

CORRIGE DE LA SERIE IV

EXERCICE IV.1 : FONCTIONS PÉRIODIQUES

Valeur moyenne : $F_m = \frac{1}{T} \int_0^T f(t) dt$ Valeur efficace : $F = \sqrt{\frac{1}{T} \int_0^T f^2(t) dt}$

Fonction 1 :

$$U_m = \frac{1}{T} \left[\int_0^{T/2} \hat{U} dt - \int_{T/2}^T \hat{U} dt \right] = \underline{0 \text{ V}} \quad U = \sqrt{\frac{1}{T} \left[\int_0^{T/2} \hat{U}^2 dt + \int_{T/2}^T \hat{U}^2 dt \right]} = \hat{U} = \underline{20 \text{ V}}$$

$$U_{\max} = \hat{U} = \underline{20 \text{ V}} \quad U_{\min} = -\underline{20 \text{ V}}$$

Fonction 2 :

$$U_m = \frac{1}{T} \int_0^{T/2} \hat{U} dt = \frac{1}{2} \hat{U} = \underline{5 \text{ V}} \quad U = \sqrt{\frac{1}{T} \int_0^{T/2} \hat{U}^2 dt} = \frac{1}{\sqrt{2}} \hat{U} = \underline{\frac{10}{\sqrt{2}} \text{ V}}$$

$$U_{\max} = \hat{U} = \underline{10 \text{ V}} \quad U_{\min} = \underline{0 \text{ V}}$$

Fonction 3 :

$$U_m = \frac{1}{T} \left[\int_0^{T/2} \hat{U}_+ dt - \int_{T/2}^T \hat{U}_- dt \right] = \frac{1}{T} \left[10 \frac{T}{2} - 20 \frac{T}{2} \right] = \underline{-5 \text{ V}}$$

$$U = \sqrt{\frac{1}{T} \left[\int_0^{T/2} \hat{U}_+^2 dt + \int_{T/2}^T \hat{U}_-^2 dt \right]} = \sqrt{\frac{1}{T} \left[10^2 \frac{T}{2} + (-20)^2 \frac{T}{2} \right]} = \sqrt{250} \approx \underline{15.8 \text{ V}}$$

$$U_{\max} = \hat{U}_+ = \underline{10 \text{ V}} \quad U_{\min} = \hat{U}_- = \underline{-20 \text{ V}}$$

Fonction 4 :

$$I_m = \frac{1}{T} \int_0^T \hat{I} \sin(\omega t) dt = -\frac{1}{T\omega} \hat{I} \cos\left(\frac{2\pi}{T} t\right) \Big|_0^T = \underline{0 \text{ A}}$$

$$I = \sqrt{\frac{1}{T} \int_0^T \hat{I}^2 \sin^2(\omega t) dt} = \sqrt{\frac{1}{T} \hat{I}^2 \left[\frac{1}{2} t - \frac{1}{4 \cdot \frac{2\pi}{T}} \sin\left(2 \frac{2\pi}{T} t\right) \right]_0^T} = \frac{\hat{I}}{\sqrt{2}} = \underline{\sqrt{2} \text{ A}}$$

$$I_{\max} = \underline{2 \text{ A}} \quad I_{\min} = \underline{-2 \text{ A}}$$

Fonction 5 :

$$I_m = \frac{2}{T} \int_0^{T/2} \hat{I} \sin(\omega t) dt = -\frac{2}{T \frac{2\pi}{T}} \hat{I} \cos\left(\frac{2\pi}{T} t\right) \Big|_0^{T/2} = \frac{2}{\pi} \hat{I} = \underline{\frac{4}{\pi} \text{ A}}$$

$$I = \sqrt{\frac{2}{T} \int_0^{T/2} \hat{I}^2 \sin^2(\omega t) dt} = \sqrt{\frac{2}{T} \hat{I}^2 \cdot \left[\frac{1}{2} t - \frac{1}{4 \frac{2\pi}{T}} \sin\left(2 \frac{2\pi}{T} t\right) \right]_0^{T/2}} = \frac{\hat{I}}{\sqrt{2}} = \underline{\sqrt{2} \text{ A}}$$

$$I_{\max} = \underline{2 \text{ A}} \quad I_{\min} = \underline{0 \text{ A}}$$

Fonction 6 :

$$I_m = \frac{1}{T} \int_0^{T/2} \hat{I} \sin(\omega t) dt = -\frac{1}{T \frac{2\pi}{T}} \hat{I} \cos\left(\frac{2\pi}{T} t\right) \bigg|_0^{T/2} = \frac{1}{\pi} \hat{I} = \frac{2}{\pi} A$$

$$I = \sqrt{\frac{1}{T} \int_0^{T/2} \hat{I}^2 \sin^2(\omega t) dt} = \sqrt{\frac{1}{T} \hat{I}^2 \cdot \left[\frac{1}{2} t - \frac{1}{4 \frac{2\pi}{T}} \sin\left(2 \frac{2\pi}{T} t\right) \right]_0^{T/2}} = \frac{\hat{I}}{2} = \underline{1 A}$$

$$I_{\max} = \underline{2 A} \qquad I_{\min} = \underline{0 A}$$

EXERCICE IV.2 : MISE EN ÉQUATION, CIRCUIT EN RÉGIME SINUSOÏDAL**A.** Définition des tensions et courants :

Le choix du sens des chutes de tension ainsi que les désignations des tensions et des courants n'est qu'une solution parmi d'autres (convention).

B.

$$u_{R_1} = R_1 i_1$$

$$u_{R_2} = R_2 i_2$$

$$u_{C_1} = \frac{1}{C_1} \int_{t_0}^t i_2 dt + u_{C_1}(t_0)$$

$$u_{C_2} = \frac{1}{C_2} \int_{t_0}^t i dt + u_{C_2}(t_0)$$

$$u_{L_1} = L_1 \frac{di_1}{dt}$$

$$u_{L_2} = L_2 \frac{di}{dt}$$

$$i = i_1 + i_2$$

$$u = u_1 + u_2$$

$$u_1 = u_{R_1} + u_{L_1}$$

$$u_1 = u_{C_1} + u_{R_2}$$

$$u_2 = u_{C_2} + u_{L_2}$$

C.

$$\underline{U}_{R_1} = R_1 \underline{I}_1$$

$$\underline{U}_{R_2} = R_2 \underline{I}_2$$

$$\underline{U}_{C_1} = \frac{\underline{I}_2}{j\omega C_1}$$

$$\underline{U}_{C_2} = \frac{\underline{I}}{j\omega C_2}$$

$$\underline{U}_{L_1} = j\omega L_1 \underline{I}_1$$

$$\underline{U}_{L_2} = j\omega L_2 \underline{I}$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2$$

$$\underline{U} = \underline{U}_1 + \underline{U}_2$$

$$\underline{U}_1 = \underline{U}_{R_1} + \underline{U}_{L_1}$$

$$\underline{U}_1 = \underline{U}_{C_1} + \underline{U}_{R_2}$$

$$\underline{U}_2 = \underline{U}_{C_2} + \underline{U}_{L_2}$$

D. Il y a 11 équations, 12 inconnues. Il faut donc connaître un courant ou une tension en plus de la valeur des composants du circuit.

E. De $u_1 = 20 \sqrt{2} \cos(\omega t)$ on tire : $\underline{U}_1 = 20e^{j0^\circ}$

$$\underline{U}_1 = \underline{U}_{R_1} + \underline{U}_{L_1} = R_1 \underline{I}_1 + j\omega L_1 \underline{I}_1 = (R_1 + j\omega L_1) \underline{I}_1$$

$$R_1 + j\omega L_1 = 4 + j 314,16 \cdot 25 \cdot 10^{-3} = 4 + j 7,854 = \underline{8,814e^{j63,0^\circ} \Omega}$$

$$\underline{I}_1 = \frac{\underline{U}_1}{R_1 + j\omega L_1} = \frac{20e^{j0^\circ}}{8,814e^{j63,0^\circ}} = \underline{2,269e^{-j63,0^\circ} A} = \underline{(1,030 - j 2,022) A}$$

$$\underline{U}_{R_1} = R_1 \underline{I}_1 = 4 \cdot 2,269e^{-j63,0^\circ} = \underline{9,076e^{-j63,0^\circ} V} = \underline{(4,12 - j 8,087) V}$$

$$\underline{U}_{L_1} = j\omega L_1 \underline{I}_1 = 7,854e^{j90,0^\circ} \cdot 2,269e^{-j63,0^\circ} = \underline{17,821e^{j27,0^\circ} V} = \underline{(15,88 + j 8,09) V}$$

$$\underline{U}_1 = \underline{U}_{R_2} + \underline{U}_{C_1} = R_2 \underline{I}_2 - j\frac{1}{\omega C_1} \underline{I}_2 = \left(R_2 - j\frac{1}{\omega C_1} \right) \underline{I}_2$$

$$R_2 - j\frac{1}{\omega C_1} = \underline{8 - j 6,366 \Omega} = \underline{10,224e^{-j38,5^\circ} \Omega}$$

$$\underline{I}_2 = \frac{\underline{U}_1}{R_2 - j\frac{1}{\omega C_1}} = \frac{20e^{j0^\circ}}{10,224e^{-j38,5^\circ}} = \underline{1,956e^{j38,5^\circ} A} = \underline{(1,531 + j 1,218) A}$$

$$\underline{U}_{R_2} = R_2 \underline{I}_2 = 8 \cdot 1,956e^{j38,5^\circ} = \underline{15,648e^{j38,5^\circ} V} = \underline{(12,246 + j 9,74) V}$$

$$\underline{U}_{C_1} = \frac{\underline{I}_2}{j\omega C_1} = \frac{1,956e^{j38,5^\circ}}{0,157e^{j90,0^\circ}} = \underline{12,452e^{-j51,5^\circ} V} = \underline{(7,75 - j 9,745) V}$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 1,030 - j 2,022 + 1,531 + j 1,218 = \underline{(2,561 - j 0,804) A} = \underline{2,684e^{-j17,4^\circ} A}$$

$$\underline{U}_{C_2} = \frac{\underline{I}}{j\omega C_2} = \frac{2,684e^{-j17,4^\circ}}{0,314e^{j90,0^\circ}} = \underline{8,543e^{-j107,4^\circ} V} = \underline{(-2,555 - j 8,152) V}$$

$$\underline{U}_{L_2} = j\omega L_2 \underline{I} = 15,71e^{j90,0^\circ} \cdot 2,684e^{-j17,4^\circ} = \underline{42,17e^{j72,6^\circ} V} = \underline{(12,61 + j 40,24) V}$$

$$\underline{U}_2 = \underline{U}_{C_2} + \underline{U}_{L_2} = -2,555 - j 8,152 + 12,61 + j 40,24 = \underline{(10,06 + j 32,09) V} = \underline{33,63e^{j72,6^\circ} V}$$

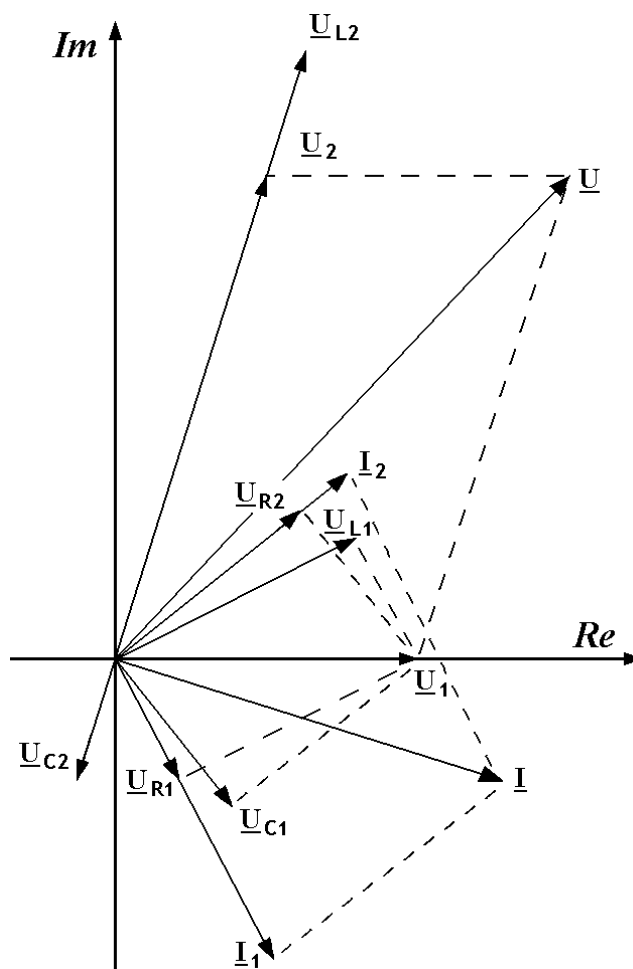
$$\underline{U} = \underline{U}_1 + \underline{U}_2 = 20 + 10,06 + j 32,09 = \underline{(30,06 + j 32,09) V} = \underline{43,97e^{j46,9^\circ} V}$$

F. Diagramme complexe des courants et des tensions

Échelles :

U : 1 cm $\leftrightarrow$ 5 V

I : 1 cm $\leftrightarrow$ 0,5 A



EXERCICE IV.3 : CALCULS D'IMPÉDANCES

A. Impédances

$$\underline{Z}_1 = R + j\omega L + \frac{1}{j\omega C} = R + j\frac{\omega^2 LC - 1}{\omega C} = \underline{(4 + j3,04) \Omega} = \underline{5e^{j37,2^\circ} \Omega}$$

$$\underline{Z}_2 = \frac{1}{j\omega C} + \frac{R \cdot j\omega L}{R + j\omega L} = \frac{\omega^3 RL^2 C - j(R^2 + \omega^2 L^2 - \omega^2 R^2 LC)}{\omega C(R^2 + \omega^2 L^2)} = \underline{(3,20 - j3,38) \Omega} = \underline{4.7e^{-j46,5^\circ} \Omega}$$

$$\underline{Z}_3 = R + \frac{\frac{1}{j\omega C} \cdot j\omega L}{\frac{1}{j\omega C} + j\omega L} = R + j\frac{\omega L}{1 - \omega^2 LC} = \underline{(4 - j13,12) \Omega} = \underline{13.7e^{-j73,0^\circ} \Omega}$$

$$\underline{Z}_4 = j\omega L + \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R + j\omega(\omega^2 R^2 LC^2 + L - R^2 C)}{1 + (\omega RC)^2} = \underline{(2,43 + j6,06) \Omega} = \underline{6.5e^{j68,2^\circ} \Omega}$$

$$\underline{Z}_5 = \frac{\frac{R \cdot j\omega L}{R + j\omega L} \cdot \frac{1}{j\omega C}}{\frac{R \cdot j\omega L}{R + j\omega L} + \frac{1}{j\omega C}} = \frac{\omega^2 R L^2 - j\omega R^2 L(\omega^2 LC - 1)}{R^2(\omega^2 LC - 1)^2 + (\omega L)^2} = (3,66 - j1,12) \Omega = 3,8e^{-j17^\circ} \Omega$$

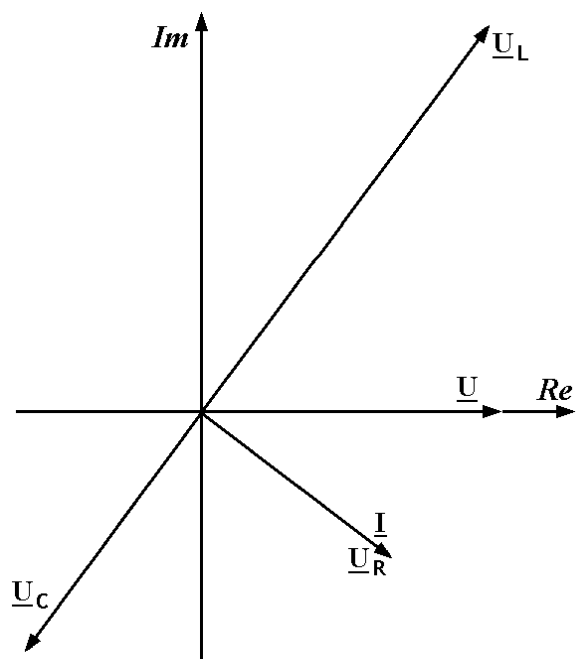
$$\underline{Z}_6 = \frac{(R + j\omega L) \cdot \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R - j\omega(\omega^2 L^2 C + R^2 C - L)}{(\omega^2 LC - 1)^2 + (\omega RC)^2} = (3,92 - j7,95) \Omega = 8,6e^{-j63,7^\circ} \Omega$$

$$\underline{Z}_7 = \frac{R \cdot \left(j\omega L + \frac{1}{j\omega C} \right)}{R + \left(j\omega L + \frac{1}{j\omega C} \right)} = \frac{R(\omega^2 LC - 1)^2 + j\omega R^2 C(\omega^2 LC - 1)}{(\omega^2 LC - 1)^2 + (\omega RC)^2} = (1,46 + j1,93) \Omega = 2,5e^{j52,8^\circ} \Omega$$

B. Courants et tensions (I désigne le courant total)

Unités : V et A. Échelles : 1 cm ↔ 1 V ; 1 cm ↔ 0,25 A

$$\begin{aligned} \underline{1} \quad \underline{I} &= \frac{\underline{U}}{\underline{Z}_1} = 0,8 \cdot e^{-j37,2^\circ} = 0,637 - j0,484 \\ \underline{U}_R &= R \cdot \underline{I} = 3,2 \cdot e^{-j37,2^\circ} = 2,549 - j1,935 \\ \underline{U}_L &= \underline{Z}_L \cdot \underline{I} = 6,409 \cdot e^{j52,8^\circ} = 3,875 + j5,105 \\ \underline{U}_C &= \underline{Z}_C \cdot \underline{I} = -3,979 \cdot e^{-j127,2^\circ} = -2,406 - j3,169 \\ \underline{I}_R &= \underline{I}_C = \underline{I}_L = \underline{I} \end{aligned}$$



$$\boxed{2} \quad \underline{I} = \frac{\underline{U}}{\underline{Z}_2} = \underline{0,85 \cdot e^{j46,5^\circ}} = \underline{0,585 + j 0,617}$$

$$\underline{U}_C = \underline{Z}_C \cdot \underline{I} = \underline{4,233 \cdot e^{-j43,5^\circ}} = \underline{3,070 - j 2,914}$$

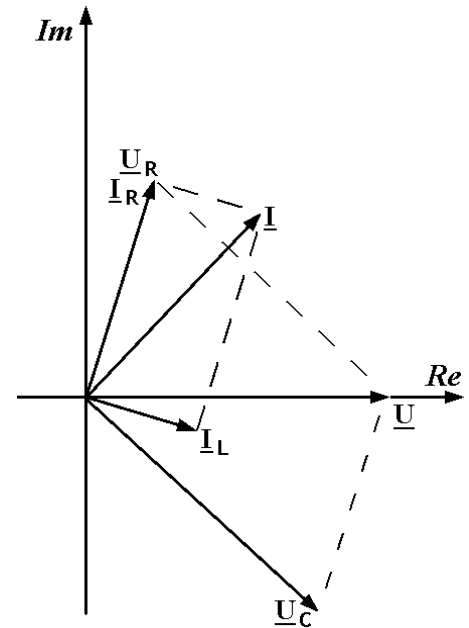
$$\underline{U}_R = \underline{U} - \underline{U}_C = \underline{3,058 \cdot e^{j72,3^\circ}} = \underline{0,930 + j 2,914}$$

$$\underline{U}_L = \underline{U}_R$$

$$\underline{I}_R = \frac{\underline{U}_R}{R} = \underline{0,764 \cdot e^{j72,3^\circ}} = \underline{0,232 + j 0,729}$$

$$\underline{I}_L = \underline{I} - \underline{I}_R = \underline{1,391 \cdot e^{-j17,6^\circ}} = \underline{0,353 - j 0,112}$$

$$\underline{I}_C = \underline{I}$$



$$\boxed{3} \quad \underline{I} = \frac{\underline{U}}{\underline{Z}_3} = \underline{0,292 \cdot e^{j73^\circ}} = \underline{0,085 + j 0,279}$$

$$\underline{U}_R = R \cdot \underline{I} = \underline{1,168 \cdot e^{j73^\circ}} = \underline{0,341 + j 1,116}$$

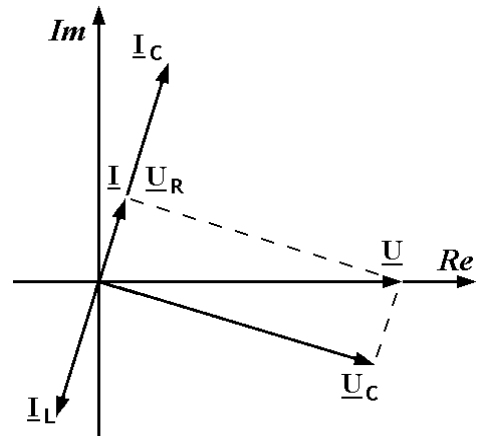
$$\underline{U}_C = \underline{U} - \underline{U}_R = \underline{3,826 \cdot e^{-j17^\circ}} = \underline{3,66 - j 1,116}$$

$$\underline{U}_L = \underline{U}_C$$

$$\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_C} = \underline{0,769 \cdot e^{j73^\circ}} = \underline{0,224 + j 0,736}$$

$$\underline{I}_L = \underline{I} - \underline{I}_C = \underline{0,478 \cdot e^{-j107^\circ}} = \underline{-0,139 - j 0,457}$$

$$\underline{I}_R = \underline{I}$$



$$\boxed{4} \quad \underline{I} = \frac{\underline{U}}{\underline{Z}_4} = \underline{0,615 \cdot e^{j-68,2^\circ}} = \underline{0,2285 - j 0,5714}$$

$$\underline{U}_L = \underline{Z}_L \cdot \underline{I} = \underline{4,93 \cdot e^{j21,8^\circ}} = \underline{4,577 + j 1,831}$$

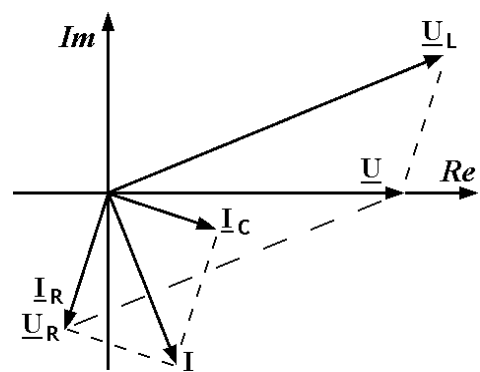
$$\underline{U}_R = \underline{U} - \underline{U}_L = \underline{1,92 \cdot e^{-j107,5^\circ}} = \underline{-0,5773 - j 1,831}$$

$$\underline{U}_C = \underline{U}_R$$

$$\underline{I}_R = \frac{\underline{U}_R}{R} = \underline{0,48 \cdot e^{-j107,5^\circ}} = \underline{-0,144 - j 0,458}$$

$$\underline{I}_C = \underline{I} - \underline{I}_R = \underline{0,386 \cdot e^{-j17,5^\circ}} = \underline{0,368 - j 0,116}$$

$$\underline{I}_L = \underline{I}$$



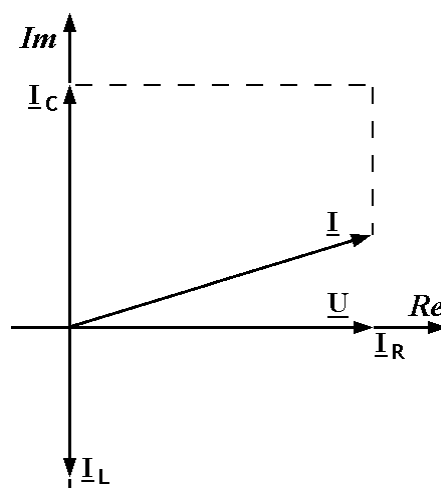
$$\boxed{5} \quad \underline{I} = \frac{\underline{U}}{\underline{Z}_5} = \underline{1,045 \cdot e^{j16,9^\circ}} = \underline{1 + j 0,304}$$

$$\underline{U}_R = \underline{U}_L = \underline{U}_C = \underline{U}$$

$$\underline{I}_R = \frac{\underline{U}}{R} = \underline{1 \cdot e^{-j0^\circ}} = \underline{1 + j 0}$$

$$\underline{I}_L = \frac{\underline{U}}{\underline{Z}_L} = \underline{0,5 \cdot e^{-j90^\circ}} = \underline{0 - j 0,5}$$

$$\underline{I}_C = \underline{I} - \underline{I}_R - \underline{I}_L = \underline{0,804 \cdot e^{j90^\circ}} = \underline{0 + j 0,804}$$



$$\boxed{6} \quad \underline{I} = \frac{\underline{U}}{\underline{Z}_6} = \underline{0,465 \cdot e^{j63,7^\circ}} = \underline{0,206 + j 0,418}$$

$$\underline{I}_C = \frac{\underline{U}}{\underline{Z}_C} = \underline{0,804 \cdot e^{j90^\circ}} = \underline{0 + j 0,804}$$

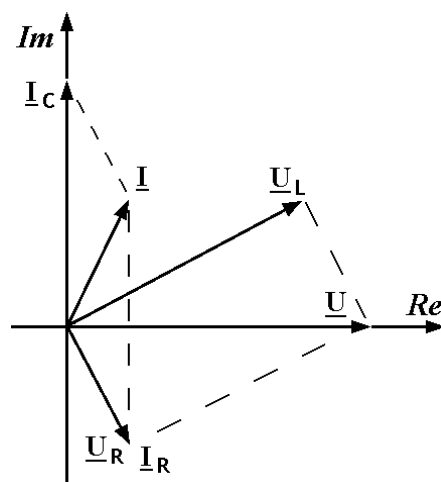
$$\underline{I}_R = \underline{I} - \underline{I}_C = \underline{0,439 \cdot e^{-j62^\circ}} = \underline{0,206 - j 0,387}$$

$$\underline{I}_L = \underline{I}_R$$

$$\underline{U}_R = R \cdot \underline{I}_R = \underline{1,754 \cdot e^{-j62^\circ}} = \underline{0,824 - j 1,548}$$

$$\underline{U}_L = \underline{U} - \underline{U}_R = \underline{3,514 \cdot e^{j28^\circ}} = \underline{3,103 + j 1,651}$$

$$\underline{U}_C = \underline{U}$$



$$\boxed{7} \quad \underline{I} = \frac{\underline{U}}{\underline{Z}_7} = \underline{1,654 \cdot e^{-j52,8^\circ}} = \underline{1 - j 1,317}$$

$$\underline{I}_R = \frac{\underline{U}}{R} = \underline{1 \cdot e^{-j0^\circ}} = \underline{1 + j 0}$$

$$\underline{I}_L = \underline{I} - \underline{I}_R = \underline{1,317 \cdot e^{-j90^\circ}} = \underline{0 - j 1,317}$$

$$\underline{I}_C = \underline{I}_L$$

$$\underline{U}_L = \underline{Z}_L \cdot \underline{I}_L = \underline{10,55 \cdot e^{j0^\circ}} = \underline{10,55 + j 0}$$

$$\underline{U}_C = \underline{U} - \underline{U}_L = \underline{6,55 \cdot e^{j180^\circ}} = \underline{-6,55 + j 0}$$

$$\underline{U}_R = \underline{U}$$

Échelles : 1 cm $\leftrightarrow$ 2 V ; 1 cm $\leftrightarrow$ 0,5 A

