EPFL

**Prof. Y. Shkel**
**Final Exam Signals and Systems - (n/a)**
**22/06/2021**
**Duration: 3h**

n/a

n/a

SCIPER: **999999**

Signature:

**Do not turn the page before the start of the exam. This document is double-sided, has 24 pages, the last ones possibly blank. Do not unstaple.**

- Place your student card on your table.
- For the **multiple choice with unique answer** questions, we give
  - +3 points if your answer is correct,
  - 0 points if your answer is incorrect.
- For the **multiple choice with multiple answers** questions, we give
  - +4 points for all correct answers,
  - +2 points for one incorrect answer and three correct answers,
  - 0 points for other possibilities of answers.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes | Read these guidelines | Beachten Sie bitte die unten stehenden Richtlinien

| choisir une réponse \| select an answer Antwort auswählen | ne PAS choisir une réponse \| NOT select an answer NICHT Antwort auswählen | Corriger une réponse \| Correct an answer Antwort korrigieren |
|---|---|---|

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

## First Part: Multiple Choice Questions

For each question, mark all boxes corresponding to the correct answers. Note that some questions are marked as having multiple correct answers – **check all that apply!** – while others have **a unique correct answer**.

**Question [MCQ-01] :** A continuous-time signal

$$x(t) = e^{j\omega_0 t} + K_0 e^{j(\omega_0 t + \pi)} + K_1 e^{j2\omega_0 t}$$

is sampled with a sampling frequency $\omega_s = 10\omega_0$ to produce a discrete-time representation $x[n] = x(nT)$ where $T$ is the corresponding sampling interval. Then **(check all that apply!)**

- [ ] $x[n] = C$, for some constant $C$
- [ ] $x[n] = (1 + K_0)e^{j\frac{\pi}{5}n} + K_1 e^{j\frac{2\pi}{5}n}$
- [x] $x[n] = (1 - K_0)e^{j\frac{\pi}{5}n} + K_1 e^{j\frac{2\pi}{5}n}$
- [x] $x[n] = (1 - K_0)(\cos(\frac{11\pi}{5}n) + j\sin(\frac{11\pi}{5}n)) + K_1(\cos(\frac{12\pi}{5}n) + j\sin(\frac{12\pi}{5}n))$

**Question [SCQ-02] :** What is the Nyquist rate of the signal

$$x(t) = x_1(t)x_2(t)$$

if $x_1(t)$ has a Nyquist rate of $\omega_0$ and $x_2(t)$ has a Nyquist rate of $\frac{\omega_0}{3}$?**(check the unique answer!)**

- [x] $\frac{4\omega_0}{3}$
- [ ] $\frac{3\omega_0}{2}$
- [ ] $\omega_0$
- [ ] $\frac{\omega_0^2}{2}$

**Question [SCQ-03] :** The Fourier Transform of the signal

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

is **(check the unique answer!)**

- [x] $X(\omega) = e^{-j\frac{1}{2}\omega}\text{sinc}\left(\frac{\omega}{2\pi}\right)$
- [ ] $X(\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$
- [ ] $X(\omega) = \text{sinc}\left(\frac{\omega - \frac{1}{2}}{2\pi}\right)$
- [ ] $X(\omega) = \frac{1}{\sqrt{2}}\text{sinc}\left(\frac{\omega - \frac{1}{2}}{2\pi}\right)$

**Question [MCQ-04] :** The Laplace transform for a signal $x(t)$ is known to have exactly two poles at $s = j$ and $s = -j$. Then $x(t)$ could be **(check all that apply!)**

- [x] a left-sided signal
- [x] a right-sided signal
- [ ] a two-sided signal
- [ ] an absolutely integrable signal

**Question [MCQ-05] :** A linear continuous-time system $\mathcal{H}$ is known to yield the following input-output pairs:

$$e^{j3\omega_0 t} = \mathcal{H}\{e^{j\omega_0 t}\} \quad \text{and} \quad e^{-j3\omega_0 t} = \mathcal{H}\{e^{-j\omega_0 t}\}$$

for some frequency $\omega_0$. Then **(check all that apply!)**

- [ ] the system $\mathcal{H}$ is time-invariant
- [x] $\cos(3\omega_0 t) = \mathcal{H}\{\cos(\omega_0 t)\}$
- [x] $\cos\left(3\omega_0(t - \frac{1}{3})\right) = \mathcal{H}\{\cos(\omega_0(t-1))\}$
- [ ] $\cos(3\omega_0(t-1)) = \mathcal{H}\{\cos(\omega_0(t-1))\}$

**Question [SCQ-06] :** Define $x_a(t) = x(at)$ and $y_a(t) = y(at)$ for $a > 0$. Let $z(t) = (x * y)(t)$. Then, we have **(check the unique answer!)**

- [x] $(x_a * y_a)(t) = \frac{z(at)}{a}$
- [ ] $(x_a * y_a)(t) = z(at)$
- [ ] $(x_a * y_a)(t) = az(at)$
- [ ] $(x_a * y_a)(t) = z(a^2 t)$

**Question [MCQ-07] :** A discrete-time LTI system has impulse response

$$h[n] = \left(\frac{2}{3}\right)^n u[n-1].$$

Which of the following claims are true? **(check all that apply!)**

- [x] if the input $x[n] = 1$ then the output is $y[n] = 2$
- [ ] if the input $x[n] = 1$ then the output $y[n]$ is undefined
- [x] the system is stable
- [x] the system is causal

**Question [MCQ-08] :** An LTI system $\mathcal{H}$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^2}{dt^2}y(t) - 2\alpha\frac{d}{dt}y(t) + (\alpha^2+1)y(t) = x(t)$$

for some real $\alpha$. Which of the following claims are true? **(check all that apply!)**

- [x] if $\mathcal{H}$ is stable and $\alpha = -1$ then $\mathcal{H}$ is causal
- [ ] if $\mathcal{H}$ is causal and $\alpha = 1$ then $\mathcal{H}$ is stable
- [ ] $\mathcal{H}$ has a two-sided impulse response
- [x] if $\alpha = 0$ then $\mathcal{H}$ must not be stable

**Question [MCQ-09] :** Consider discrete-time signals $x[n] = (\frac{1}{3})^n u[n] + 2^n u[n]$ and $y[n] = -(\frac{1}{2})^n u[-n-1] - 2^n u[n]$. Which of the following claims are true? **(check all that apply!)**

- [x] The signal $x[n] + y[n]$ is not absolutely integrable
- [x] The signal $x[n]$ is not absolutely integrable
- [x] $z$-transform of $x[n] + y[n]$ exists (that is, has a non-empty ROC)
- [ ] $z$-transform of $y[n]$ exists (that is, has a non-empty ROC)

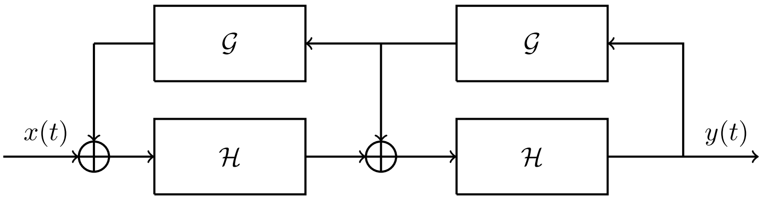
**Question** [MCQ-10] **:** A continuous-time LTI system has a transfer function

$$H(s) = \frac{1}{s^2 + s}, Re(s) > 0.$$

Then, $H(s)$ could be constructed by composing two LTI systems $H_1(s)$ and $H_2(s)$ where **(check all that apply!)**

- [x] $H_1(s) = \frac{1}{s}, Re(s) > 0$, $H_2(s) = \frac{1}{s+1}, Re(s) > -1$, and the systems are composed in series
- [ ] $H_1(s) = \frac{1}{s}, Re(s) > 0$, $H_2(s) = \frac{1}{s+1}, Re(s) > -1$, and the systems are composed in parallel
- [x] $H_1(s) = \frac{1}{s}, Re(s) > 0$, $H_2(s) = -\frac{1}{s+1}, Re(s) > -1$, and the systems are composed in parallel
- [ ] $H_1(s) = \frac{1}{s}, Re(s) > 0$, $H_2(s) = -\frac{1}{s+1}, Re(s) > -1$, and the systems are composed in series

**Question** [SCQ-11] **:** Consider the continuous-time system interconnect shown as follows, where the component systems $\mathcal{G}$ and $\mathcal{H}$ are LTI systems with transfer functions $G(s)$ and $H(s)$, respectively.




What is the overall transfer function of the system as a function of $G(s)$ and $H(s)$? **(check the unique answer!)**

- [ ] $\frac{G^2(s)}{1-H(s)G(s)-H^2(s)G^2(s)}$
- [ ] $\frac{H^2(s)}{(1-H(s)G(s))^2}$
- [x] $\frac{H^2(s)}{1-H(s)G(s)-H^2(s)G^2(s)}$
- [ ] $\frac{G^2(s)}{(1-H(s)G(s))^2}$

**Question** [MCQ-12] **:** Which of the following systems are invertible? **(check all that apply!)**

- [x] An LTI system with an impulse response $h(t) = \delta(t-7)$
- [ ] A system defined by the input-output relation $y[n] = |n-1|x[n-1]$
- [x] A system defined by the input-output relation $y(t) = x(3t)$
- [x] An LTI system with an impulse response $h[n] = \delta[n] + \delta[n-1]$

Q12.4: Both answers could be accepted. See @202 on piazza for more details.