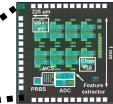
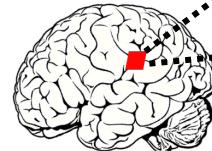


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## Analog IC design (EE-320), Lecture 5

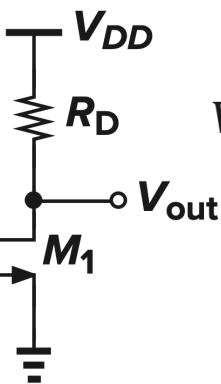
Prof. Mahsa Shoaran

Institute of Electrical and Micro Engineering, School of Engineering, EPFL

# EDADK Statement

- **Assignment for EDADK document**, to be signed and uploaded via moodle, please complete the assignment **by Nov 6**
- **First TP session on Nov 11 in BC07-08** from **3:15pm to 6pm**
- **Homework 1 assignment** will be posted after lecture 6
- **Homework2 assignment** will be posted after TP3

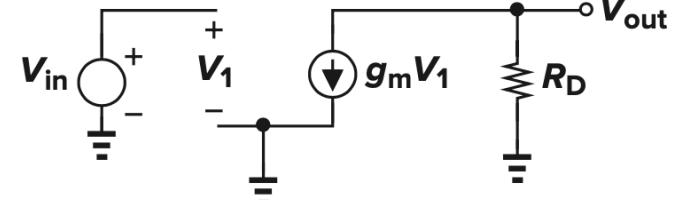
# Review: Common-Source Amplifier



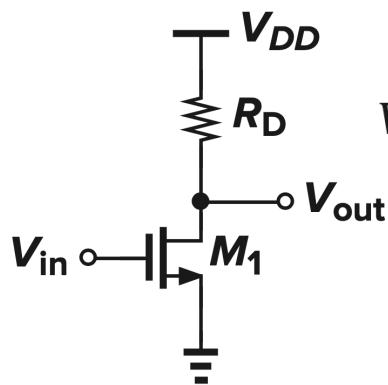
*large-signal analysis:*

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

*small-signal analysis:*



# Review: Common-Source Amplifier



**large-signal analysis:**

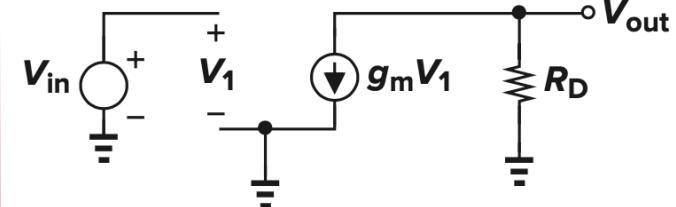
$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}}$$

$$= -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})$$

$$= -g_m R_D$$

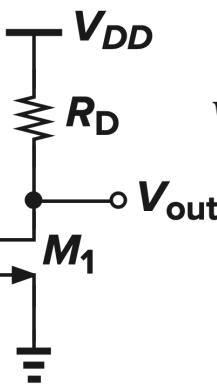
**small-signal analysis:**



$$V_{out} = -g_m V_1 R_D = -g_m V_{in} R_D$$

$$A_v = -g_m R_D$$

# Review: Common-Source Amplifier



**large-signal analysis:**

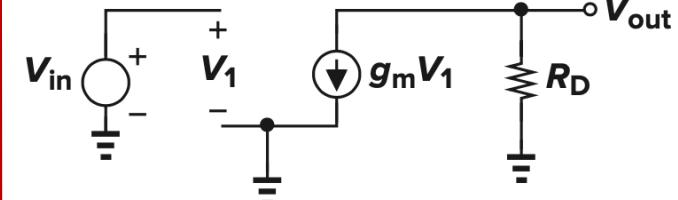
$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}}$$

$$= -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})$$

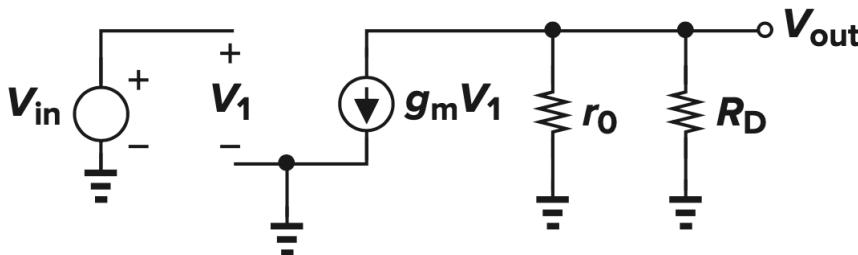
$$= -g_m R_D$$

**small-signal analysis:**



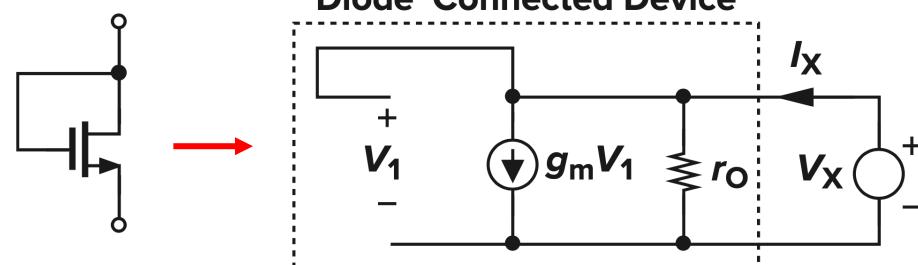
$$V_{out} = -g_m V_1 R_D = -g_m V_{in} R_D$$

$$A_v = -g_m R_D$$



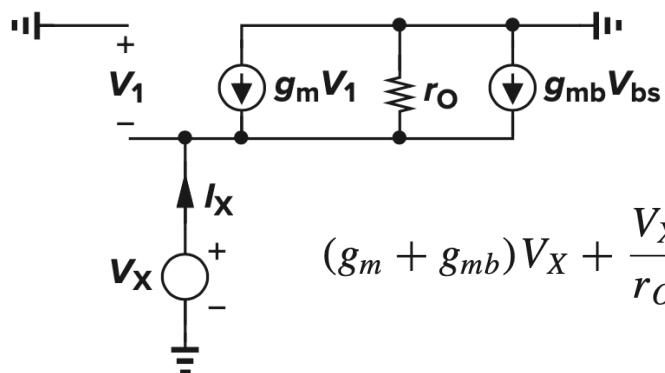
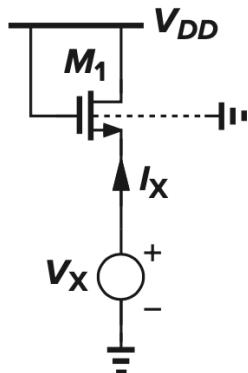
$$g_m V_1 (r_o \parallel R_D) = -V_{out}$$

$$V_{out} / V_{in} = -g_m (r_o \parallel R_D)$$



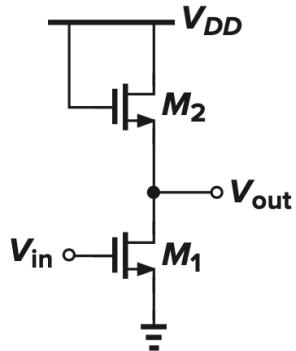
$$V_X / I_X = (1/g_m) \parallel r_o \approx 1/g_m$$

# Review: Common-Source Amplifier

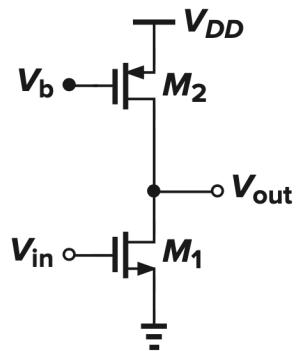


$$(g_m + g_{mb})V_X + \frac{V_X}{r_O} = I_X \quad \rightarrow$$

$$\begin{aligned} \frac{V_X}{I_X} &= \frac{1}{g_m + g_{mb} + r_O^{-1}} \\ &= \frac{1}{g_m + g_{mb}} \| r_O \\ &\approx \frac{1}{g_m + g_{mb}} \end{aligned}$$

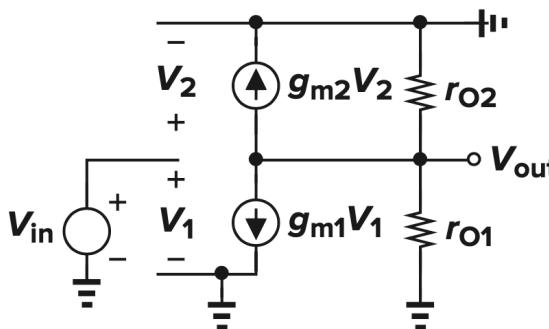
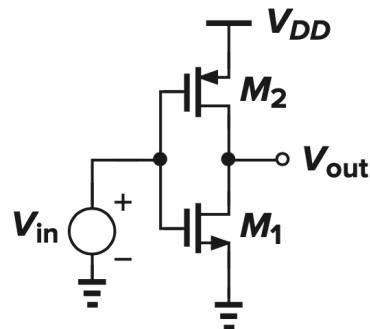


$$\begin{aligned} &= -g_{m1} \frac{1}{g_{m2} + g_{mb2}} \\ &= -\frac{g_{m1}}{g_{m2}} \frac{1}{1 + \eta} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1 + \eta} \end{aligned}$$

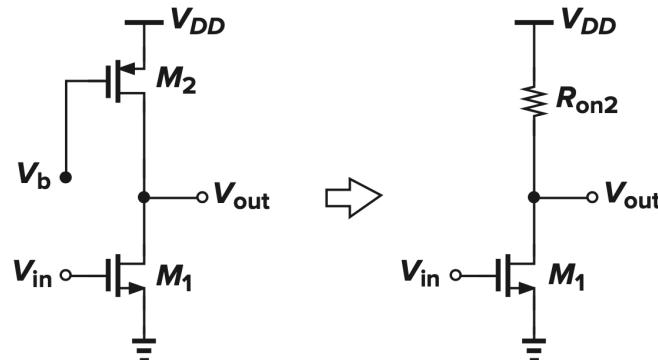
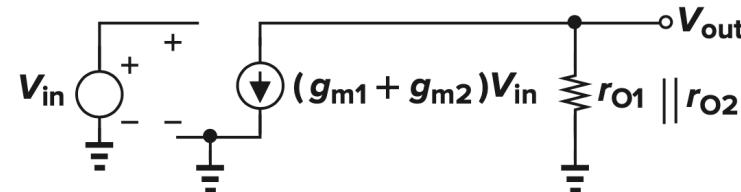


$$\begin{aligned} &\rightarrow \quad A_v = -g_{m1}(r_{O1} \| r_{O2}) \\ &|V_{DS2,min}| = |V_{GS2} - V_{TH2}| \end{aligned}$$

# Review: Common-Source Amplifier



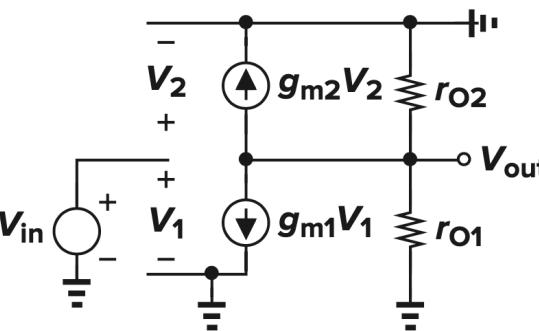
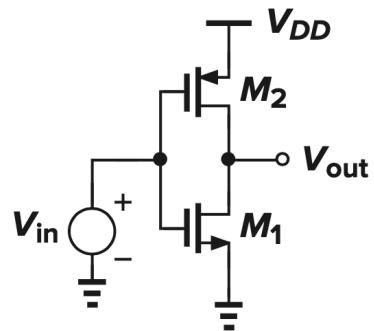
$$A_v = -(g_{m1} + g_{m2})(r_{O1} || r_{O2})$$



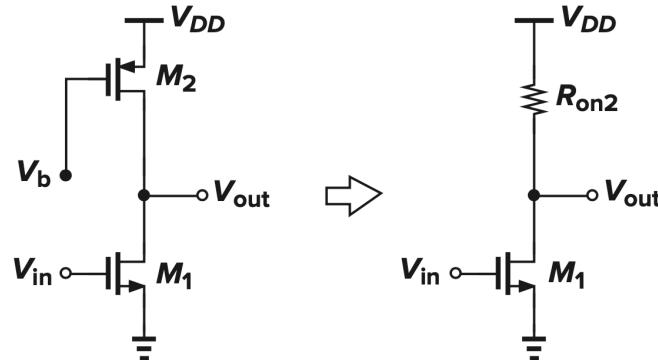
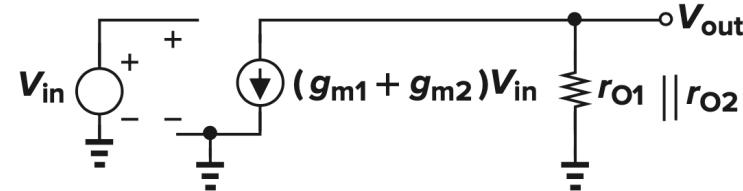
$$A_v = -g_{m1} R_{on2}$$

$$R_{on2} = \frac{1}{\mu_p C_{ox} (W/L)_2 (V_{DD} - V_b - |V_{THP}|)}$$

# Review: Common-Source Amplifier



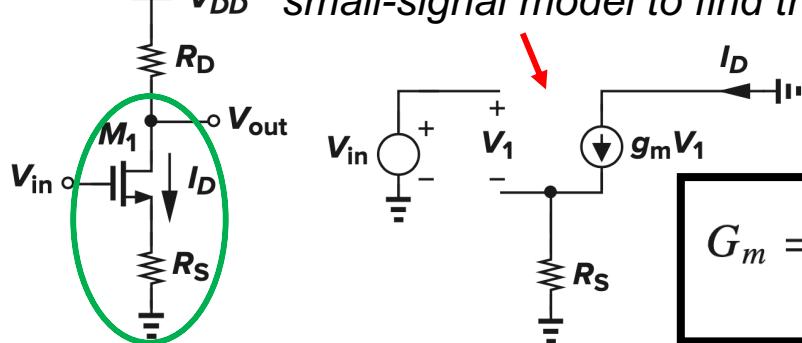
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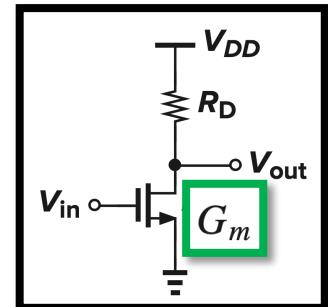
$$R_{on2} = \frac{1}{\mu_p C_{ox} (W/L)_2 (V_{DD} - V_b - |V_{THP}|)}$$

$V_{DD}$  small-signal model to find the **equivalent  $G_m$**  of a source-degenerated CS



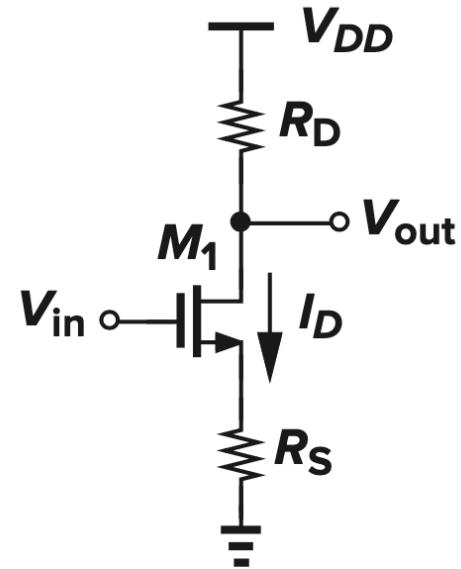
$$G_m = \frac{g_m}{1 + g_m R_S}$$

$$A_v = -G_m R_D \rightarrow = \frac{-g_m R_D}{1 + g_m R_S}$$



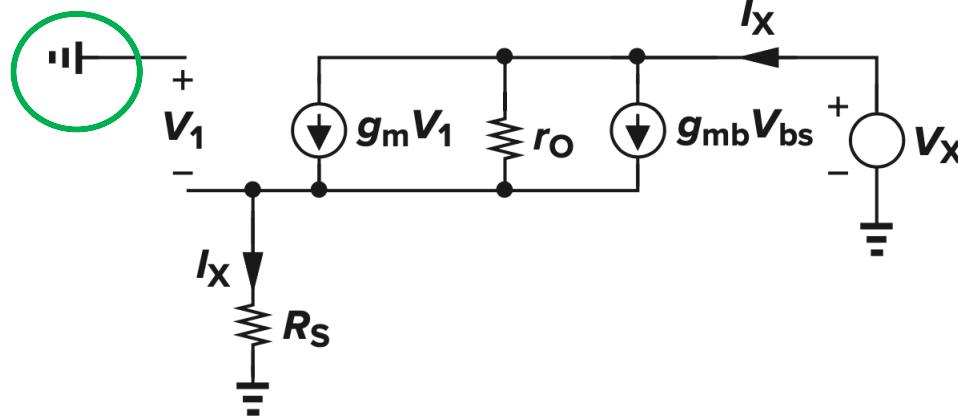
# Output Resistance of a degenerated CS stage

- Source degeneration **increases** the output resistance



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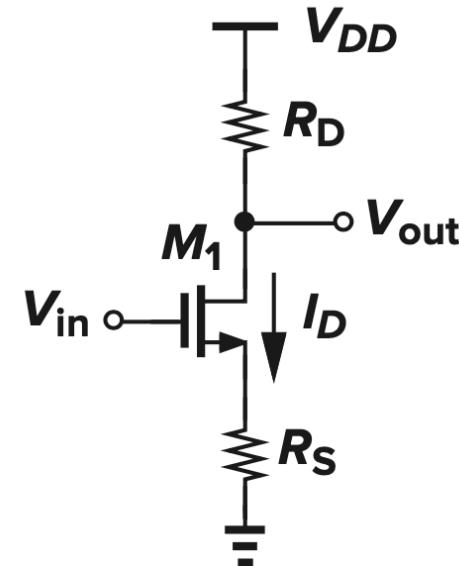
$$r_O[I_X + (g_m + g_{mb})R_S I_X] + I_X R_S = V_X$$

$$R_{out} = [1 + (g_m + g_{mb})R_S]r_O + R_S$$

$$= [1 + (g_m + g_{mb})r_O]R_S + r_O$$

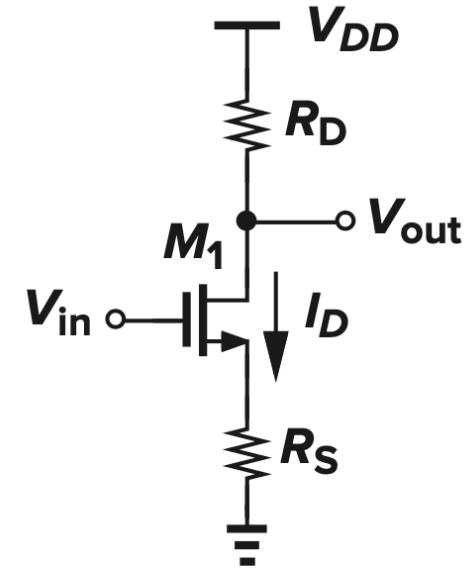
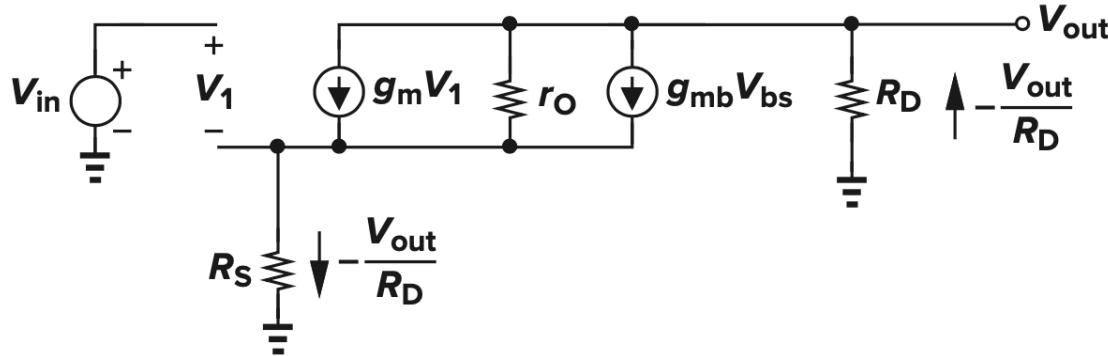
$$\approx (g_m + g_{mb})r_O R_S + r_O$$

$$= [1 + (g_m + g_{mb})R_S]r_O$$

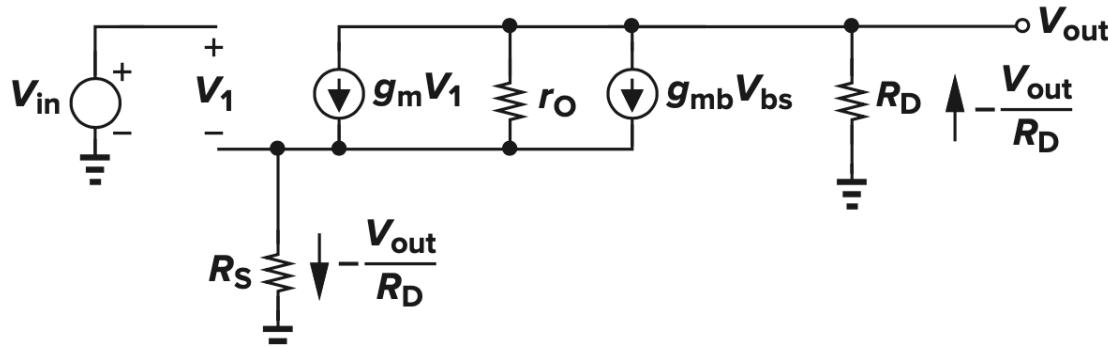


- The **overall output resistance** is the **parallel combination** of  $R_{out}$  and  $R_D$

# Gain of a degenerated CS stage

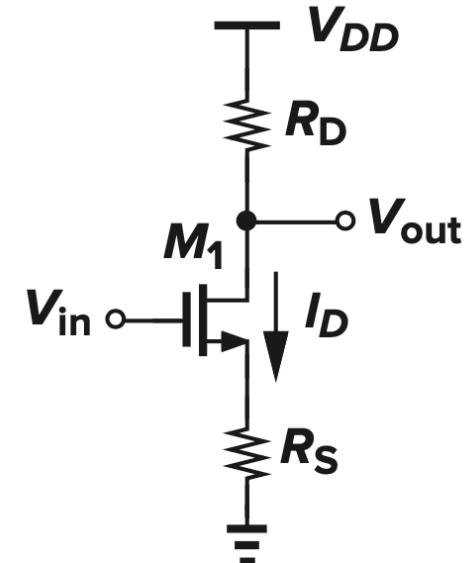


# Gain of a degenerated CS stage



$$\begin{aligned}
 I_{ro} &= -\frac{V_{out}}{R_D} - (g_m V_1 + g_{mb} V_{bs}) \\
 &= -\frac{V_{out}}{R_D} - \left[ g_m \left( V_{in} + V_{out} \frac{R_S}{R_D} \right) + g_{mb} V_{out} \frac{R_S}{R_D} \right]
 \end{aligned}$$

$$\begin{aligned}
 V_{out} &= I_{ro} r_O - \frac{V_{out}}{R_D} R_S \\
 &= -\frac{V_{out}}{R_D} r_O - \left[ g_m \left( V_{in} + V_{out} \frac{R_S}{R_D} \right) + g_{mb} V_{out} \frac{R_S}{R_D} \right] r_O - V_{out} \frac{R_S}{R_D}
 \end{aligned}$$



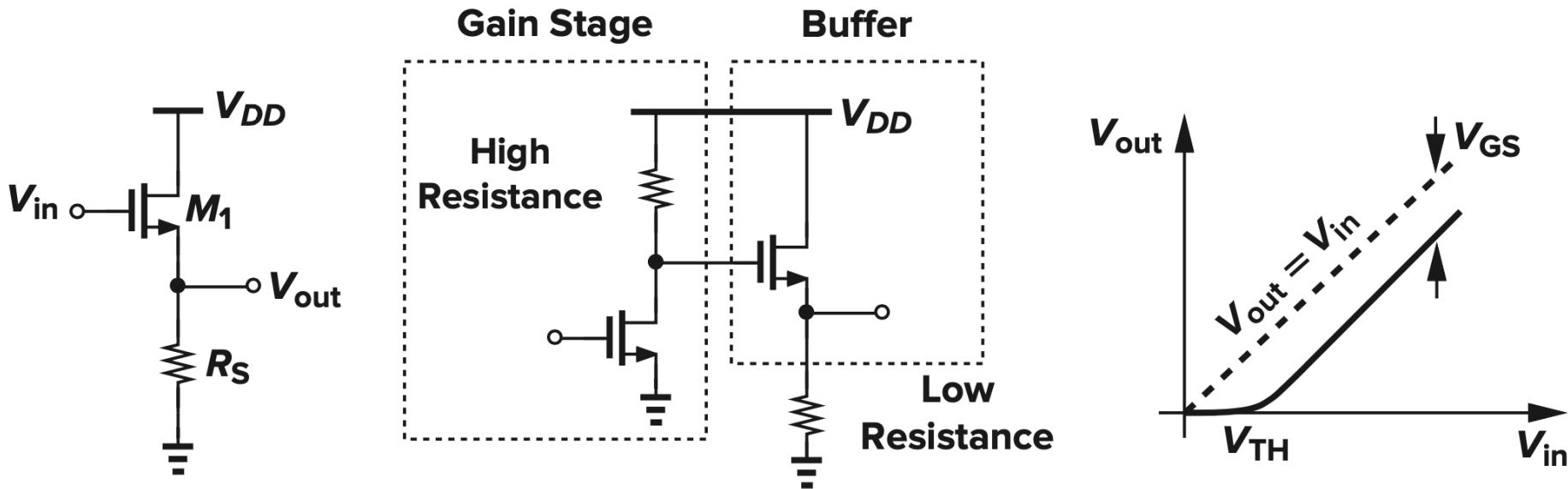
$$\frac{V_{out}}{V_{in}} = \frac{-g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

# Source Follower stage

- To achieve a high voltage gain, the load impedance must be large
- If such a stage is to drive a **low-impedance** load, then a “**buffer**” must be placed after the amplifier to avoid gain reduction
- A **source follower** (or “**common-drain**” stage) can operate as a **voltage buffer**

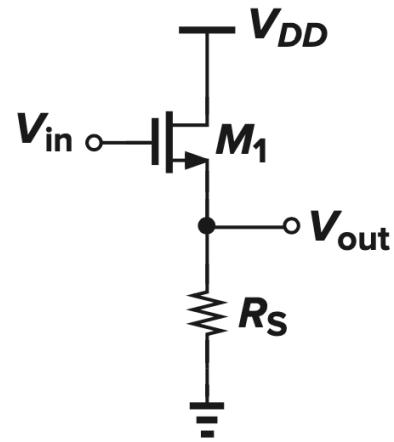
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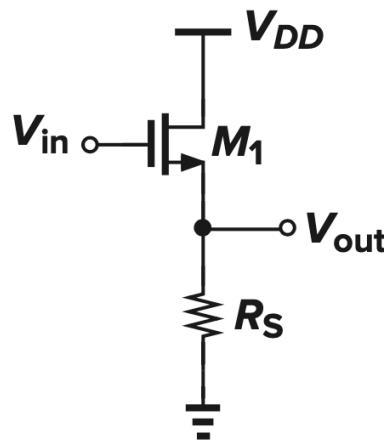


- It senses the signal at the gate, while presenting a **high input impedance**, and drives the load at the source, with a source potential “**following**” the gate voltage

# Source Follower: large signal analysis



# Source Follower: large signal analysis



$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out})^2 R_S = V_{out}$$

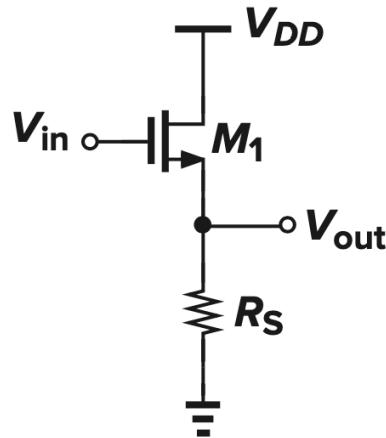
$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_{in} - V_{TH} - V_{out}) \left( 1 - \frac{\partial V_{TH}}{\partial V_{in}} - \frac{\partial V_{out}}{\partial V_{in}} \right) R_S = \frac{\partial V_{out}}{\partial V_{in}}$$

$$\boxed{\frac{\partial V_{TH}}{\partial V_{in}} = (\frac{\partial V_{TH}}{\partial V_{SB}})(\frac{\partial V_{SB}}{\partial V_{in}}) = \eta \frac{\partial V_{out}}{\partial V_{in}}}$$

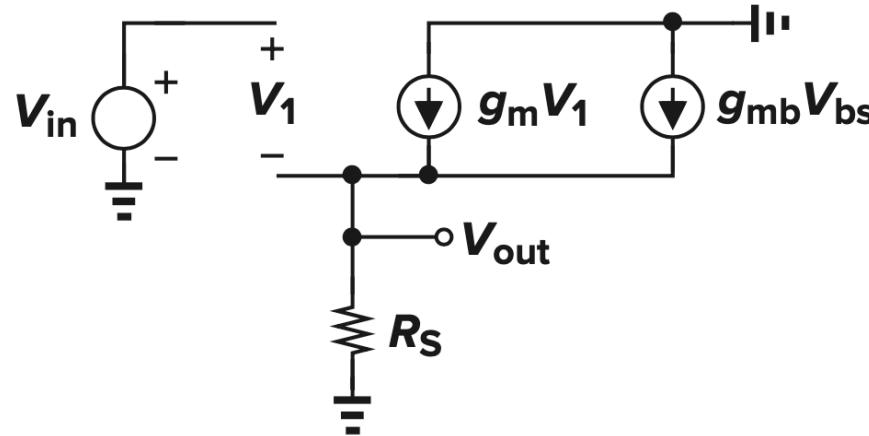
$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out}) R_S}{1 + \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out}) R_S (1 + \eta)}$$

$$\boxed{A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}}$$

# Source Follower: small signal model



Small-signal equivalent of source follower:



$$V_{in} - V_1 = V_{out}$$

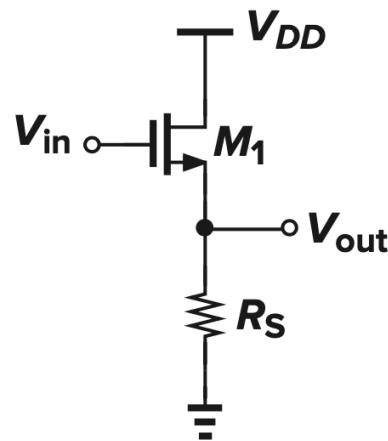
$$V_{bs} = -V_{out}$$

$$\frac{V_{out}}{V_{in}}$$

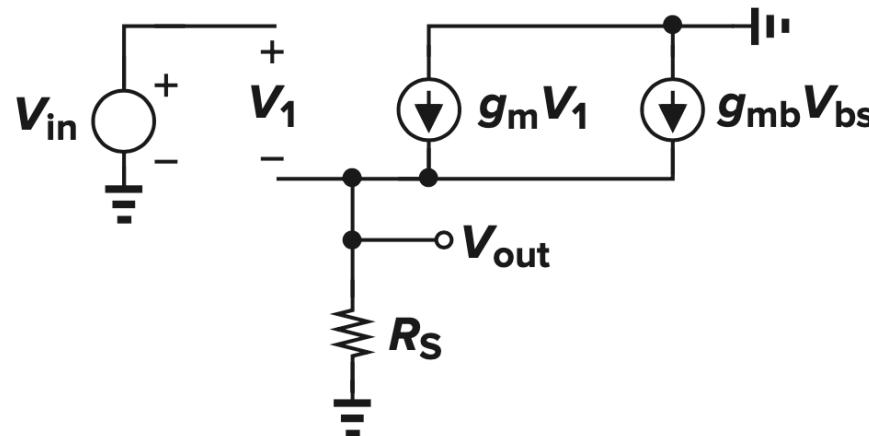
$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$

$$g_m V_1 - g_{mb} V_{out} = V_{out} / R_S$$

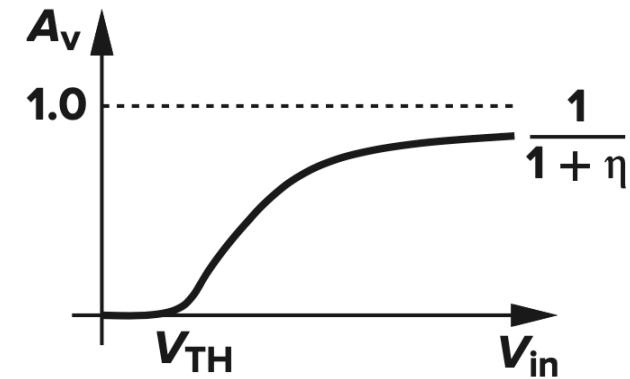
# Source Follower: small signal model



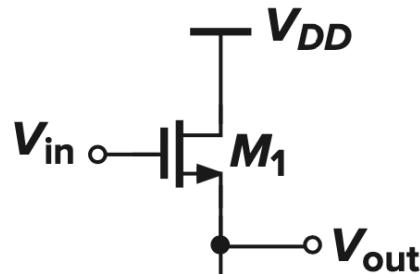
Small-signal equivalent of source follower:



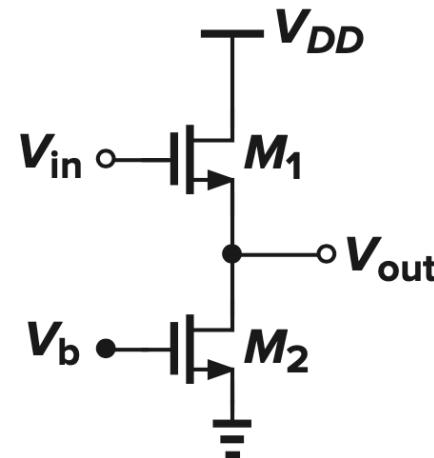
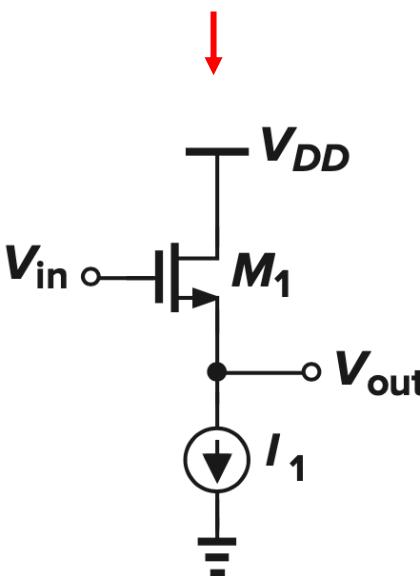
$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$



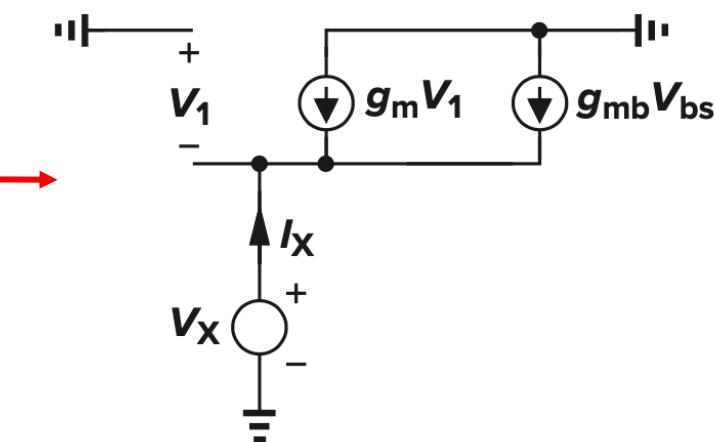
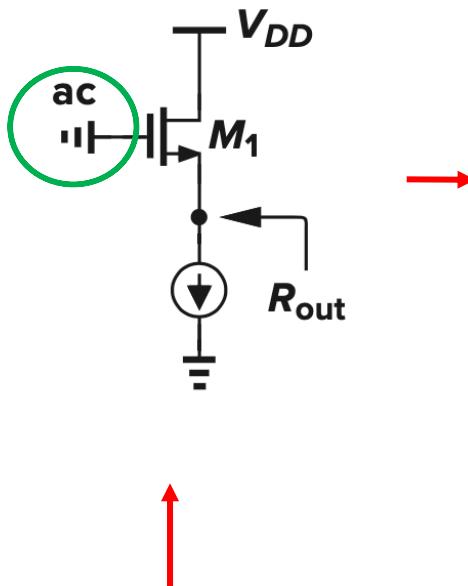
# Source follower with a current source



$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out})^2 R_S = V_{out}$$

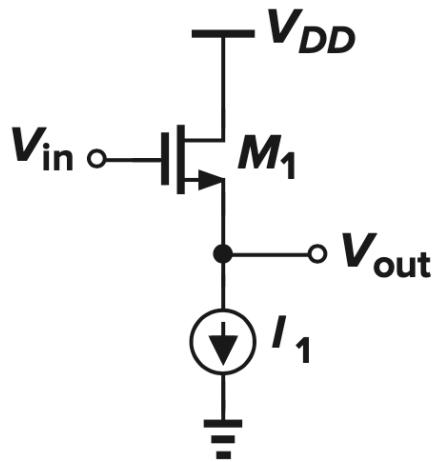


# Output resistance of a Source Follower

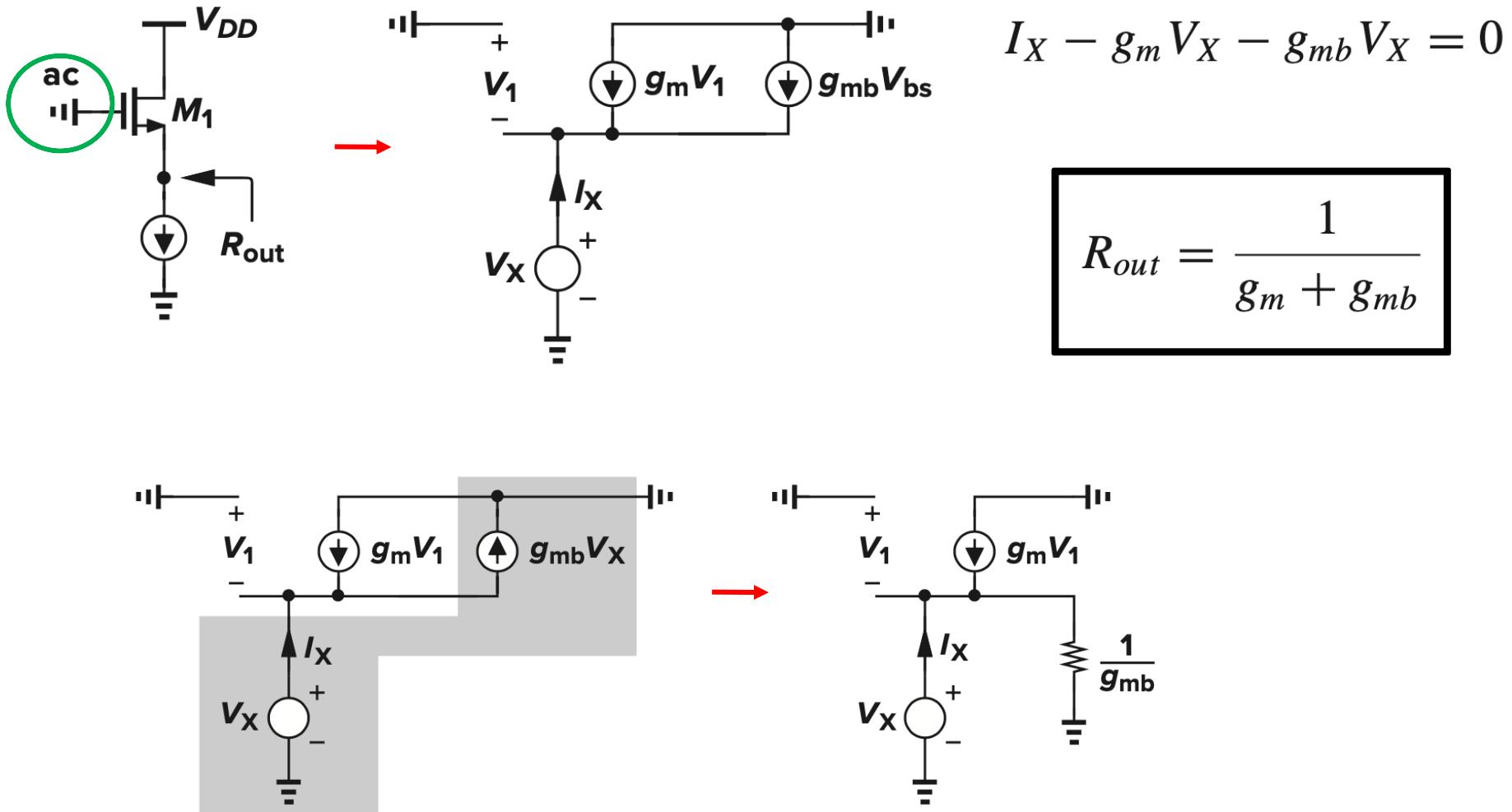


$$I_X - g_m V_X - g_{mb} V_X = 0$$

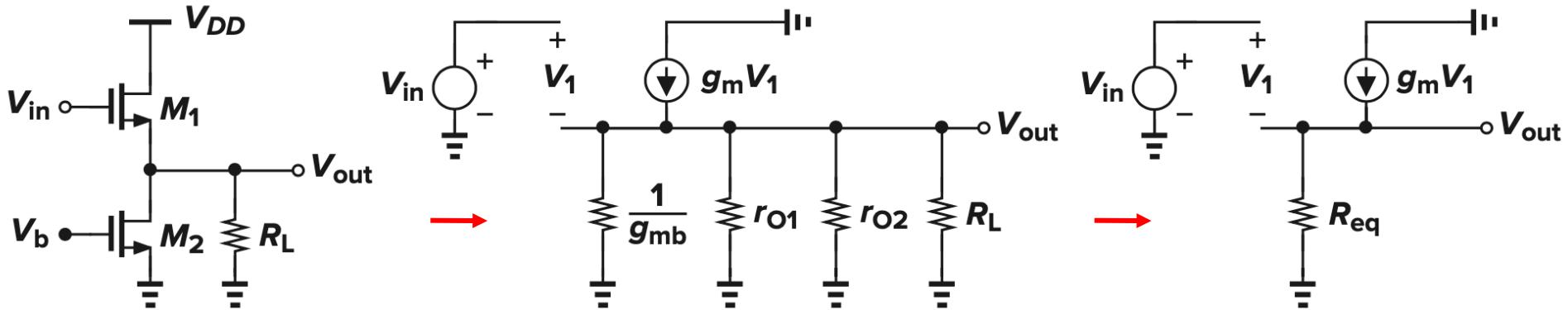
$$R_{out} = \frac{1}{g_m + g_{mb}}$$



# Output resistance of a Source Follower



# Source Follower with a finite load resistance



$$A_v = \frac{R_{eq}}{R_{eq} + \frac{1}{g_m}}$$

$$R_{eq} = (1/g_{mb}) || r_{O1} || r_{O2} || R_L$$