EPFL

n/a

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**Final Exam Signals and Systems - (n/a)**
**22/06/2021**
**Duration: 3h**

n/a

SCIPER : **999999**    Signature :

**Do not turn the page before the start of the exam. This document is double-sided, has 20 pages, the last ones possibly blank. Do not unstaple.**

- Place your student card on your table.
- For the **multiple choice with unique answer** questions, we give
  - +3 points if your answer is correct,
  - 0 points if your answer is incorrect.
- For the **multiple choice with multiple answers** questions, we give
  - +4 points for all correct answers,
  - +2 points for one incorrect answer and three correct answers,
  - 0 points for other possibilities of answers.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes | Read these guidelines | Beachten Sie bitte die unten stehenden Richtlinien

| choisir une réponse \| select an answer Antwort auswählen | ne PAS choisir une réponse \| NOT select an answer NICHT Antwort auswählen | Corriger une réponse \| Correct an answer Antwort korrigieren |
|---|---|---|

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

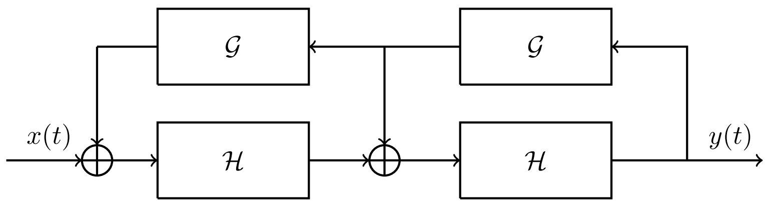
## First Part: Multiple Choice Questions

For each question, mark all boxes corresponding to the correct answers. Note that some questions are marked as having multiple correct answers – **check all that apply!** – while others have **a unique correct answer**.

**Question 1 :** Consider discrete-time signals $x[n] = (\frac{1}{3})^n u[n] + 2^n u[n]$ and $y[n] = -(\frac{1}{2})^n u[-n-1] - 2^n u[n]$. Which of the following claims are true? **(check all that apply!)**

- ☐ The signal $x[n]$ is not absolutely integrable
- ☐ The signal $x[n] + y[n]$ is not absolutely integrable
- ☐ $z$-transform of $x[n] + y[n]$ exists (that is, has a non-empty ROC)
- ☐ $z$-transform of $y[n]$ exists (that is, has a non-empty ROC)

**Question 2 :** Consider the continuous-time system interconnect shown as follows, where the component systems $\mathcal{G}$ and $\mathcal{H}$ are LTI systems with transfer functions $G(s)$ and $H(s)$, respectively.




What is the overall transfer function of the system as a function of $G(s)$ and $H(s)$? **(check the unique answer!)**

- ☐ $\frac{G^2(s)}{1-H(s)G(s)-H^2(s)G^2(s)}$
- ☐ $\frac{G^2(s)}{(1-H(s)G(s))^2}$
- ☐ $\frac{H^2(s)}{(1-H(s)G(s))^2}$
- ☐ $\frac{H^2(s)}{1-H(s)G(s)-H^2(s)G^2(s)}$

**Question 3 :** Define $x_a(t) = x(at)$ and $y_a(t) = y(at)$ for $a > 0$. Let $z(t) = (x * y)(t)$. Then, we have **(check the unique answer!)**

- ☐ $(x_a * y_a)(t) = \frac{z(at)}{a}$
- ☐ $(x_a * y_a)(t) = az(at)$
- ☐ $(x_a * y_a)(t) = z(a^2 t)$
- ☐ $(x_a * y_a)(t) = z(at)$

**Question 4 :** Which of the following systems are invertible? **(check all that apply!)**

- ☐ A system defined by the input-output relation $y(t) = x(3t)$
- ☐ An LTI system with an impulse response $h[n] = \delta[n] + \delta[n-1]$
- ☐ An LTI system with an impulse response $h(t) = \delta(t-7)$
- ☐ A system defined by the input-output relation $y[n] = |n-1|x[n-1]$

**Question 5 :** A discrete-time LTI system has impulse response

$$h[n] = \left(\frac{2}{3}\right)^n u[n-1].$$

Which of the following claims are true? **(check all that apply!)**

- ☐ if the input $x[n] = 1$ then the output $y[n]$ is undefined
- ☐ the system is stable
- ☐ if the input $x[n] = 1$ then the output is $y[n] = 2$
- ☐ the system is causal

**Question 6 :** The Laplace transform for a signal $x(t)$ is known to have exactly two poles at $s = j$ and $s = -j$. Then $x(t)$ could be **(check all that apply!)**

- ☐ a left-sided signal
- ☐ a two-sided signal
- ☐ a right-sided signal
- ☐ an absolutely integrable signal

**Question 7 :** An LTI system $\mathcal{H}$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^2}{dt^2}y(t) - 2\alpha\frac{d}{dt}y(t) + (\alpha^2+1)y(t) = x(t)$$

for some real $\alpha$. Which of the following claims are true? **(check all that apply!)**

- ☐ if $\mathcal{H}$ is stable and $\alpha = -1$ then $\mathcal{H}$ is causal
- ☐ $\mathcal{H}$ has a two-sided impulse response
- ☐ if $\alpha = 0$ then $\mathcal{H}$ must not be stable
- ☐ if $\mathcal{H}$ is causal and $\alpha = 1$ then $\mathcal{H}$ is stable

**Question 8 :** What is the Nyquist rate of the signal

$$x(t) = x_1(t)x_2(t)$$

if $x_1(t)$ has a Nyquist rate of $\omega_0$ and $x_2(t)$ has a Nyquist rate of $\frac{\omega_0}{3}$?**(check the unique answer!)**

- ☐ $\omega_0$
- ☐ $\frac{3\omega_0}{2}$
- ☐ $\frac{\omega_0^2}{2}$
- ☐ $\frac{4\omega_0}{3}$

**Question 9 :** A linear continuous-time system $\mathcal{H}$ is known to yield the following input-output pairs:

$$e^{j3\omega_0 t} = \mathcal{H}\{e^{j\omega_0 t}\} \quad \text{and} \quad e^{-j3\omega_0 t} = \mathcal{H}\{e^{-j\omega_0 t}\}$$

for some frequency $\omega_0$. Then **(check all that apply!)**

- ☐ $\cos(3\omega_0 t) = \mathcal{H}\{\cos(\omega_0 t)\}$
- ☐ $\cos\left(3\omega_0(t-\frac{1}{3})\right) = \mathcal{H}\{\cos(\omega_0(t-1))\}$
- ☐ $\cos(3\omega_0(t-1)) = \mathcal{H}\{\cos(\omega_0(t-1))\}$
- ☐ the system $\mathcal{H}$ is time-invariant

**Question 10 :** The Fourier Transform of the signal

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

is **(check the unique answer!)**

- ☐ $X(\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$
- ☐ $X(\omega) = \frac{1}{\sqrt{2}}\text{sinc}\left(\frac{\omega-\frac{1}{2}}{2\pi}\right)$
- ☐ $X(\omega) = \text{sinc}\left(\frac{\omega-\frac{1}{2}}{2\pi}\right)$
- ☐ $X(\omega) = e^{-j\frac{1}{2}\omega}\text{sinc}\left(\frac{\omega}{2\pi}\right)$

**Question 11 :** A continuous-time signal

$$x(t) = e^{j\omega_0 t} + K_0 e^{j(\omega_0 t+\pi)} + K_1 e^{j2\omega_0 t}$$

is sampled with a sampling frequency $\omega_s = 10\omega_0$ to produce a discrete-time representation $x[n] = x(nT)$ where $T$ is the corresponding sampling interval. Then **(check all that apply!)**

- ☐ $x[n] = C$, for some constant $C$
- ☐ $x[n] = (1 - K_0)e^{j\frac{\pi}{5}n} + K_1 e^{j\frac{2\pi}{5}n}$
- ☐ $x[n] = (1 + K_0)e^{j\frac{\pi}{5}n} + K_1 e^{j\frac{2\pi}{5}n}$
- ☐ $x[n] = (1 - K_0)(\cos(\frac{11\pi}{5}n) + j\sin(\frac{11\pi}{5}n)) + K_1(\cos(\frac{12\pi}{5}n) + j\sin(\frac{12\pi}{5}n))$

**Question 12 :** A continuous-time LTI system has a transfer function

$$H(s) = \frac{1}{s^2 + s}, Re(s) > 0.$$

Then, $H(s)$ could be constructed by composing two LTI systems $H_1(s)$ and $H_2(s)$ where **(check all that apply!)**

- ☐ $H_1(s) = \frac{1}{s}, Re(s) > 0,\ H_2(s) = -\frac{1}{s+1}, Re(s) > -1$, and the systems are composed in series
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- ☐ $H_1(s) = \frac{1}{s}, Re(s) > 0,\ H_2(s) = \frac{1}{s+1}, Re(s) > -1$, and the systems are composed in parallel
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## Second Part: Open Problems

Answer in the empty space below. Your answer should be carefully justified, and all the steps of your argument should be discussed in detail. Leave the check-boxes empty, they are used for the grading.

**Question 13:** *(12 Points)*

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 ☐11 ☐12 *Do not write here.*

Determine the impulse response for each system described below. Justify your answer.

(a) A casual continuous-time LTI system with transfer function $H(s) = \frac{1}{s}$

(b) A stable discrete-time LTI system with transfer function $H(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$

(c) An LTI system described by the input-output relationship

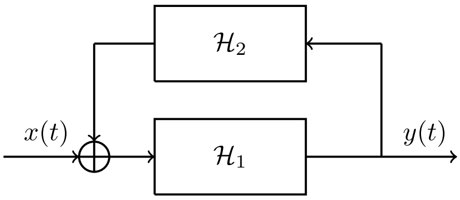
$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau + 2) d\tau.$$

(d) An LTI system with a step response $s[n] = \alpha^n u[n]$ where recall that the step response is given by $s[n] = \mathcal{H}\{u[n]\}$.

**Question 14:** *(10 Points)*

0 1 2 3 4 5 6 7 8 9 10 *Do not write here.*

Consider the feedback composition of two continuous-time LTI systems with transfer functions $H_1(s)$ and $H_2(s)$:




(a) Suppose $H_1(s) = 1$ and $H_2(s) = \frac{1}{s}$, what is the transfer function $H(s)$ of the overall system $\mathcal{H}$? The overall system $\mathcal{H}$ is known to be causal, is it also stable? Find its impulse response $h(t)$.

(b) Suppose the overall system $\mathcal{H}$ is known to be causal and to have a causal inverse system $\mathcal{G}$. Find the transfer function $G(s)$ and the impulse response $g(t)$ of the inverse system.

**Question 15:** *(10 Points)*

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Do not write here.*

An LTI system is described by the following difference equation:

$$y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n].$$

(a) Find the transfer function $H(z)$.

(b) If the system is known to be stable, determine the impulse response. Is this system causal, anti-causal or neither?

(c) If the system is known to be causal, determine the impulse response. Is this system stable?

**Question 16:** *(12 Points)*

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 ☐11 ☐12 *Do not write here.*

In this problem we study an operation called *correlation* which is widely used in signal analysis. The correlation is defined, in discrete-time, as

$$R_{xt}[n] = x[n] \star t[n] = \sum_{m=-\infty}^{\infty} x^*[m]t[n+m]$$

where $x[n]$ is the signal of interest, $t[n]$ is called *the template* signal, and $x^*[n]$ denotes the complex conjugate of $x[n]$.

(a) The correlation operation could be transformed into classical convolution. Express $R_{xt}[n]$ as a convolution between functions of $t[n]$ and $x[n]$.

(b) Given the signal

$$x[n] = \begin{cases} 1, & if\ 0 \leq n \leq 2 \\ 2, & if\ 3 \leq n \leq 5 \\ 0, & otherwise \end{cases}$$

and the template

$$t[n] = max\left(1 - \left|\frac{n}{2}\right|, 0\right)$$

find the correlation product $R_{xt}$.

(c) Finally, the cross-correlation operation is defined as the correlation of the signal with itself:

$$R_{xx}[n] = x[n] \star x[n] = \sum_{m=-\infty}^{\infty} x^*[m]x[n+m].$$

Find $k$ such that

$$E = R_{xx}[k]$$

where $E$ denotes the energy of the signal $x[n]$.

Find the energy of the signal in part (b).