Midterm Exam

EXAM INFORMATION

General Note: In our view, the most important issue is to know how to address a particular problem. Therefore, there will be partial credit for good solution outlines even if not all the mathematical manipulations are completed.

*** Good Luck! ***

First Part *(Multiple Choice Questions)* 32 points

For each statement, select a single correct answer.

(Question 1) (4 Pts): Which of the following claims about the Fourier Transform is true?

- If $x(t) = \frac{1}{2}(u(t+1) - u(t-1))$, then $X(\omega) = \text{sinc}(w)$
- If $X(w) = 2\pi\delta(\omega - 2)$, then $x(t) = e^{-2t}u(t)$
- Signals $u(t)$ and $u(-t)$ have the same Fourier Transform
- If $x(t) = 2\sin(7t + \pi/3)$, then $X(w) = \frac{2\pi}{j}(\delta(w-7)e^{\frac{j\pi}{3}} - \delta(w+7)e^{-\frac{j\pi}{3}})$

(Question 2) (4 Pts): Which of the following claims about the Fourier Transform is false?

- If $x(t)$ is an energy signal with non-zero energy, $X(\omega)$ must be a power signal
- If $x(t) = \int_{-\infty}^{\infty} y(\tau)z(t-\tau)d\tau$ then $X(\omega) = Y(\omega)Z(\omega)$
- If $X(w)$ is a Fourier transform of the signal $x(t)$, the Fourier transform of the signal $x(2t)$ is $\frac{1}{2}X(\frac{w}{2})$
- If $|X(w)|$ is an even function and $\arg(X(w))$ is an odd function, signal $x(t)$ with a Fourier transform $X(w)$ is a real function.

(Question 3) (4 Pts):

Which of the following claims about convolution properties is true for signals $x(t)$, $y(t)$ and $z(t) = x(t) * y(t)$?

- $x(t-1) * y(t-2) = x(t-2) * y(t-1)$
- $x(t) * y(-t) = z(-t)$
- $x(2t) * y(2t) = z(2t)$
- $\frac{dx(t)}{dt} * \frac{dy(t)}{dt} = \frac{dz(t)}{dt}$

(Question 4) (4 Pts): The input-output relationship for an LTI system S satisfies $y[n] - \frac{3}{2}y[n-1] = x[n]$. Which of the following statements about the system is false?

- If this system is stable, then it is not invertible
- The system could be causal and have an impulse response $h[n] = \left(\frac{3}{2}\right)^n u[n]$
- The system could be stable and have the frequency response $H(e^{j\omega}) = \frac{1}{1-\frac{3}{2}e^{-j\omega}}$
- If this system is causal, then it is not stable

(Question 5) (4 Pts): Let the Discrete-Time Fourier Transform of the signal $x(t)$ be given by $X(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}}$ where α is a real number with $|\alpha| < 1$. Which of the following claims is always true?

- The integral $\int_{-\infty}^{+\infty} |X(e^{j\omega})| d\omega$ exists (that is, it is not infinite)
- The shifted frequency version $X(e^{j(\omega-\omega_0)})$ is a Fourier Transform of a complex valued signal
- $\lim_{\omega \to \infty} X(e^{j\omega}) = 0$
- $X(e^{j\omega})$ is conjugate symmetric; that is, $X(e^{j\omega}) = X^*(e^{-j\omega})$

(Question 6) (4 Pts): Let $x[n] = \delta[-n-1] - \delta[-n]$, then

- $X(e^{j\omega}) = e^{-j\omega} - 1$
- $X(e^{j\omega}) = e^{-j\omega} + 1$
- $X(e^{j\omega}) = e^{j\omega} - 1$
- $X(e^{j\omega}) = 1 - e^{-j\omega}$

(Question 7) (4 Pts): A discrete-time system is defined by an input-output relationship $y[n] = x[-n]$. Which of the following claims about the systems is true?

- The system is linear
- The system is time-invariant
- The system is causal
- The system is memoryless

(Question 8) (4 Pts): Given an input $x(t) = e^{j3t}$, a stable LTI system produces an output $y(t) = e^{j3t}$. Based on this information, which of the following could NOT be the frequency response of the system?

- $H(\omega) = \delta(\omega)$
- $H(\omega) = 1$
- $H(\omega) = \frac{3j+1}{j\omega+1}$
- $H(\omega) = \begin{cases} 1, & \omega < \frac{3\pi}{2} \\ 0 & \text{otherwise.} \end{cases}$

Problem 2 *(Difference Equations.)* 20 points

(a)(4pts) Find the frequency response of a stable LTI system characterized by

$$y(t) + 4\frac{d}{dt}y(t) + 3\frac{d^2}{dt^2}y(t) = 2\frac{d^2}{dt^2}x(t) \tag{1}$$

(b)(8pts) Find the impulse response of this system.

(c)(8pts) Suppose that an input $x(t) = e^{-t}u(t)$ is applied to the system. What is the output $y(t)$?

Problem 3 *(System Composition)* 24 points

In this problem, we study the system composition illustrated in Figure 1 with input $x(t)$ and output $y(t)$. System $\mathcal{H}_1$ is described by the impulse response $h_1(t) = \delta(t-2)$, while system $\mathcal{H}_2$ is described as $y_2(t) = 2x_2(t-1)$.

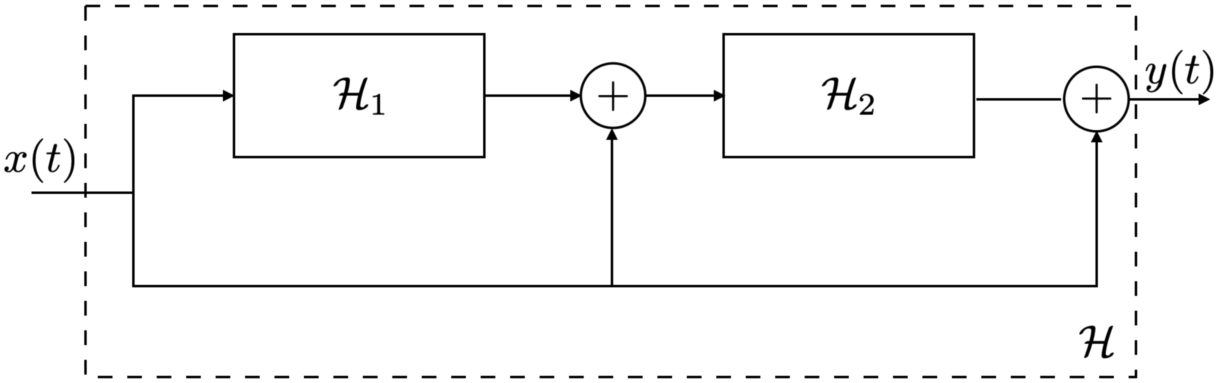

Figure 1: A composed system

(a)(8pts) Find the impulse response of the composed system $\mathcal{H}$.

(b)(8pts) Is the composed system $\mathcal{H}$ stable? Is it causal?

(c)(8pts) Signal $x(t)$ is given as

$$x(t) = \begin{cases} 2, -2 < t < 2 \\ 0, \text{otherwise} \end{cases} \quad (2)$$

Calculate and sketch the output of the system $y(t)$.

Problem 4 *(LTI Systems)* 24 points

(a)(7pts) First, let $a > 0$ be a real number and let us define an LTI system with the following impulse response:

$$h_a(t) = \begin{cases} \frac{1}{2a}, & |t| \leq \text{a} \\ 0, & \text{otherwise.} \end{cases}$$

A signal $x_1(t) = \sin(\omega_0 t)$ is applied to the system. Find the output $y_{1,a}(t)$ using convolution in the time domain.

(b)(7pts) A signal $x_2(t) = u(t)$ is applied to the system in part (a). Find the output $y_{2,a}(t)$ using convolution in the time domain.

(c)(6pts) Find the frequency response $H_a(\omega)$ of the system in part (a).

(d)(4pts) We now decide to push our system to its limit and make a very small. What should happen to our system as we do this? That is, what will happen to the frequency response and the impulse response as a goes to zero? What do you expect $y_{1,a}(t)$ and $y_{2,a}(t)$ to converge to as a goes to zero?