EPFL

n/a

Prof. Y. Shkel
Final Exam Signals and Systems - (n/a)
23/06/2022
Duration: 3h

n/a

SCIPER : **999999** Signature :

Do not turn the page before the start of the exam. This document is double-sided, has 24 pages, the last ones possibly blank. Do not unstaple.

- Place your student card on your table.
- For the **multiple choice with unique answer** questions, we give
  - +3 points if your answer is correct,
  - 0 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes | Read these guidelines | Beachten Sie bitte die unten stehenden Richtlinien

choisir une réponse | select an answer Antwort auswählen	ne PAS choisir une réponse | NOT select an answer NICHT Antwort auswählen	Corriger une réponse | Correct an answer Antwort korrigieren

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

First Part: Multiple Choice Questions

For each question, mark a box corresponding to the single correct answer.

Question 1 : A discrete-time system is defined by an input-output relationship $y[n] = nx[n]$. Which of the following claims about this system is false?

- ☐ This system is causal
- ☐ This system is linear
- ☐ This system is time-variant (not time-invariant)
- ☐ This system is stable

Question 2 : If the signal $x[n]$ has a z-transform of $X(z)$, what is the z-transform of the signal $a^n x[n]$?

- ☐ $X(a^{-1}z)$
- ☐ $X(a)X(z)$
- ☐ $X(az)$
- ☐ $X(az^{-1})$

Question 3 : Suppose that two causal LTI systems $\mathcal{H}$ and $\mathcal{G}$ have transfer functions $H(s) = \frac{s-a}{s-b}$ and $G(s) = \frac{1}{s-a}$ where a and b are real constants. Which of the following claims is true?

- ☐ If $a < 0$ and $b > 0$, then $\mathcal{H}$ and $\mathcal{G}$ composed in series will be stable
- ☐ If $a > 0$ and $b < 0$, then $\mathcal{H}$ and $\mathcal{G}$ composed in series will be stable
- ☐ If $a > 0$ and $b < 0$, then $\mathcal{H}$ and $\mathcal{G}$ composed in parallel will be stable
- ☐ If $a < 0$ and $b > 0$, then $\mathcal{H}$ and $\mathcal{G}$ composed in parallel will be stable

Question 4 : An LTI system $\mathcal{H}$ has its input $x(t)$ and output $y(t)$ related through a linear constant coefficient differential equation of the form

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

Which of the following combination of properties for this system is not possible?

- ☐ $\mathcal{H}$ is anti-causal and unstable
- ☐ $\mathcal{H}$ is stable
- ☐ $\mathcal{H}$ is not causal, not anti-causal, and unstable
- ☐ $\mathcal{H}$ is causal and unstable

Question 5 : The input-output relationship of an LTI system satisfies $y[n-2] - 6y[n-1] + 8y[n] = x[n]$. Which of the following statements about the system is false?

- ☐ Transfer function of this system is $H(z) = \frac{z^2}{(1-4z)(1-2z)}$
- ☐ This system is invertible
- ☐ If this system is causal, it is unstable
- ☐ This system is not memoryless

Question 6 : What is the Laplace transform and the associated region of convergence of the signal $x(t) = 3e^{-3t}u(t) - e^{2t}u(-t)$?

- ☐ $X(s) = \frac{3}{s+3} + \frac{1}{s-2}$, $2 < Re(s)$
- ☐ $X(s) = \frac{3}{s+3} + \frac{1}{s-2}$, $Re(s) < -3$
- ☐ The Laplace transform does not exist for this signal
- ☐ $X(s) = \frac{3}{s+3} + \frac{1}{s-2}$, $-3 < Re(s) < 2$

Question 7 : Which of the following statements about an impulse response of a continuous-time system is false?

- ☐ We are always able to deduce the impulse response of an LTI system by inputing $x(t) = e^{j2t}$ into the system and observing the output
- ☐ For an LTI system, we can calculate the output of the system by convolving the input signal and the impulse response of the system
- ☐ Given two stable LTI systems in series with impulse responses $h_1(t)$ and $h_2(t)$, the overall impulse response of the composite system is given by $h_1(t) * h_2(t)$
- ☐ Impulse response of a system described by $y(t) = \int_0^1 x(t-\lambda)d\lambda$ is $h(t) = \begin{cases} 0, t < 0 \\ 1, 0 < t < 1 \\ 0, t > 1 \end{cases}$

Question 8 : What is the Nyquist rate of the signal

$$x(t) = x_1(t)\cos(\omega_0 t) + x_2(t)\sin(\omega_0 t)$$

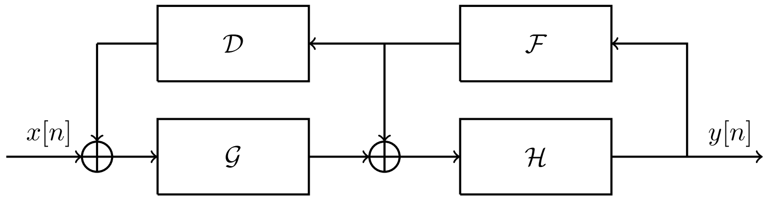
if $x_1(t)$ has a Nyquist rate of ω_0 and $x_2(t)$ has a Nyquist rate of $\frac{\omega_0}{2}$?

- ☐ ω_0
- ☐ $\frac{3\omega_0}{2}$
- ☐ $\frac{7\omega_0}{2}$
- ☐ $3\omega_0$

Question 9 : Which of the following LTI systems is a high-pass filter?

- ☐ A system with an impulse response $h(t) = \delta(t - t_0)$ where $t_0 < 0$
- ☐ A system with an impulse response $h(t) = \text{sinc}(t)$
- ☐ A system with an impulse response $h(t) = \delta(t) - \text{sinc}(t)$
- ☐ A system with an impulse response $h(t) = \delta(t - t_0)$ where $t_0 > 0$

Question 10 : Consider the discrete-time system interconnect shown as follows, where the component systems $\mathcal{D}$, $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ are LTI systems with transfer functions $D(z)$, $F(z)$, $G(z)$ and $H(z)$, respectively.



What is the overall transfer function of the system as a function of $D(z)$, $F(z)$, $G(z)$ and $H(z)$?

- ☐ $\frac{D(z)F(z)}{1-G^2(z)(1+H^2(z))}$
- ☐ $\frac{G(z)H(z)}{1-D(z)G(z)(1+H(z)F(z))}$
- ☐ $\frac{G(z)H(z)}{1-H(z)F(z)(1+D(z)G(z))}$
- ☐ $\frac{D(z)F(z)}{1-H(z)G(z)-H^2(z)G^2(z)}$

Question 11 : A continuous-time signal

$$x(t) = 1 + \sin(1000\pi t) + \cos(2000\pi t)$$

is sampled with a sampling frequency $\omega_s = 3000\pi$ to produce a discrete-time representation $x[n] = x(nT)$ where T is the corresponding sampling interval. Then

- ☐ $x[n] = (-1)^n$
- ☐ $x[n] = 1 + \sin\left(\frac{2}{3}\pi n\right) + \cos\left(\frac{1}{3}\pi n\right)$
- ☐ $x[n] = 1 + \sin\left(\frac{1}{3}\pi n\right) + \cos\left(\frac{2}{3}\pi n\right)$
- ☐ $x[n] = 1 + \sin\left(\frac{2}{3}\pi n\right) + \cos\left(\frac{2}{3}\pi n\right)$

Question 12 : Given two signals $x_1[n] = 3^{-n}u[n] - 2^{-n}u[n]$ and $x_2[n] = 3^n u[-n] - 2^{-n}u[n]$, which of the following statements is false?

- ☐ The signal $x_2[n]$ is absolutely summable
- ☐ The signal $x_1[n]$ is right-sided
- ☐ The signal $x_1[n]$ is absolutely summable
- ☐ The signal $x_1[n] - x_2[n]$ is left-sided

Second Part: Open Problems

Answer in the empty space below. Your answer should be carefully justified, and all the steps of your argument should be discussed in detail. Leave the check-boxes empty, they are used for the grading.

Question 13: *(12 Points)*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐ 11 ☐ 12 *Do not write here.*

Find the transfer function for each system described below. Remember to specify the algebraic expression and the ROC. Justify your answer.

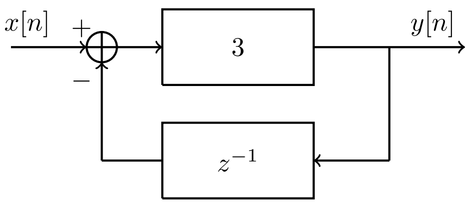
(a) An LTI system described by the input-output relationship

$$y(t) = \int_{-\infty}^{t} x(\tau - 3)d\tau$$

(b) A stable continuous-time LTI system with the frequency response $H(\omega) = \frac{1}{(j\omega)^2 - j\omega - 6}$

(c) A discrete-time LTI system with an impulse response $h[n] = \delta[n - n_0] + a_0\delta[n] + \delta[n + n_0]$ for some real constant $a_0 > 0$ and integer $n_0 > 0$

(d) A causal LTI system described by the following block diagram



where recall that the labels 3 and z^{-1} denote the transfer functions for the two composed systems

Question 14: *(12 Points)*

0 1 2 3 4 5 6 7 8 9 10 11 12 *Do not write here.*

An LTI system is described by the following difference equation:

$$y[n+2] - 5y[n+1] + 6y[n] = x[n+1].$$

(a) Find the transfer function $H(z)$ and draw the pole-zero plot for it.

(b) If the system is known to be stable, determine the impulse response of the system. Could this system be causal?

(c) If the system is known to be causal, determine the impulse response of the system. Could this system be stable?

Question 15: *(12 Points)*

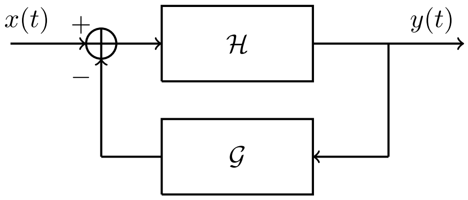
0 1 2 3 4 5 6 7 8 9 10 11 12 *Do not write here.*

(a) A causal LTI system $\mathcal{H}$ satisfies the following input-output relationship

$$\frac{dy(t)}{dt} = K_0 x(t) + K_1 y(t),$$

where K_0 and K_1 are positive real numbers. Determine if the system $\mathcal{H}$ is stable and find its transfer function.

(b) Next, we are going to analyze the feedback system composition given as follows, where $\mathcal{H}$ is the system from part (a) and $\mathcal{G}$ is another causal, LTI system



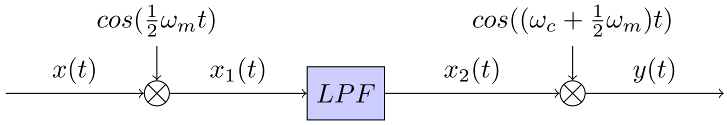
Find the transfer function of the overall composite system. Your answer should be in terms of K_0, K_1 and $G(s)$.

(c) Suppose that the system $G(s)$ is constrained to be an amplifier; that is, $G(s) = K_2$ where K_2 is any real constant. Is it possible to design $G(s)$ so that the overall system in (b) is stable? If yes, determine the constraints on K_2 (in terms of K_1 and K_0). If not, explain why.

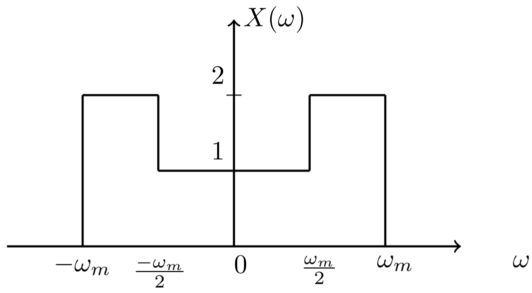
Question 16: *(14 Points)*

0 1 2 3 4 5 6 7
8 9 10 11 12 13 14 *Do not write here.*

(a) Consider the following system where $\omega_c >> \omega_m$ and LPF represents an ideal low-pass filter with cutoff frequency $\frac{\omega_m}{2}$



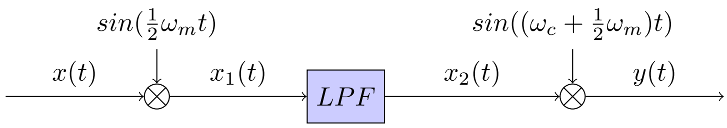
The input signal $x(t)$ has the following Fourier transform,



What is the Fourier transform of $y(t)$? Sketch the Fourier transforms of $x_1(t)$ and $x_2(t)$ first to show your work.

Hint: The notation $\omega_c >> \omega_m$ means that ω_c is much larger than ω_m. If you find that confusing, you may assume, for simplicity, that $\omega_c = 3\omega_m$.

(b) Next, we construct the same system, but with cosine signals replaced by sines. That is



What is the Fourier Transform of $y(t)$ given that the Fourier transform of $x(t)$ is the same as in part (a)?

(c) If the systems in parts (a) and (b) are connected in parallel, what is the output given the same input $x(t)$? What do you observe about the output?

Question 17: *(14 Points)*

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7
☐8 ☐9 ☐10 ☐11 ☐12 ☐13 ☐14

Do not write here.

We are given a certain LTI system with an impulse response $h_0(t)$. We are told that when the input is $x_0(t)$, the output is

$$y_0(t) = \begin{cases} 0, & t \leq 0 \\ t, & 0 < t \leq 1 \\ 1, & t > 1. \end{cases}$$

(a) Suppose that the input signal $x(t) = x_0(-t)$ is applied to this LTI system. Do we have enough information to determine the output signal $y(t)$? If yes, find and sketch the output signal $y(t)$. If no, prove that this is not possible.

(b) Suppose that the input signal $x(t) = x_0(t-2)$ is applied to an LTI system with the impulse response $h(t) = h_0(t+1)$. Do we have enough information to determine the output signal $y(t)$? If yes, find and sketch the output signal $y(t)$. If no, prove that this is not possible.

(c) Suppose that the input signal $x(t) = x_0(t)$ is applied to an LTI system with the impulse response $h(t) = \int_{t-1}^{t} h_0(\tau)d\tau$. Do we have enough information to determine the output signal $y(t)$? If yes, find and sketch the output signal $y(t)$. If no, prove that this is not possible.