)
- One zettabyte is the equivalent of 36,000,000 years of high-definition video. (Thomas Barnett Jr., Cisco)

*The key device behind information processing and storage: FET*



**Micro & Nanoelectronic  
devices in all computing,  
communication and sensing  
systems**

# Moore's law and nano-processors

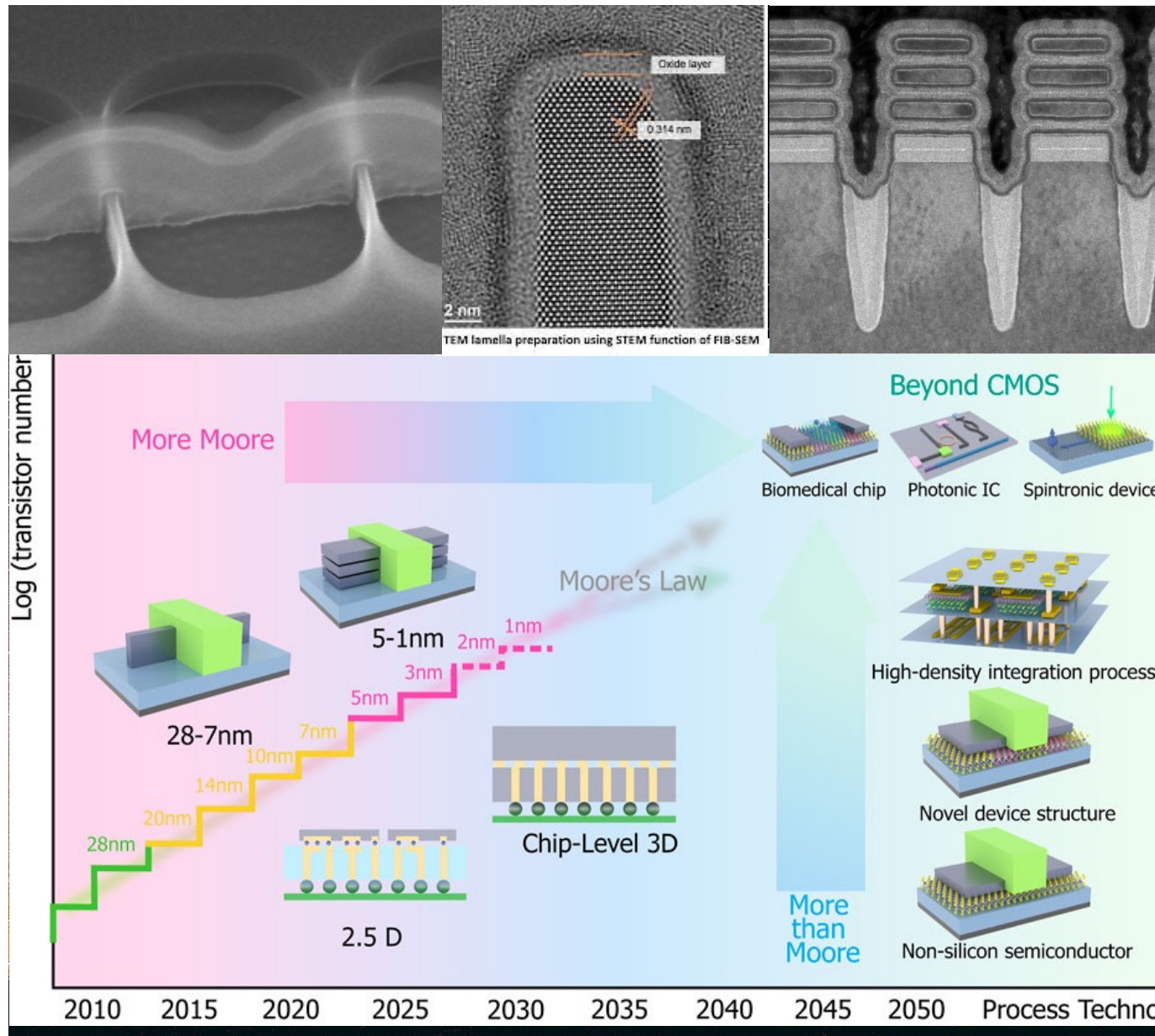
- From 100 micrometer down to several nm in 60 years
- 100M to 1B transistors/mm<sup>2</sup>

-Same device concept (FET)

-Same materials system:

Si/SiO<sub>2</sub>

-Changing technology



# Main topics and goals of the course

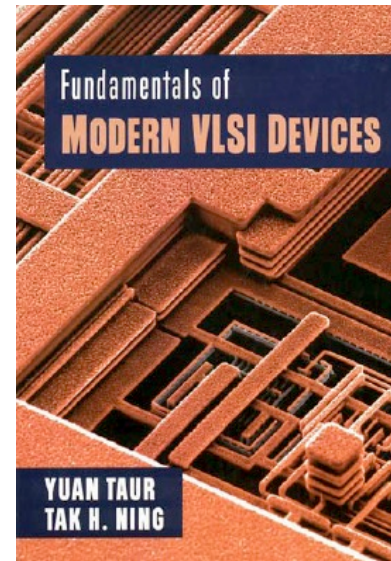
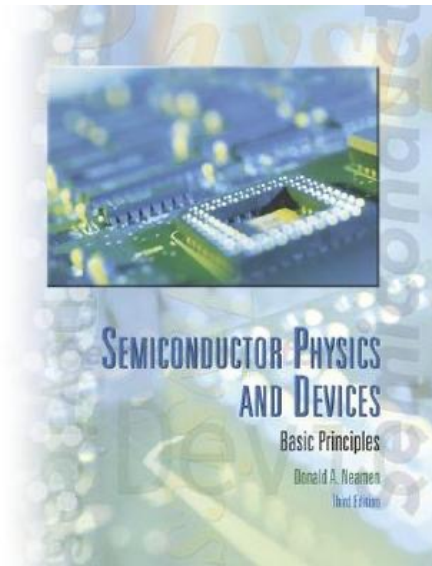
- Functional elements of semiconductor electronics: transistors
  - key elements of transistors :pn junctions, metal/insulator junctions and others
- Basics of semiconductors: energy bands, charge transport, space-charge layers
- How to calculate basic figures of device performance using simple models
- Device architectures, operation principles and main device technologies
- Main applications of the microelectronic devices (digital, analog, memory, etc.)
- Beyond silicon electronics: other materials, different concepts and principles
- Materials physics behind information processing and storage: dielectric response, polarization, conduction, phase transitions...

# Organisation and practical aspects

- Lecture presentation, slides = **ENGLISH**,
- **Questions/discussions = ENGLISH / FRENCH**
- **4 hours lecture + exercises per week (may vary, exercise time 1-2h)**
- **Test: a set of exercises to be completed and returned in class, time - 3h, anticipated date 7 April, or 14 April. The test will be evaluated, and contribute to the final exam note: full mark equals to 1 point out of 6 for the exam**
- **Exam: written, exercises + questions of different types (open book)**

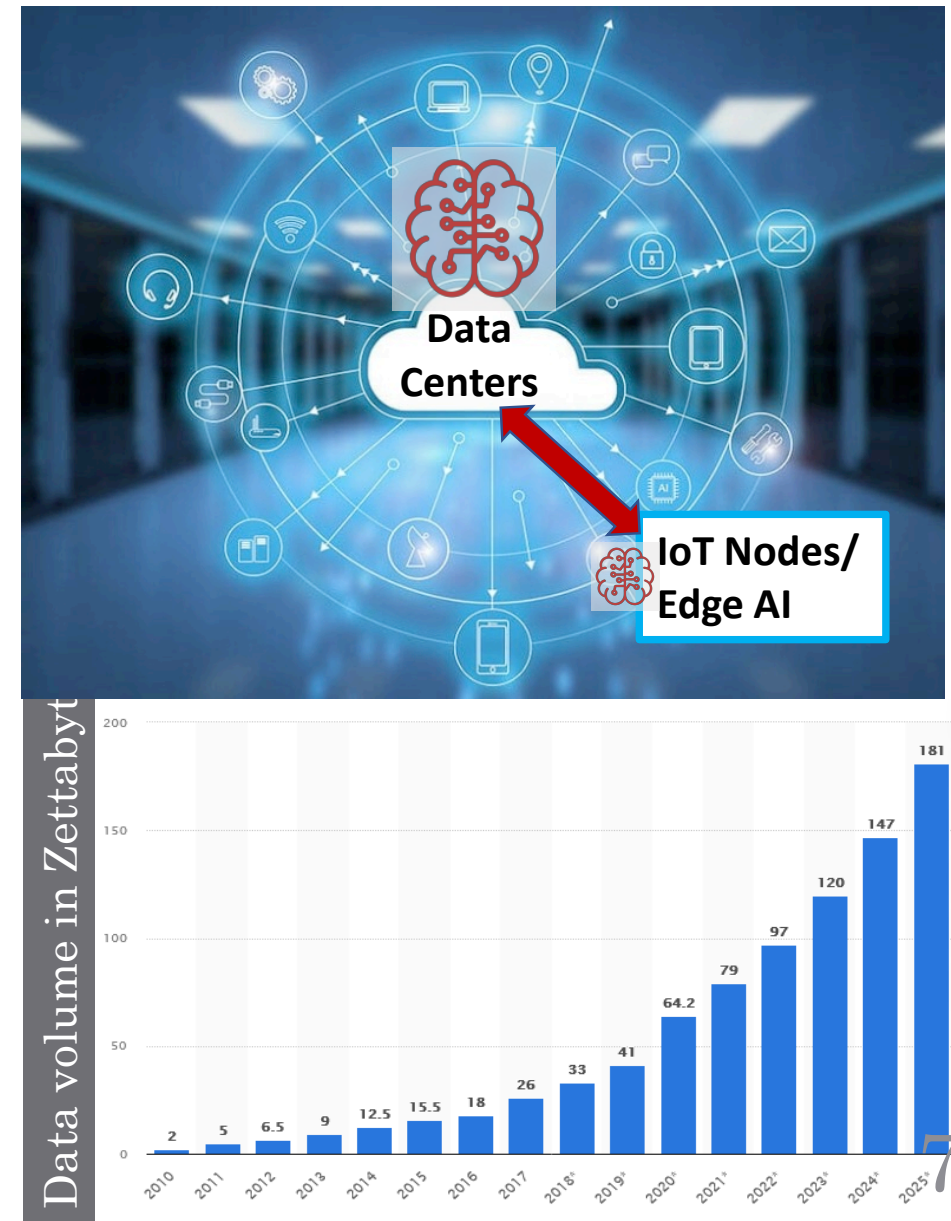
# lecture support & and optional reading

- 1) Lecture handouts (slides) in paper and pdf formats, exercises, references wherein (should be self-sufficient)
- 2) Yuan Taur, Tak. H. Ning, *Fundamentals of Modern VLSI Devices*, Cambridge University Press, 3<sup>rd</sup> edition, 2021.
- 3) Donald A. Neamen, *Semiconductor Physics and Devices*, 4<sup>th</sup> edition, McGraw Hill, 2011.



# Energy efficiency and data proliferation

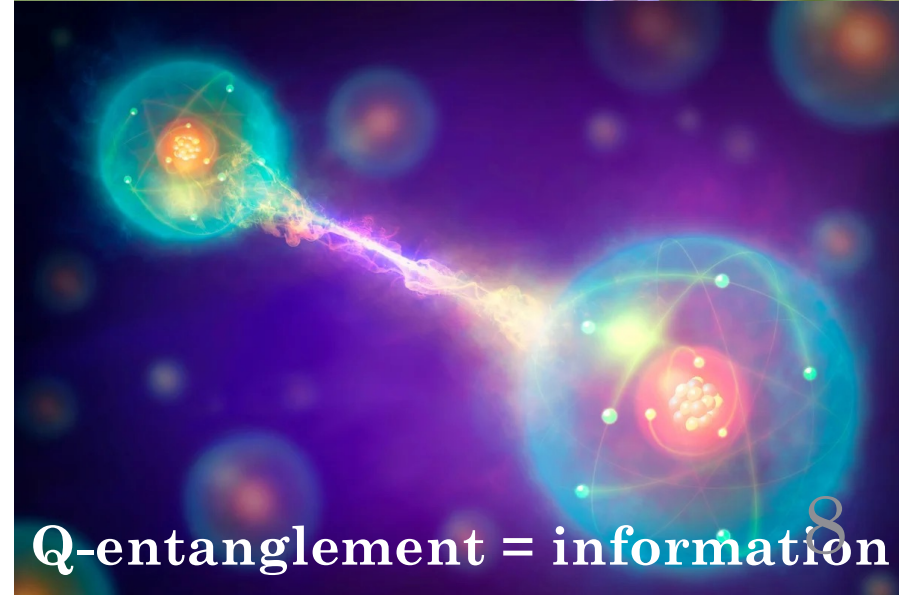
- By 2030, about 25% of the world's energy consumed by electronic devices if nothing is done to make them more energy efficient.
- Data volume is exploding.
- AI – even higher demand for data processing and storage



# Beyond CMOS...

*“The future of computing will not be based on ever-increasing processing power... it will rely on **understanding and drawing inferences from massive collections of data.**”*

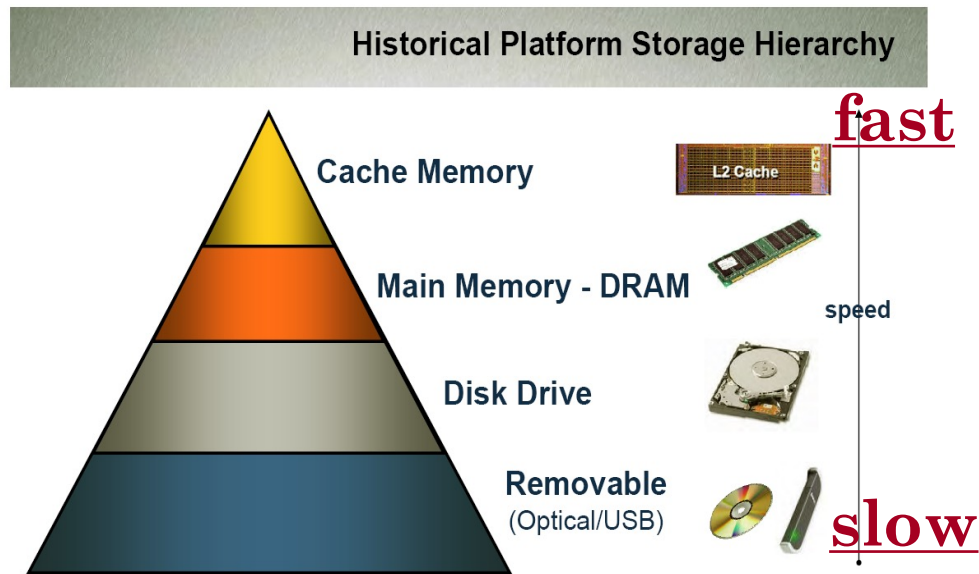
Breakthroughs in biological sciences and physics for **novel information processing hardware** – “tiny brains” for IoT



Q-entanglement = information

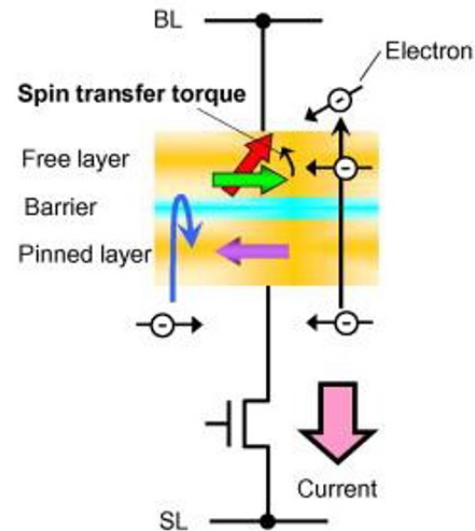
# The actual memory solutions are less than ideal...

Quest for the ideal memory:

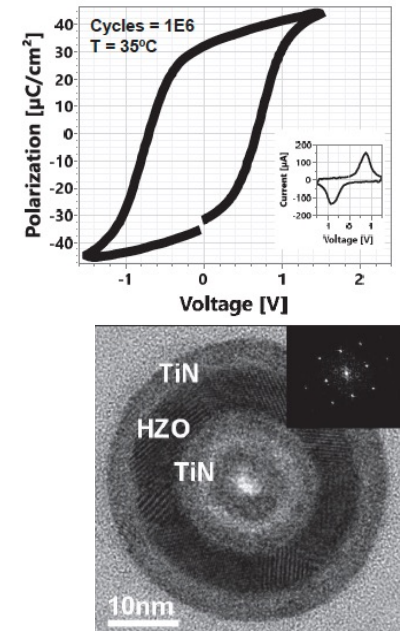


- volatility
- energy consumption
- speed

## STT MRAM Spin-Transfer Torque magnetic RAM

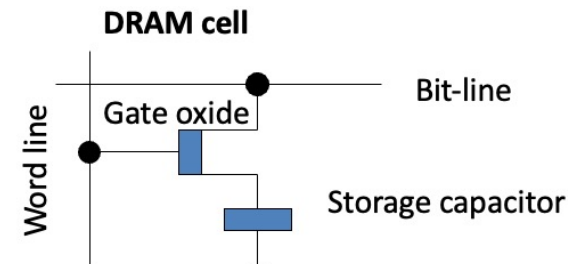
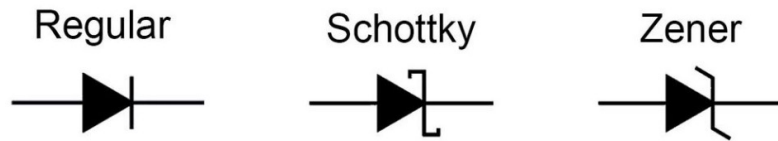


## FRAM (ferroelectric RAM)



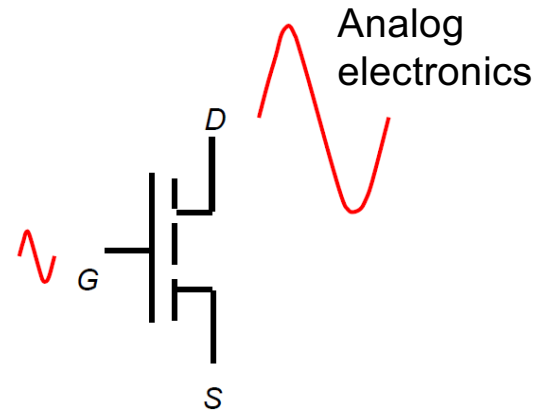
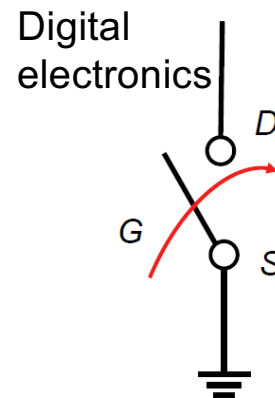
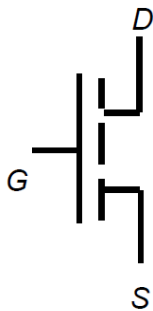
# Functional elements of semiconductor electronics

## Diodes, capacitors = 2 terminal devices



*"The interface is the device", Herbert Kroemer, Nobel lecture*

## Transistor = 3 terminal devices



# Planning and schedule

Lectures 1-2 : basics of semiconductors

Lectures 3-5: doped semiconductors, p-n junctions, diodes of different kinds

Lecture 6-8: FETs (Field Effect Transistors), basics, simple models and performance, modern FETs , and aggressive scaling

Lecture 9 – test (tentative date, to be confirmed later)

Lecture 10 bipolar transistors

Lecture 11-13: materials, concepts and devices beyond silicon electronics

- other semiconductors
- different device concepts
- Physical phenomena behind new concepts of information processing

# Vacuum tubes

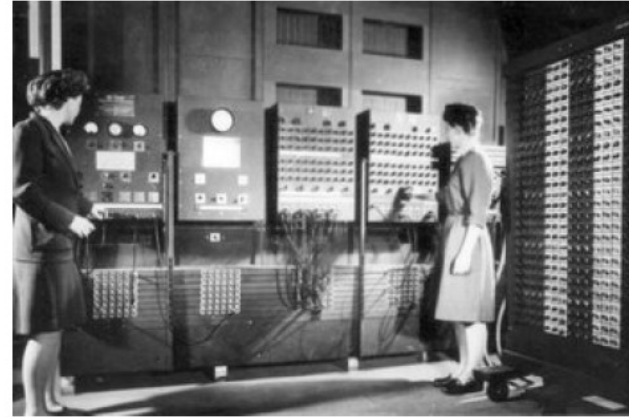
Golden age of radio  
1935 - 1950



<http://history.sandiego.edu/GEN/recording/images5/radio11.jpg>

ENIAC

(1945, Mauchly and Eckert, U Penn)



<http://en.wikipedia.org>

17,468 vacuum tubes  
1000 sq. feet of floor space  
30 tons  
150 KW

Until 1950s electronics was dominated by vacuum tubes, then semiconductors came...

# Core device of modern electronics: electronic switch = transistor



**One shouldn't work on semiconductors, that is a filthy mess; who knows if they really exist!**

**God created the solids, the devil their surfaces**

Wolfgang Pauli, **1931** (Nobel Prize, Physics, 1945)

**“The most important moment since mankind emerged as a life form.”**

**Isaac Asimov**

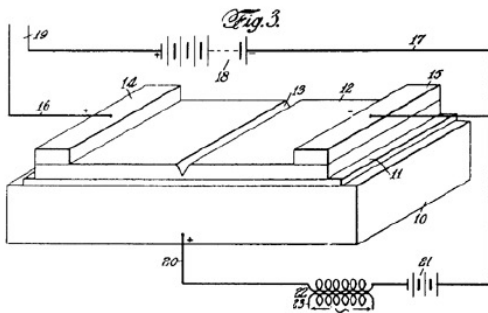
**(speaking about the “planar process” used to manufacture ICs - invented by Jean Hoerni, Fairchild Semiconductor, 1959)**

# Early days of transistors

Field-Effect Transistor

Lilienfeld, 1925

Heil, 1935



Bell Labs: a quest to seek a solid-state alternative to fragile glass vacuum tube amplifiers.

transistor



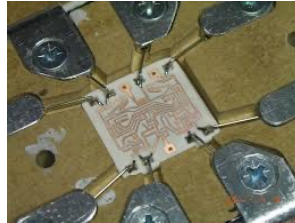
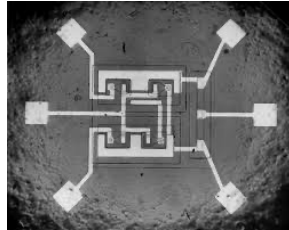
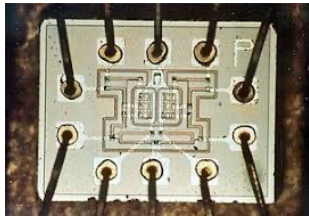
Bardeen, Brattain,  
Shockley, 1947

## Silicon Valley

In 1957, decades before Steve Jobs dreamed up Apple, a group of eight brilliant young men defected from the Shockley Semiconductor Company in order to start their own transistor business.

Their leader was 29-year-old Robert Noyce, who would **co-invent the microchip** -- an essential component of nearly all modern electronics today, including computers, motor vehicles, cell phones and household appliances.

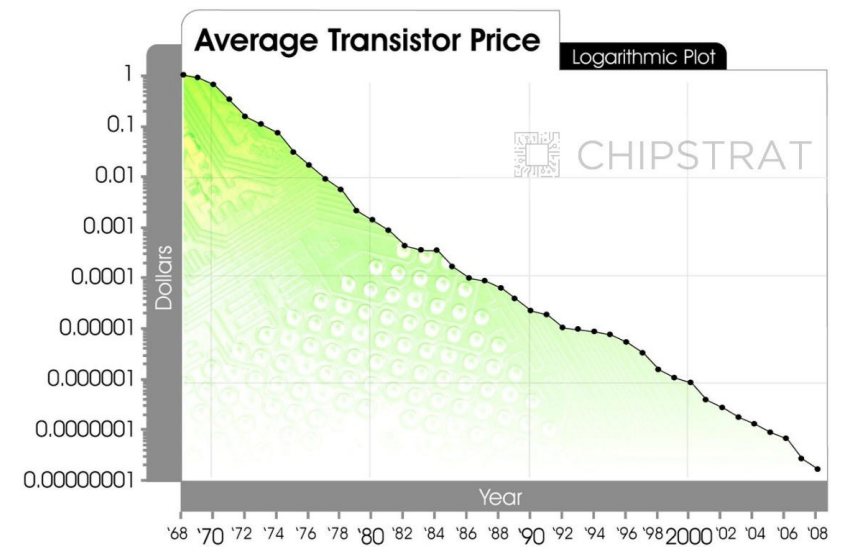
- 1965:  
a single transistor cost more than a dollar



- Chip Apple A17 pro (3nm technology TSMC)

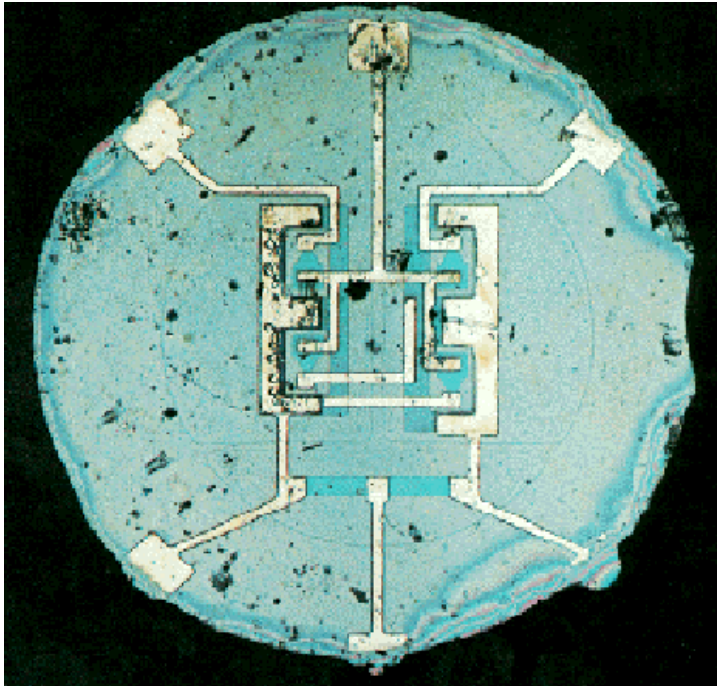


The A17 Pro contains 19 billion transistors

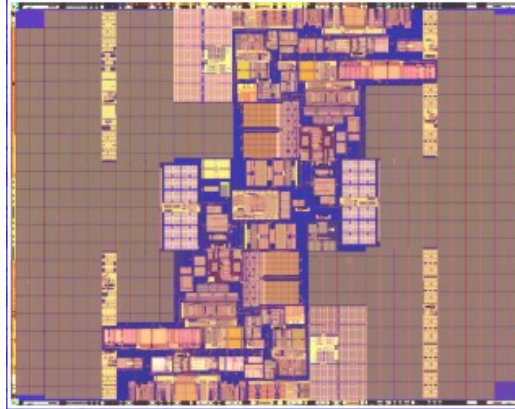


# Integrated circuits: from first sub-mm demo to 3nm technology

First planar integrated circuit (1961)



90 nm Intel's processor  
Montecito (2004)  
Itanium Processor Family



Transistors: 1.72 Billion  
Frequency: >1.7GHz  
Power: ~100W

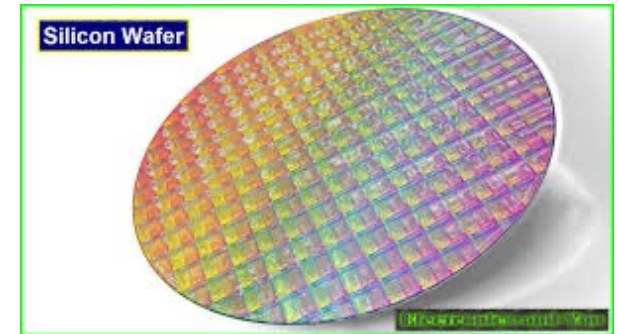
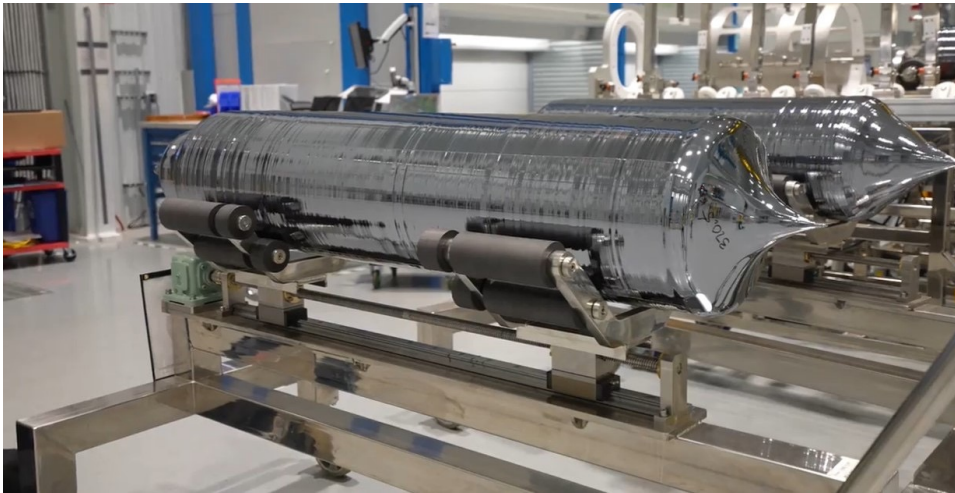
TSMC 3nm technology  
(in fact, transistors are larger than 3nm!)  
Apple M4 (2024)



Transistors: 28 Billion  
Frequency: up to 4.4 GHz  
Power: ~20-40W (up to 60)

# Silicon is the king: high quality crystals + price!

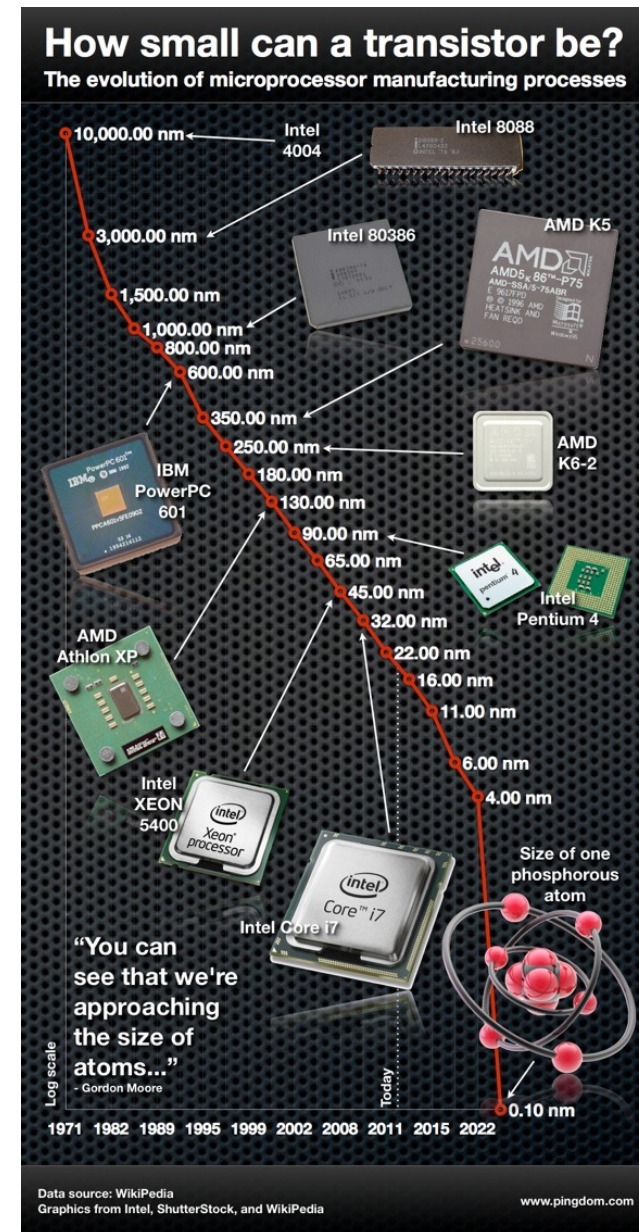
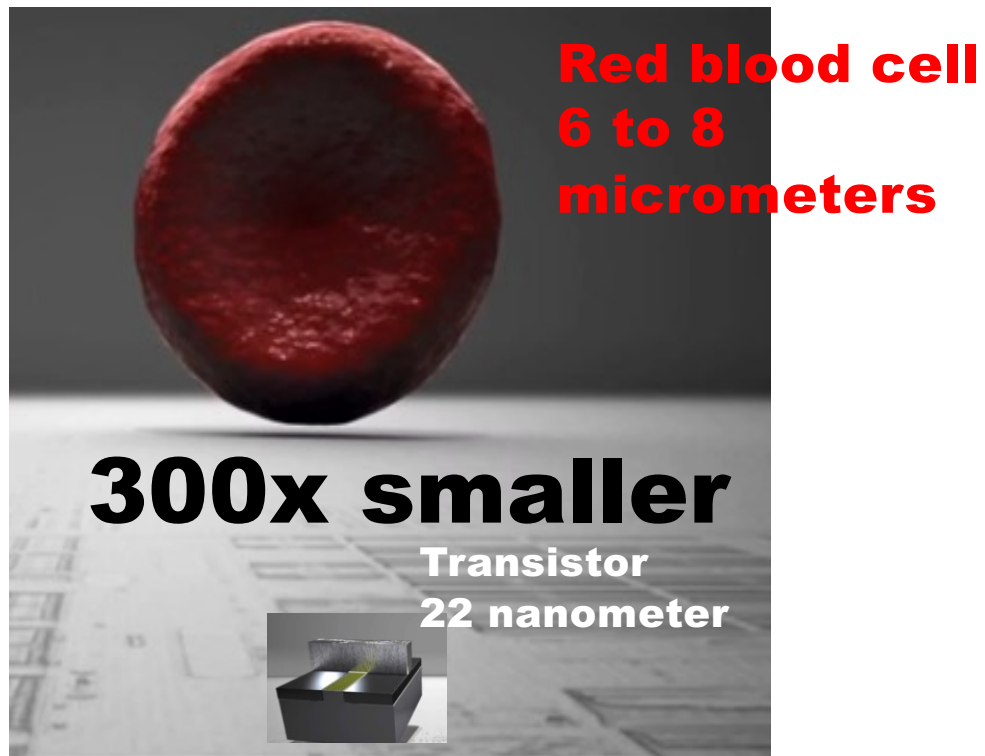
at least 99.9999999% pure



- Wafers are usually 300 mm (almost 12 in.) in diameter, which typically holds 148 chips, each 20 mm x 20 mm
- Wafer surfaces are polished to the atomic level, and wafer edges are shaped to remove cracks and chips.

# >2000: Nano Era

- Size of a transistor < 100nm
- Today: 3D 22nm transistors



# Moore's Law: The number of transistors on microchips doubles every two years

Our World  
in Data

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.

## Transistor count

50,000,000,000

10,000,000,000

5,000,000,000

1,000,000,000

500,000,000

100,000,000

50,000,000

10,000,000

5,000,000

1,000,000

500,000

100,000

50,000

10,000

5,000

1,000

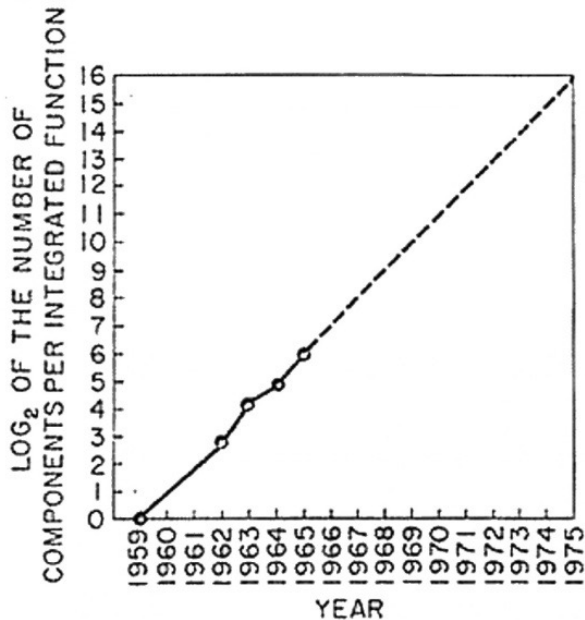
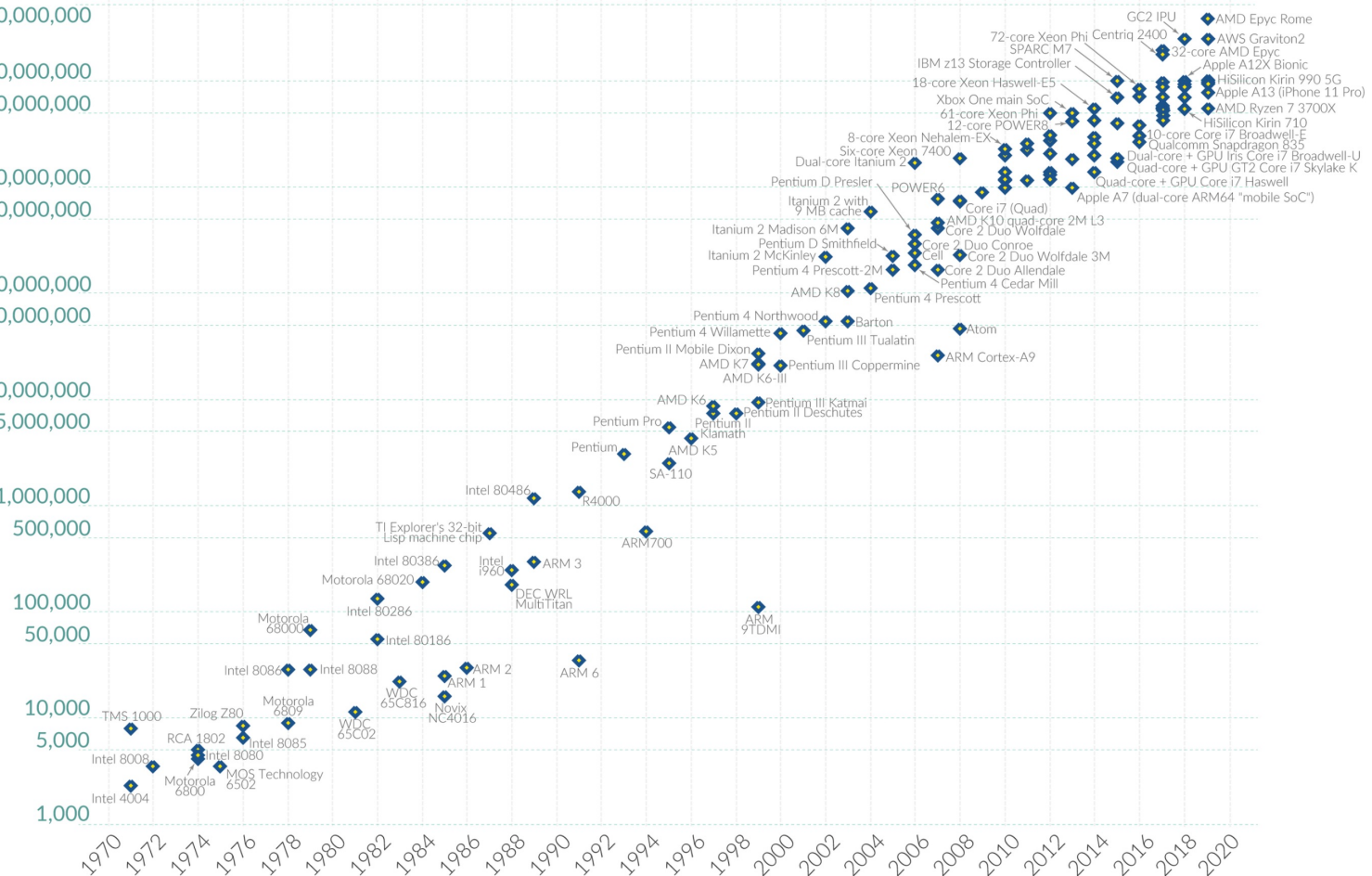


Fig. 2 Number of components per integrated function for minimum cost per component extrapolated vs time.



Data source: Wikipedia ([wikipedia.org/wiki/Transistor\\_count](https://wikipedia.org/wiki/Transistor_count))

OurWorldinData.org – Research and data to make progress against the world's largest problems.

Year in which the microchip was first introduced

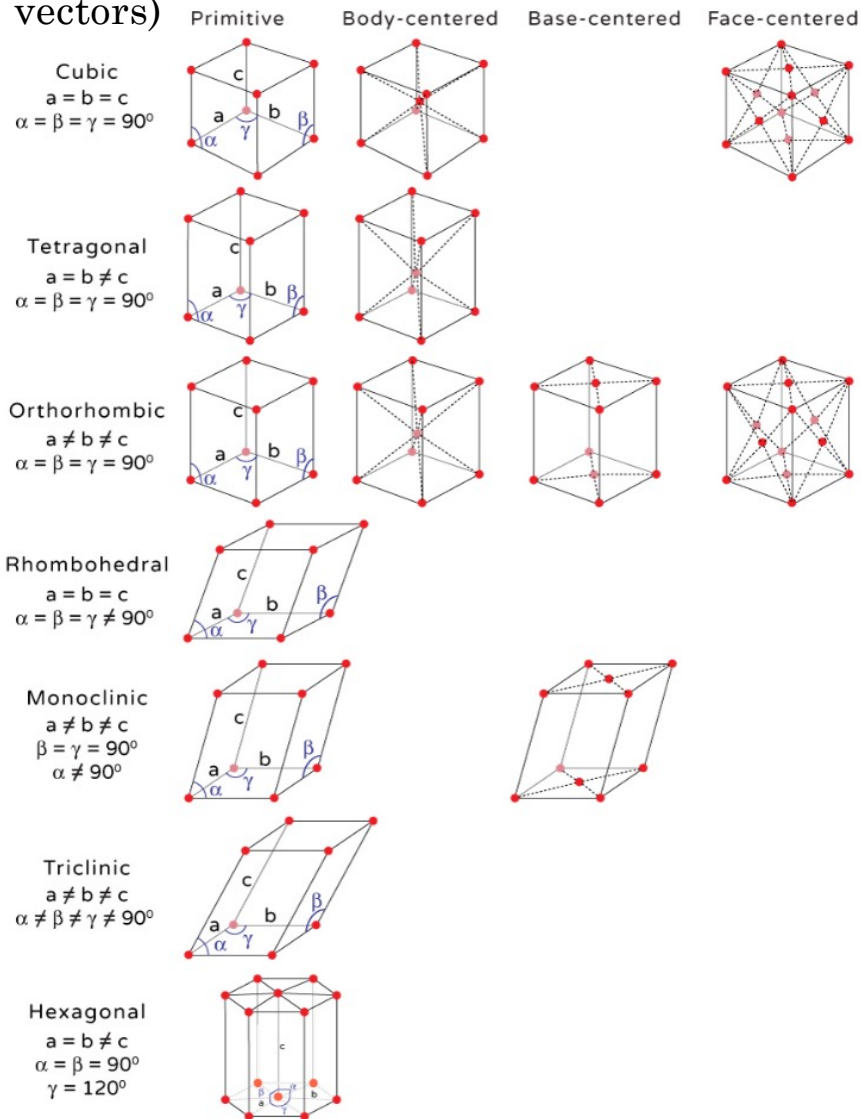
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# Chapter 1: Basics of semiconductor devices physics:

Materials: Silicon and other semiconductors – structure, properties, application-relevant features

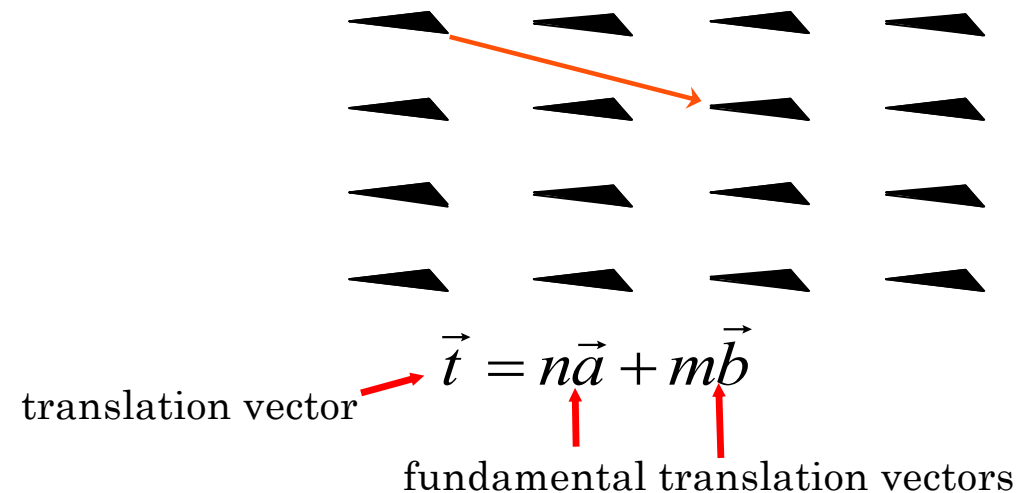
- Most important semiconductors are crystals. What are crystals?
- What are semiconductors
- How do semiconductors conduct electricity?
- electron and hole conduction
- How many electrons and holes are there in a semiconductor in thermal equilibrium at a certain temperature?
- How can one engineer the conductivity of semiconductors?

## Bravais lattices (formed with translation vectors)



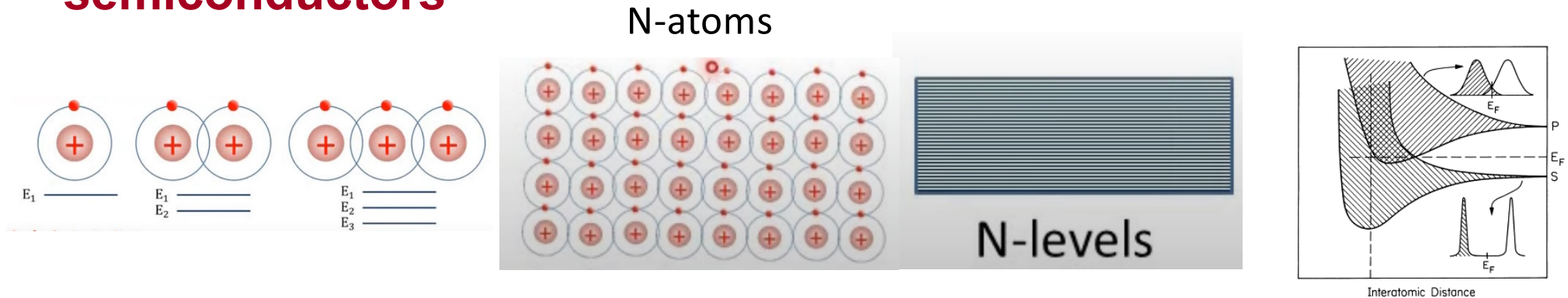
## What is a crystal?

- defined by a motif (an atom or group of atoms) + translation law



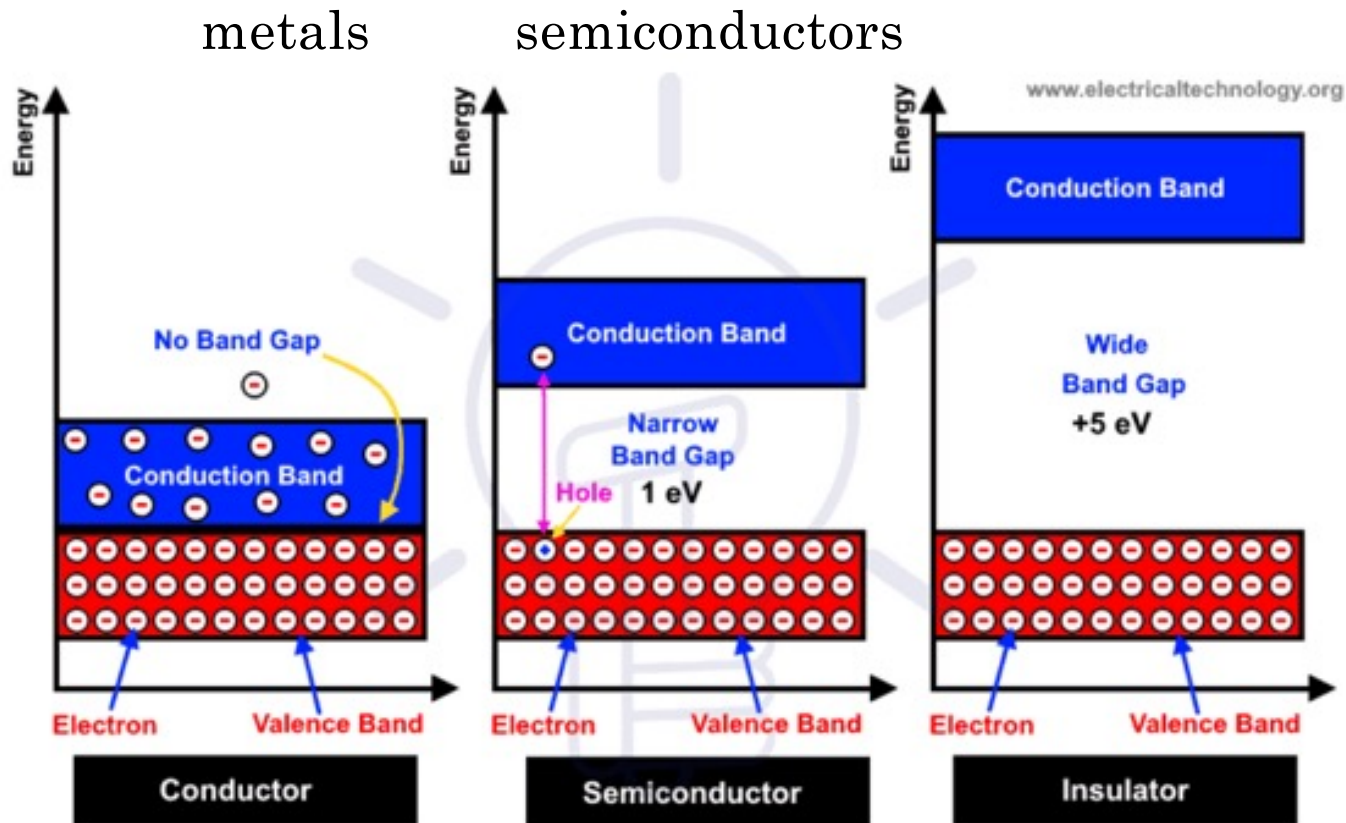
Crystal is characterized by its symmetry (the property of an object to remain unchanged under application of certain operations e.g. rotations, reflections)  
Symmetry is important for properties!

# Electrons in crystals and energy bands: metals, insulators and semiconductors



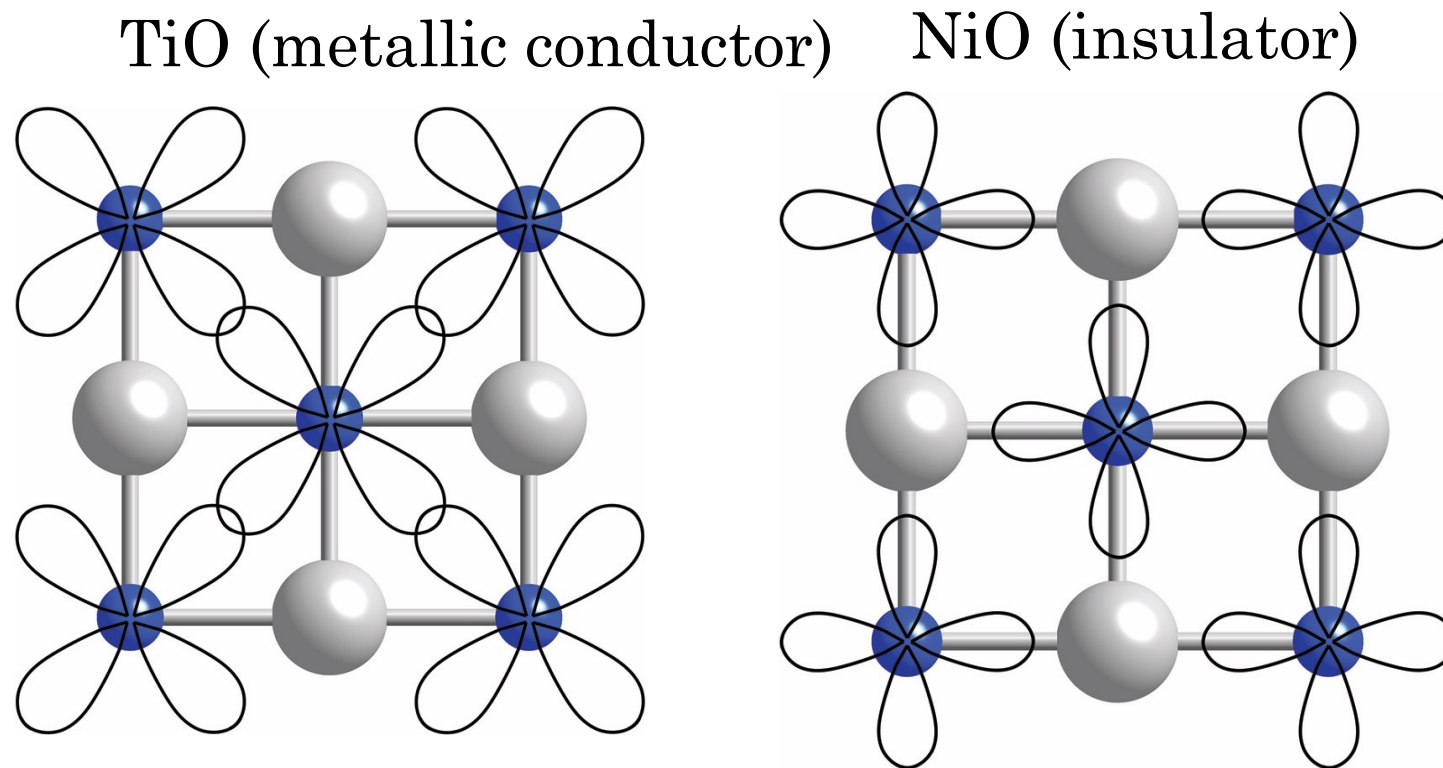
- As a result of interaction of a big number of atoms the energy levels of electrons form continuous energy bands
- For electronic applications most important are *valence band* and *conduction band*
- The energy band which comprises of valence electrons energy levels is referred to as the valence band. This band is present below the conduction band. The electrons of this band are loosely bound to the atom's nucleus
- Conduction band includes free electrons energy level. The electrons get free when external energy pushes them to the conduction band
- Forbidden band the *energy gap* between the valence band and the conduction

# Electrons in solids and energy bands: metals, insulators and semiconductors



- Semiconductors have the **energy gap** between the valence band and the conduction band (the gap is typically of order of 1eV, in some wide-gap semiconductors up to 3eV)

# Metallic conduction from compounds including non-metals



In TiO metallic conduction originates from overlapping d-orbitals that form partially filled conduction band. In NiO d-orbitals stay localized

Si – semiconductor that dominates modern electronics

**Other semiconductors:**

- Ge
- C (diamond form)
- Carbon Nanotubes – some have semiconductor properties with the bangap depending on the diameter
- GaAs, InP, InGaAs, InGaAsP, GaN, ZnSe, CdTe (on the average, 4 valence electrons per atom)

**Introduction of the new materials: the key question – is it Si compatible? (or is it CMOS-compatible?)**

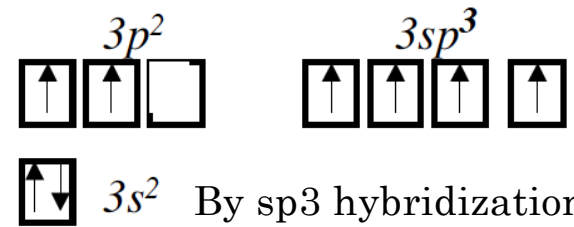
## Silicon - Si

Si is in Column IV of the periodic table:

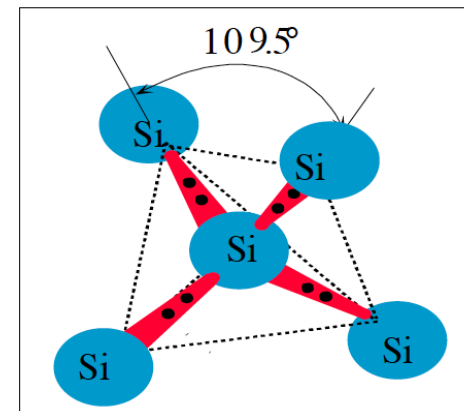
	IIIA	IVA	VA	VIA
	5 B	6 C	7 N	8 O
	13 Al	14 Si	15 P	16 S
IIB	30 Zn	31 Ga	32 Ge	33 As
	48 Cd	49 In	50 Sn	51 Sb
			52 Te	

Electronic structure of Si atom:

- 10 core electrons (tightly bound)
- 4 valence electrons (loosely bound, responsible for most chemical properties)

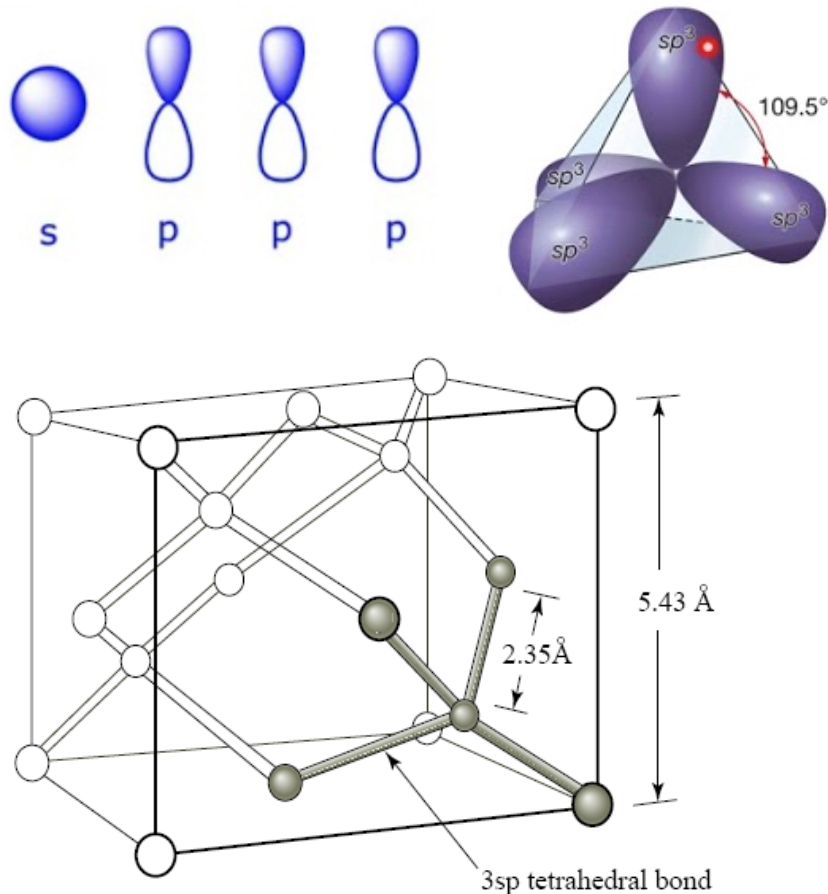


By  $sp^3$  hybridization  
two new bonds are formed



the bond takes shape of tetrahedron

## Silicon crystalline structure



- Silicon is a **crystalline material**:
  - long range atomic arrangement
- Diamond lattice:
  - atoms tetrahedrally bonded by sharing valence electrons (covalent bonding)
- Each atom shares 8 electrons:
  - the distance between the neighboring covalently bonded silicon atoms is  $2.35 \text{ \AA}$
- Lattice parameter  $5.43 \text{ \AA}$
- Si atomic density:  
 **$5 \times 10^{22} \text{ cm}^{-3}$**

Cubic structure: isotropic conduction and dielectric response

# Why silicon is a king?

- abundance



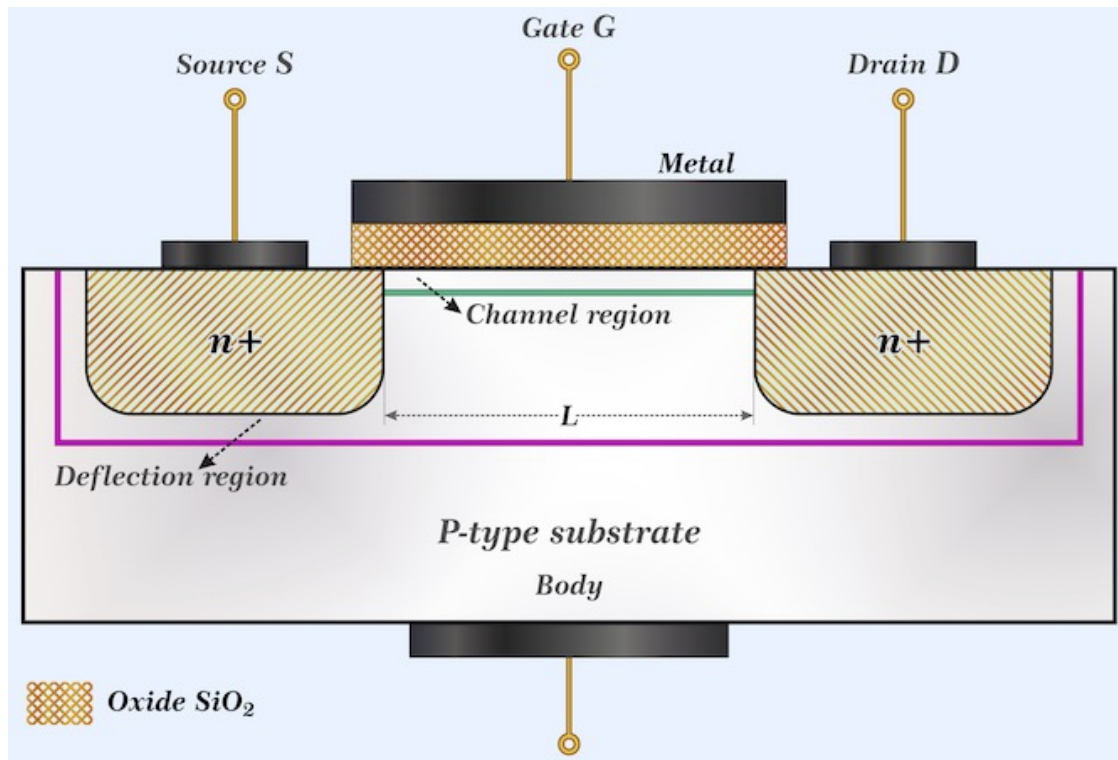
Element	Approximate % by weight
Oxygen	46.6
Silicon	27.7
Aluminum	8.1
Iron	5.0
Calcium	3.6
Sodium	2.8
Potassium	2.6
Magnesium	2.1
All others	1.5

Element	Symbol	Atomic Number	Percent in Universe	Percent in Earth	Percent in Human Body
Hydrogen	H	1	91	0.14	9.5
Helium	He	2	9	Trace	Trace
Carbon	C	6	0.02	0.03	18.5
Nitrogen	N	7	0.04	Trace	3.3
Oxygen	O	8	0.06	47	65
Sodium	Na	11	Trace	2.8	0.2
Magnesium	Mg	12	Trace	2.1	0.1
Phosphorus	P	15	Trace	0.07	1
Sulfur	S	16	Trace	0.03	0.3
Chlorine	Cl	17	Trace	0.01	0.2
Potassium	K	19	Trace	2.6	0.4
Calcium	Ca	20	Trace	3.6	1.5
Iron	Fe	26	Trace	5	Trace

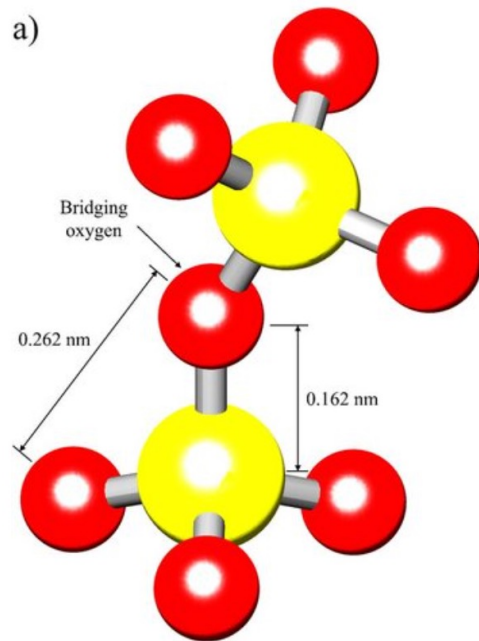
Silicate perovskite( $\text{Mg,FeSiO}_3$ ) may form up to 93% of the lower mantle  
 Magnesium iron silicate perovskite is considered to be the most abundant mineral in the Earth, making up 38% of its volume

# Why silicon is a king?

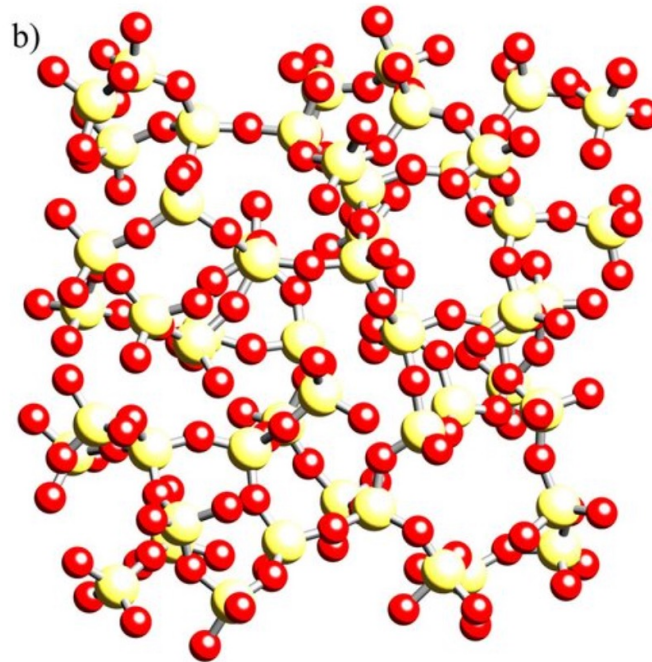
- semiconductor Si,  $E_g = 1.1\text{eV}$
- insulating properties of  $\text{SiO}_2$
- good Si/ $\text{SiO}_2$  interface can be created
- easy Si/ $\text{SiO}_2$  transformations
- oxygen and silicon make up about 75% of the earth's crust  
Si is the second most abundant element on earth after oxygen - cheap!



# Why silicon is a king? Role of Silica



bridging oxygen between two neighbouring tetrahedra



Amorphous silica: random arrangement of tetrahedra

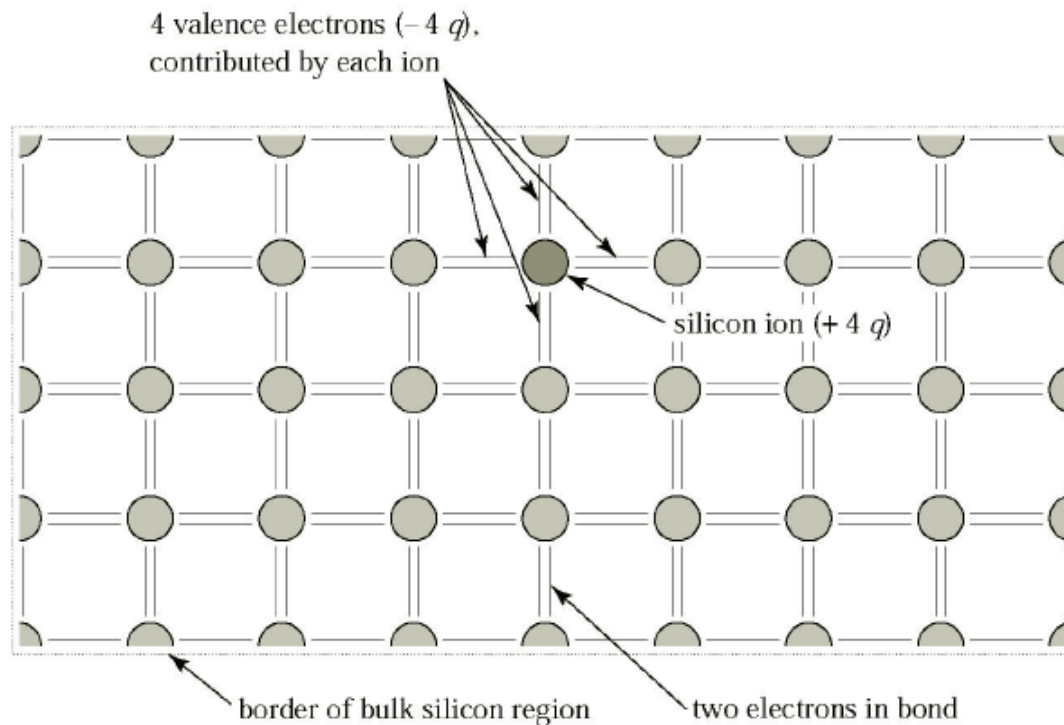
## Amorphous $\text{SiO}_2$ :

- Good insulator,  $E_g > 9\text{eV}$ , high breakdown strength,  $\epsilon=3.9$
- Stable Si/SiO<sub>2</sub> interface with very few defects
- SiO<sub>2</sub> is easily grown thermally or deposited by other techniques

Crystalline silica: can be in the form of quartz (important, but not discussed here)



# Mobile charge carriers in Si: $T=0\text{K}$ – virtually no free charges

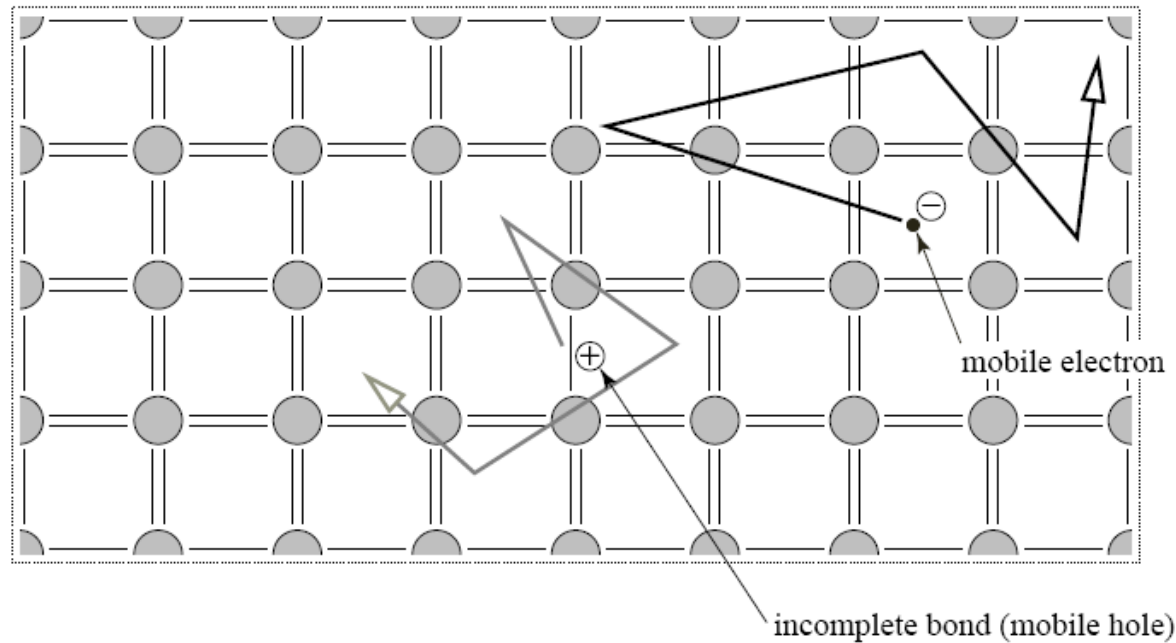


All bonds are satisfied

⇒ all valence electrons engaged in bonding

- Almost no “free” electrons (some defects are there because of entropy considerations)

At  $T = \text{finite} > 0\text{K}$



- finite thermal energy
- some bonds are broken
- **"free" electrons**  $\rightarrow$  mobile negative charge,  $-1.6 \times 10^{-19} \text{ C}$
- **"free" holes**  $\rightarrow$  mobile positive charge,  $1.6 \times 10^{-19} \text{ C}$

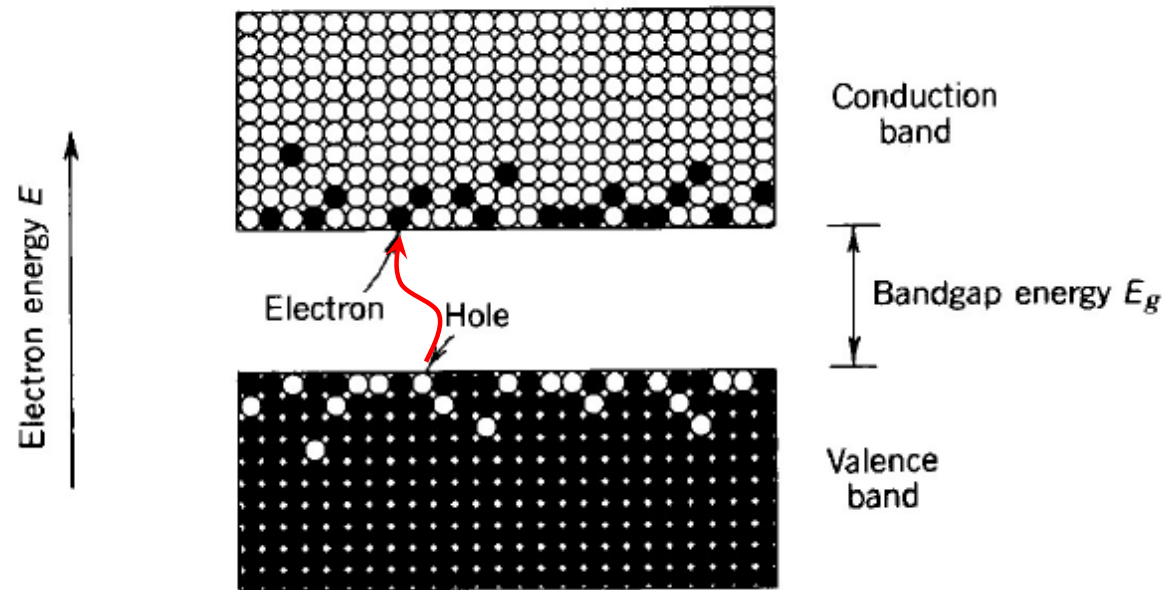
## Important for this course

- electron means free electron
- Not concerned with bonding electrons or core electrons
- Define:

$n \equiv$  (free) electron concentration [ $\text{cm}^{-3}$ ]

$p \equiv$  hole concentration [ $\text{cm}^{-3}$ ]

## Si and other semiconductors: Energy Bands and Charge Carriers



- At finite temperatures, electrons are promoted from the valence band to the conduction band resulting in electrons in the conduction band and holes in the valence band
- Motion of electrons in the conduction band (CB) and holes in the valence band (VB) at  $T > 0$  K generates electrical current

# Generation and Recombination (1)

**Generation** = break up of covalent bond to form electron and hole

- requires energy from thermal or optical sources (or other external sources)
- generation rate (thermal, optical,...):

$$G = G_{th} + G_{opt} + ... [\text{cm}^{-3} \cdot \text{s}^{-1}]$$

- in general, atomic density  $\gg n, p$ 
  - supply of breakable bonds virtually inexhaustible

## Generation and Recombination (2)

Recombination = formation of bond by bringing together electron and hole

- releases energy in thermal or optical form
- recombination rate:  $\mathbf{R \text{ [cm}^{-3} \cdot \text{s}^{-1}]}$
- a recombination event requires 1 electron + 1 hole:

$$\mathbf{R \sim n \cdot p}$$

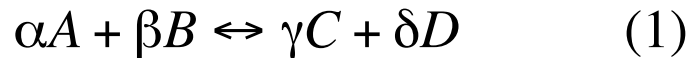
- Generation and recombination most likely at surfaces where periodic crystalline structure is broken: surface recombination velocity

**Thermal equilibrium: a balance between the generation and recombination processes**

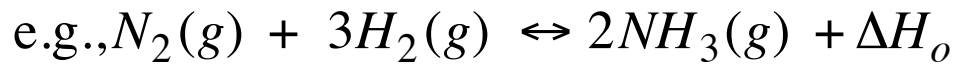
# Thermal equilibrium: Law of Mass action (like for chemical reactions)

It is possible to consider formation of electron/hole pair like a chemical reaction

**For a general chemical reaction**



Law of Mass action for defects



**The equilibrium constant is given by**

$$K = \frac{[C]^\gamma [D]^\delta}{[A]^\alpha [B]^\beta}, \text{ eg. } K_{eq} = \frac{[NH_3]^2}{[N_2][H_2]^3} \quad (2)$$

The concentrations  $[A]$ ,  $[B]$ ,  $[C]$ ,  $[D]$  represent activities (or conc., or partial pressure) of different species, and exponents are their stoichiometric coefficients, respectively  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ .

Equation (2) is known as the »*Law of mass action*» for the chemical reaction (1).

- **The balance between the reagents and products is determined by the equilibrium constant**
- **The equilibrium constant depends on the temperature only**
- **The balance between electrons and holes is governed by the same law**

## Thermal equilibrium

**Thermal equilibrium = steady state + absence of external energy sources.**

- Generation rate in thermal equilibrium (suffix **0** means equilibrium):

**$G_0 = f(T)$**       R (recombination)= formation of bond by bringing together electron and hole

- Recombination rate in thermal equilibrium:  **$R_0 \sim n_0 \cdot p_0$**

In thermal equilibrium (absence of external energy sources):

$$G_0 = R_0 \rightarrow \boxed{n_0 p_0 = n_i^2 (T)} \quad (\text{suffix } 0 \text{ means thermal equilibrium})$$

Important consequence:

In thermal equilibrium and for a given semiconductor,  
 $n \times p$  product is a constant that depends only on temperature!

## Intrinsic semiconductor

**Question:** In a perfectly pure semiconductor in thermal equilibrium at finite temperature, **how many electrons and holes** are there?

In a **sufficiently** pure Si wafer at 300K, called “***intrinsic***” semiconductor:

Enough?

$$n_0 = p_0$$

$$n_0 \times p_0 = n_i^2$$

$$n_i = 1 \times 10^{10} \text{ cm}^{-3}$$

$n_i \equiv$  intrinsic carrier concentration [ $\text{cm}^{-3}$ ]

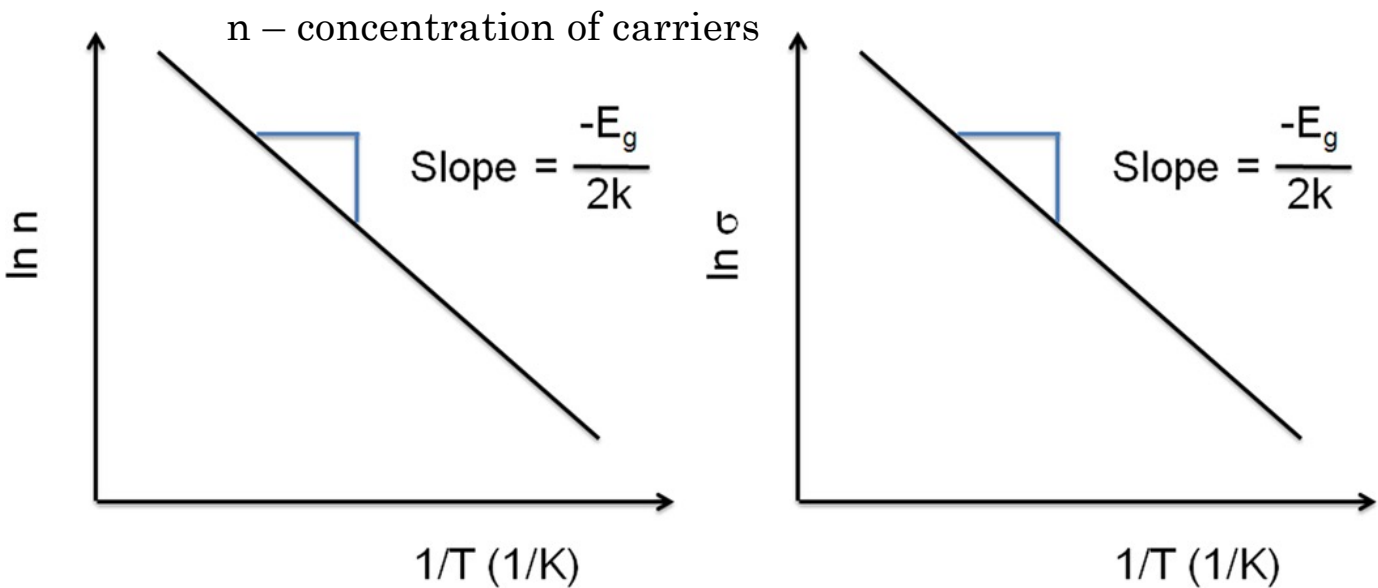
Remark:  $n_i$  is a strong function of temperature (increases when temperature increases)

# Electrical conductivity in intrinsic (undoped) semiconductors vs. T

$n_i$  is proportional to  $\exp\left(\frac{-E_g}{2KT}\right)$

(Boltzmann approximation), more details in L2

	Germanium	Silicon	Gallium Arsenide
300 K	$2.02 \times 10^{13}$	$8.72 \times 10^9$	$2.03 \times 10^6$
400 K	$1.38 \times 10^{15}$	$4.52 \times 10^{12}$	$5.98 \times 10^9$
500 K	$1.91 \times 10^{16}$	$2.16 \times 10^{14}$	$7.98 \times 10^{11}$
600 K	$1.18 \times 10^{17}$	$3.07 \times 10^{15}$	$2.22 \times 10^{13}$



Semiconductor	InSb	Ge	Si	GaAs	GaP	ZnSe	Diamond
$E_g \text{ (eV)}$	0.18	0.67	1.12	1.42	2.25	2.7	6.0

# Doping

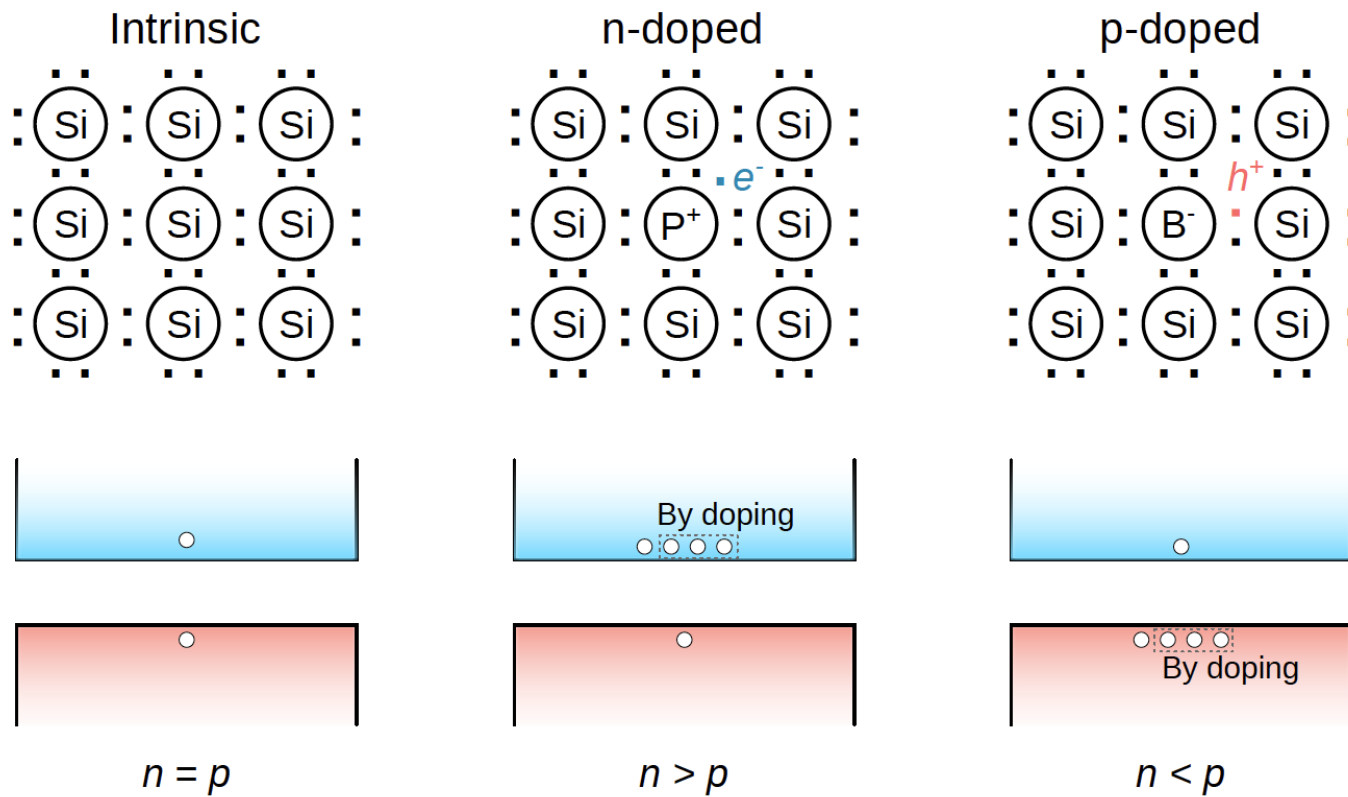
Doping = engineered introduction of **foreign atoms** to modify semiconductor electrical properties

## Donors

- Introduce electrons into the semiconductor (but not holes)
- For Si, group V elements with 5 valence electrons (As, P, Sb)

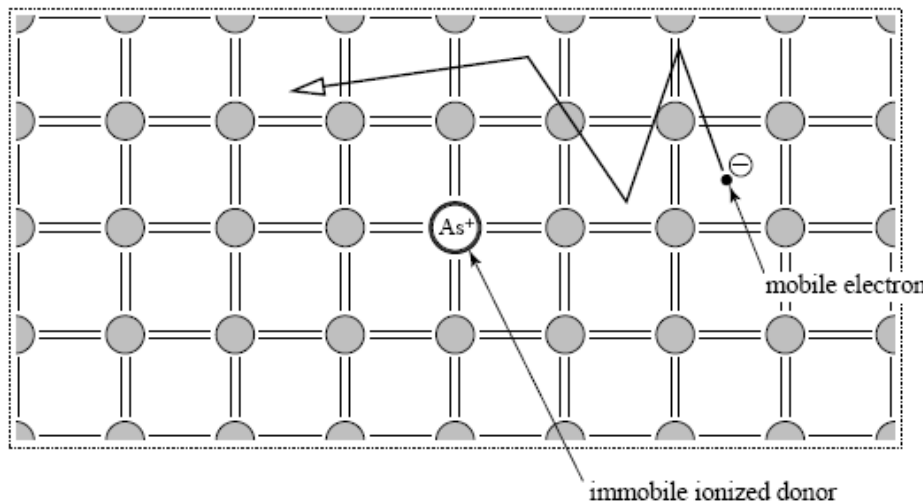
	IIIA	IVA	VA	VIA
	<sup>5</sup> B	<sup>6</sup> C	<sup>7</sup> N	<sup>8</sup> O
	<sup>13</sup> Al	<sup>14</sup> Si	<sup>15</sup> P	<sup>16</sup> S
IIB	<sup>30</sup> Zn	<sup>31</sup> Ga	<sup>32</sup> Ge	<sup>33</sup> As
	<sup>48</sup> Cd	<sup>49</sup> In	<sup>50</sup> Sn	<sup>51</sup> Sb
				<sup>52</sup> Te

## Doping: donors and acceptors



## Donors – how it works?

- 4 electrons of donor atom participate in bonding
- 5th electron easy to release
  - at room temperature, each donor releases 1 electron that is available for conduction
- donor site become positively charged (fixed charge)



$N_d$  = donor concentration [ $cm^{-3}$ ]

- If  $N_d \ll n_i$  doping irrelevant (intrinsic semiconductor) !

$$n_0 = p_0 = n_i$$

## Donors – how it works?

- If  $N_d \gg n_i$  doping controls carrier concentration and the semiconductor is extrinsic

$$n_o = N_d \qquad p_o = \frac{n_i^2}{N_d}$$

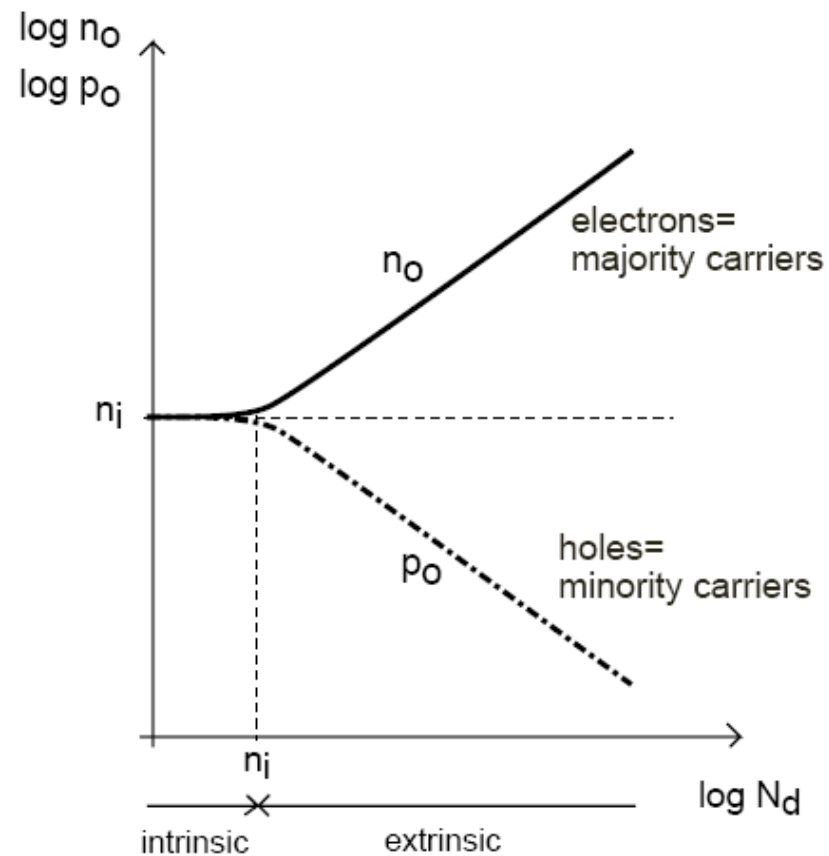
$n_o \gg p_o \rightarrow$  n-type semiconductor

Example:

$$N_d = 10^{17} \text{ cm}^{-3} \rightarrow n_o = 10^{17} \text{ cm}^{-3}, p_o = 10^3 \text{ cm}^{-3}.$$

In general:  $N_d \sim 10^{15} - 10^{20} \text{ cm}^{-3}$

## Donors – majority versus minority carriers



$$n_0 \times p_0 = n_i^2$$

# Doping

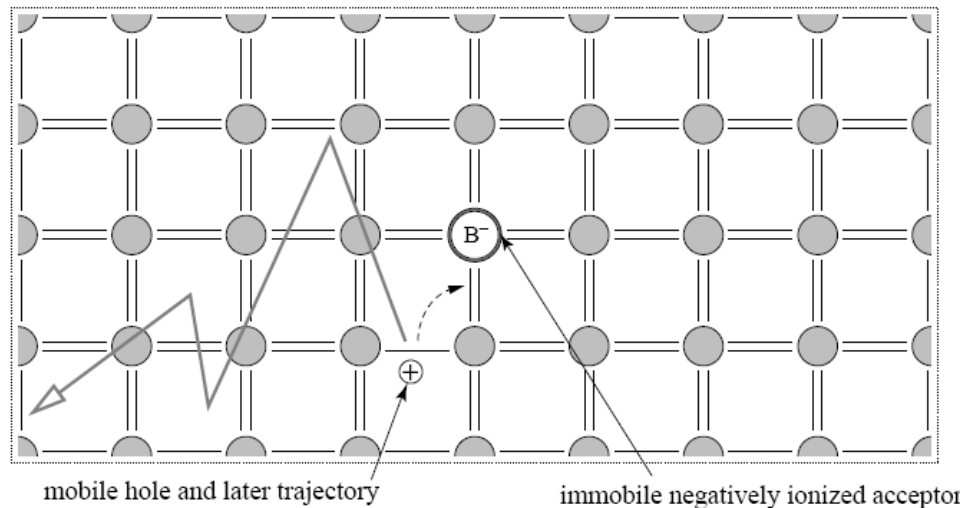
## Acceptors

- Introduce holes into the semiconductor (but not electrons)
- For Si, group-III atoms with 3 valence electrons (B)

	IIIA	IVA	VA	VIA
	5 B	6 C	7 N	8 O
	13 Al	14 Si	15 P	16 S
IIB	30 Zn	31 Ga	32 Ge	33 As
	48 Cd	49 In	50 Sn	51 Sb
			52 Te	

## Acceptors – how it works?

- 3 electrons used in bonding to neighboring Si atoms
- 1 bonding site "unsatisfied":
  - easy to "accept" neighboring bonding electron to complete all bonds
  - at room temperature, each acceptor releases 1 hole that is available to conduction
- acceptor site become negatively charged (fixed charge)



$N_a$  = acceptor concentration [ $\text{cm}^{-3}$ ]

- If  $N_a \ll n_i$  doping irrelevant (intrinsic semiconductor) !

$$n_0 = p_0 = n_i$$

## Acceptors – how it works?

- If  $N_a \gg n_i$  doping controls carrier concentration and the semiconductor is extrinsic

$$p_o = N_a \qquad n_o = \frac{n_i^2}{N_a}$$

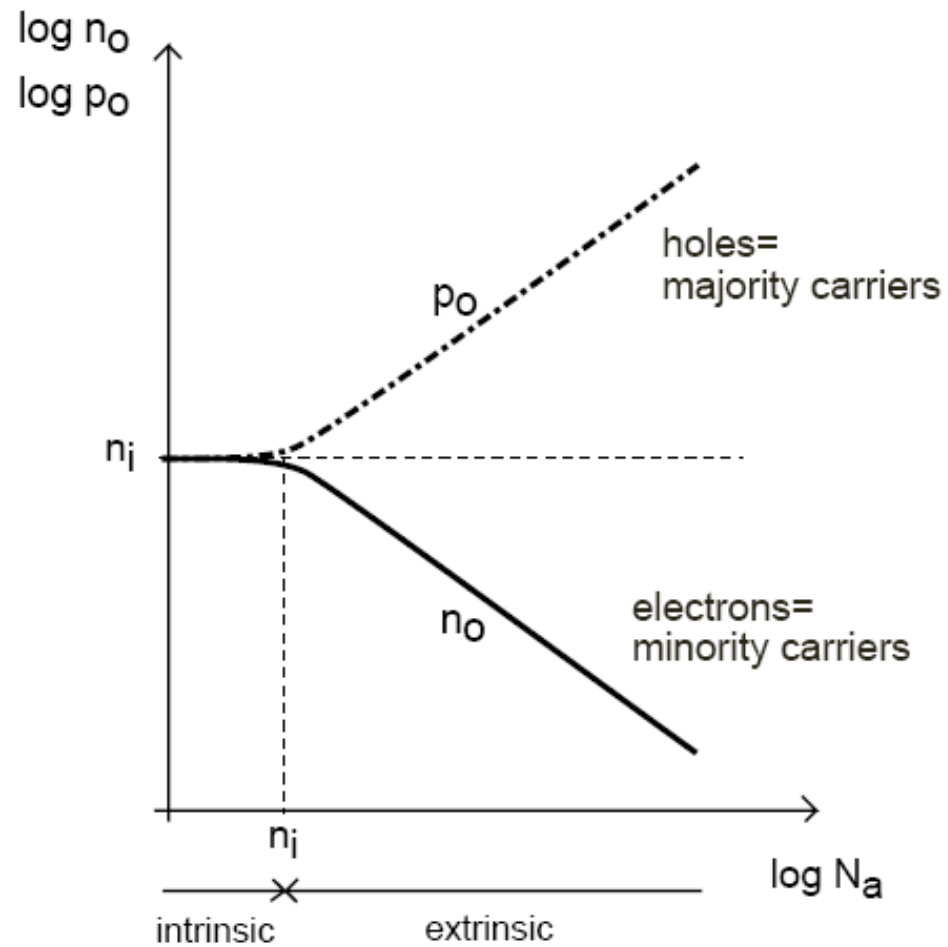
$p_o \gg n_o \rightarrow$  p-type semiconductor

Example:

$$N_a = 10^{16} \text{ cm}^{-3} \rightarrow p_o = 10^{16} \text{ cm}^{-3}, n_o = 10^4 \text{ cm}^{-3}.$$

In general:  $N_a \sim 10^{15} - 10^{20} \text{ cm}^{-3}$

## Acceptors – majority versus minority carriers



## Charge neutrality

- Every single atom in a semiconductor (doped & undoped) is charge neutral

⇒ Overall charge neutrality must be satisfied

- In general (for donor-doped semiconductor):

$$\rho = q(p_o - n_o + N_d - N_a)$$

$$n_o \approx N_d = 10^{17} \text{ cm}^{-3}, \quad p_o \approx \frac{n_i^2}{N_d} = 10^3 \text{ cm}^{-3} \quad \rho \neq 0!$$

**What is wrong?** Nothing, but  $n_o \sim N_d$  is an approximation  
exact relation:  **$p_o - n_o + N_d = 0$**

## Compensation doping

- What happens if a semiconductor contains both donors and acceptors?
  - Compensational doping: a p-type semiconductor is converted to n-type by adding  $N_d > N_a$
  - The electron concentration is  $N_d - N_a$  (provided the latter is  $> n_i$ )
  - What happens: the electrons from donors recombine with the holes from acceptors, so that the mass action law is respected
  - one cannot simultaneously increase concentration of electrons and holes (recombination rate increases,  $n \times p = n_i^2$  maintained)
  - More donors  $N_d - N_a \gg n_i$ ,  $n = N_d - N_a$ ,  $p = n_i^2 / (N_d - N_a)$
  - More acceptors  $N_a - N_d \gg n_i$ ,  $p = N_a - N_d$ ,  $n = n_i^2 / (N_a - N_d)$
  - This logic is valid for nearly completely ionized dopants (this is the case for typical dopants at the room temperature. At low temperatures, we have to consider donor/acceptor statistics and charge neutrality)

## Intermediate Conclusions (1)

- In a semiconductor, there are two types of "carriers":  
**electrons and holes**
- In thermal equilibrium and for a given semiconductor  $n_0 \times p_0$  is a constant that only depends on temperature:

$$n_0 \times p_0 = n_i^2$$

- For Si at room temperature:

$$n_i = 1 \times 10^{10} \text{ cm}^{-3}$$

- Intrinsic semiconductor = "pure" semiconductor

$$n_0 = p_0$$

## Conclusions (2)

- Carrier concentrations can be engineered by addition of "dopants" (selected foreign atoms):

- n-type semiconductor:

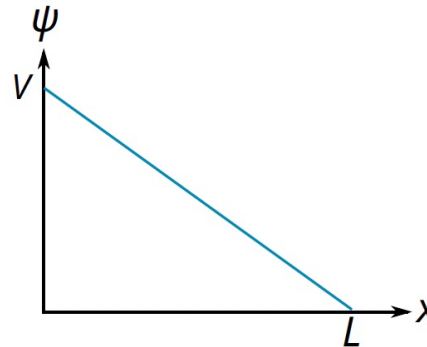
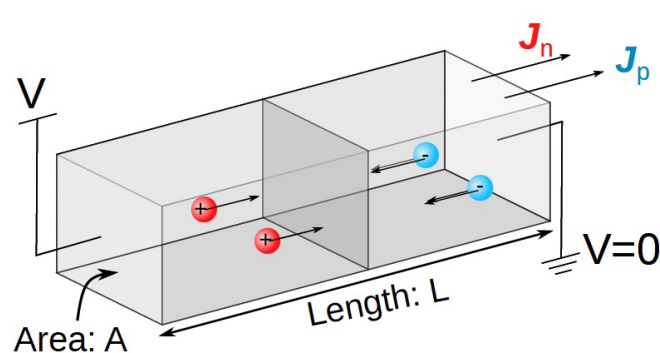
$$n_o = N_d \qquad p_o = \frac{n_i^2}{N_d}$$

- p-type semiconductor:

$$p_o = N_a \qquad n_o = \frac{n_i^2}{N_a}$$

# Electrical conduction: Drude model

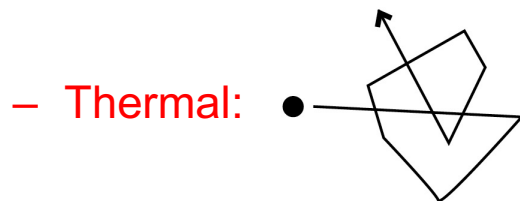
- a widely used classical model of charge transport
  - Works well for describing conductivity in semiconductors



Constant voltage  $V$

Uniform electric field  $E = V/L$

- Types of movement:



stochastic, time between scattering events  $\tau_c$ ,  $\lambda = v_{th}\tau_c$   
mean free path  $\lambda$  depends on the thermal velocity

- Drift: movement under the external electric field

## Electrical conduction: Drude model - 2



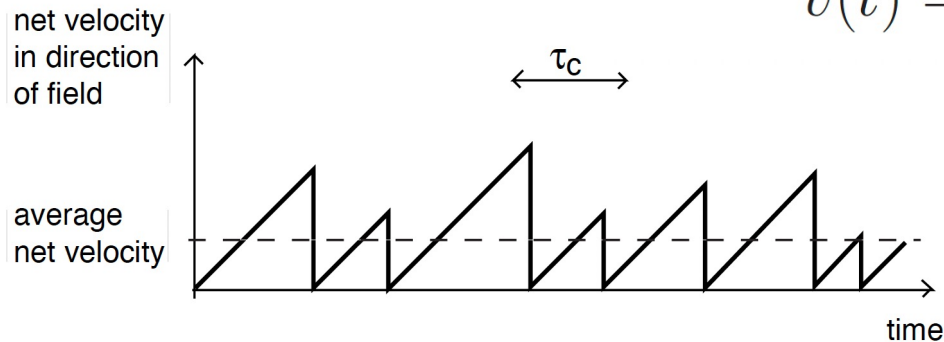
$E \equiv \text{electric field [V/cm]}$

$F = \pm qE$       *The net force acting on charge carrier*



$v(t) = at = -\frac{qE}{m_n}t$       *For electrons*

$v(t) = \frac{qE}{m_p}t$       *For holes (holes also have effective mass)*



*Drift velocity:*

$$\bar{v} = v_d = \pm \frac{qE}{2m_{n,p}}\tau_c = \pm \frac{q\tau_c}{2m_{n,p}}E$$

$$v_{dn} = -\mu_n E$$

$$v_{dp} = \mu_p E$$

$$\mu_{n,p} = \frac{q\tau_c}{2m_{n,p}} \equiv \text{mobility [cm}^2/\text{V} \cdot \text{s]}$$

## Electrical conduction: Drude model - 3

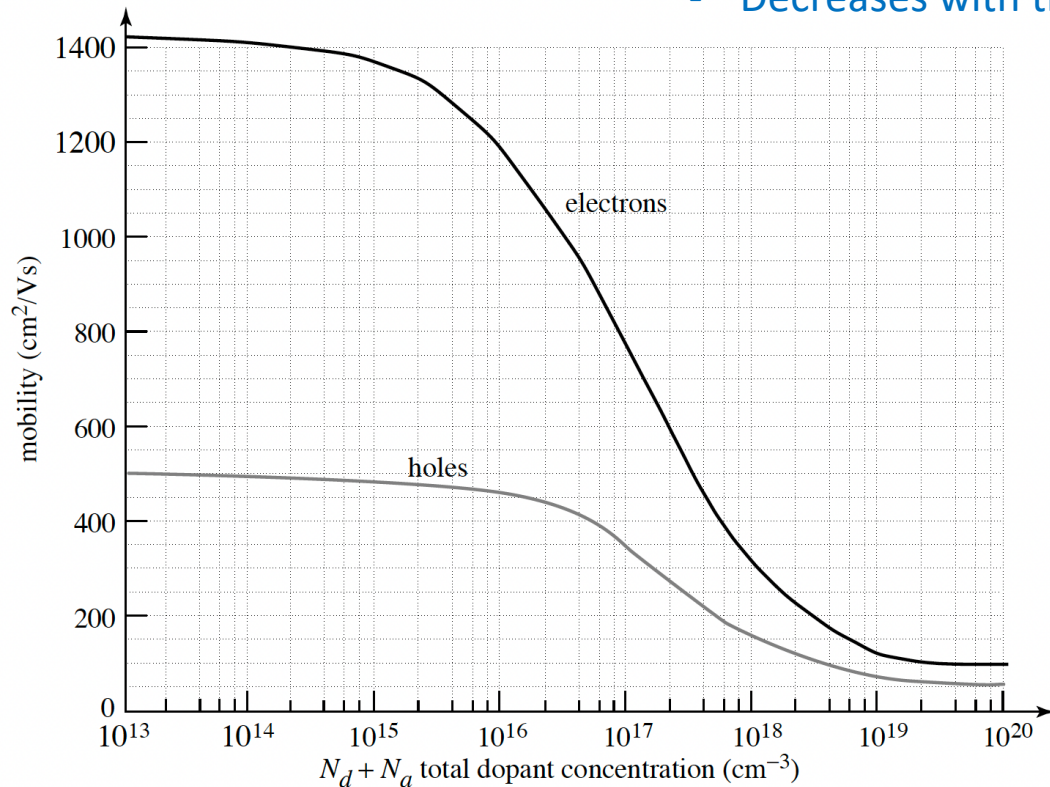
$$v_{dn} = -\mu_n E$$

$$v_{dp} = \mu_p E$$

$$\mu_{n,p} = \frac{q\tau_c}{2m_{n,p}} \equiv \text{mobility} [cm^2/V \cdot s]$$

Mobility is measure of *ease* of carrier drift

- Increases with longer time between collisions
- Decreases with the higher effective mass



### Mobility for Si at 300K

mobility depends on doping

Holes are “heavier” than electrons

$$\tau_c \simeq 10^{-14} \sim 10^{-13} \text{ s} \ll 1 \text{ ps}$$

$$v_{th} \simeq 10^7 \text{ cm/s}$$

$$\Rightarrow \lambda \simeq 1 \sim 10 \text{ nm}$$

## Electrical conduction: Drude model - 4

*Drift current density:*

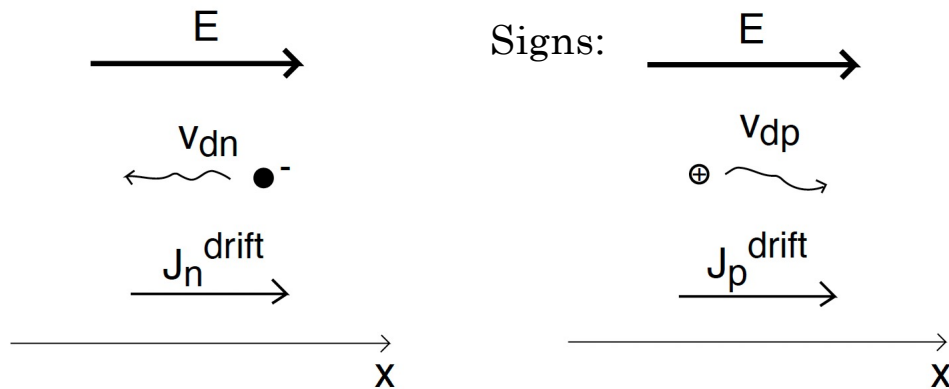
$\propto$  carrier drift velocity

$\propto$  carrier concentration

$\propto$  carrier charge

$$J_n^{drift} = -qn v_{dn} = qn \mu_n E$$

$$J_p^{drift} = qp v_{dp} = qp \mu_p E$$



$$J^{drift} = J_n^{drift} + J_p^{drift} = q(n\mu_n + p\mu_p)E$$

**Ohm's law:**

$$J = \sigma E = \frac{E}{\rho}$$

$$\sigma \equiv \text{conductivity} [\Omega^{-1} \cdot \text{cm}^{-1}]$$

$$\rho \equiv \text{resistivity} [\Omega \cdot \text{cm}]$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

# Electrical conductivity – simple relations, numbers

Ohm's law:  $\mathbf{J} = \sigma \mathbf{E}$  where  $\mathbf{J}$  is the current density,  $\mathbf{E}$  is the electric field and  $\sigma$  is the conductivity. In microscopic terms,

$\sigma = nq\mu$  where  $\mu$  is mobility of charge carriers.

$$\bar{v} = v_d = \pm \frac{qE}{2m_{n,p}} \tau_c = \pm \frac{q\tau_c}{2m_{n,p}} E \quad \mu_{n,p} = \frac{q\tau_c}{2m_{n,p}} \equiv \text{mobility} [cm^2/V \cdot s]$$

**Si at 300K: what is the mean free path?**

$$\tau_c \simeq 10^{-14} \sim 10^{-13} \text{ s} \ll 1\text{ps}$$

$$v_{th} \simeq 10^7 \text{ cm/s}$$

$$\Rightarrow \lambda \simeq 1 \sim 10 \text{ nm}$$

Electron mobility at 300K:

Si (300K) – about 1000 cm<sup>2</sup>/V s

Metals like Cu, Ag – 30-50 cm<sup>2</sup>/V s

GaAs - 10000 cm<sup>2</sup>/V s

Ionic conducton - <1 cm<sup>2</sup>/V s

GaAs/AlGaAs 2D gas:

Mobility below 4K can be 10<sup>6</sup>-10<sup>7</sup> cm<sup>2</sup>/V s

# Electrical conductivity – limitations of the classic theory (Drude model)

- Drude model is a useful and important tool for understanding/describing charge transport, however, it has limitations
- Electrons are considered classic particles (like a gas), and their wave nature is not taken into account – problems at short distances
- Ballistic transport (small number of scattering events) is described differently
- trans
- Transport through barriers e.g. tunneling is described differently
- Transport at high electric fields may not be accurately described with Drude theory
- The concept of electron/hole mobility is widely used in semiconductor device physics, very often, the mobility is considered as an empirical parameter (without any relation with electron/hole mass)