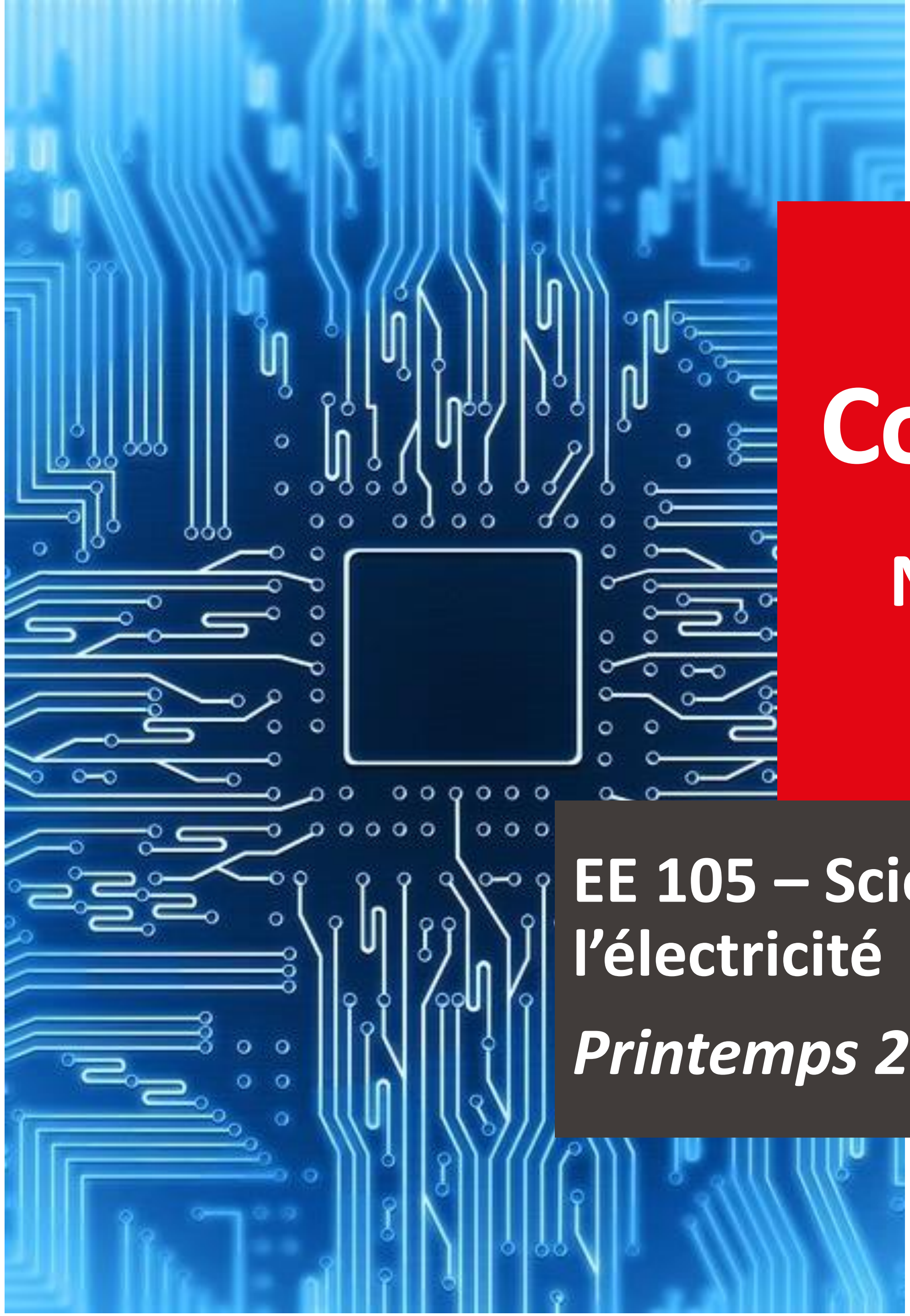
EPFL



Complément de cours

Nombres complexes

EE 105 – Sciences et Technologies de l'électricité

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Définitions

- Forme algébrique

$$z = x + jy$$

- Forme trigonométrique

$$z = \rho\big(\cos(\theta) + j\sin(\theta)\big)$$

- Forme exponentielle

$$z = \rho e^{j\theta}$$

- $\mathrm{Re}(z) = x = \rho \cdot \cos(\theta) \quad ; \quad \mathrm{Im}(z) = y = \rho \cdot \sin(\theta)$

  $|z| = \rho = \sqrt{x^2 + y^2} \quad ; \quad \arg(z) = \theta$

- $\cos(\theta) = \frac{y}{\rho} \quad ; \quad \sin(\theta) = \frac{y}{\rho} \quad ; \quad \tan(\theta) = \frac{y}{x}$

Complexe conjugué

- Forme algébrique

$$z = x + jy \Rightarrow z^* = x - jy$$

- Forme trigonométrique

$$z = \rho\big(\cos(\theta) + j\sin(\theta)\big) \Rightarrow z^* = \rho(\cos(\theta) - j\sin(\theta))$$

- Forme exponentielle

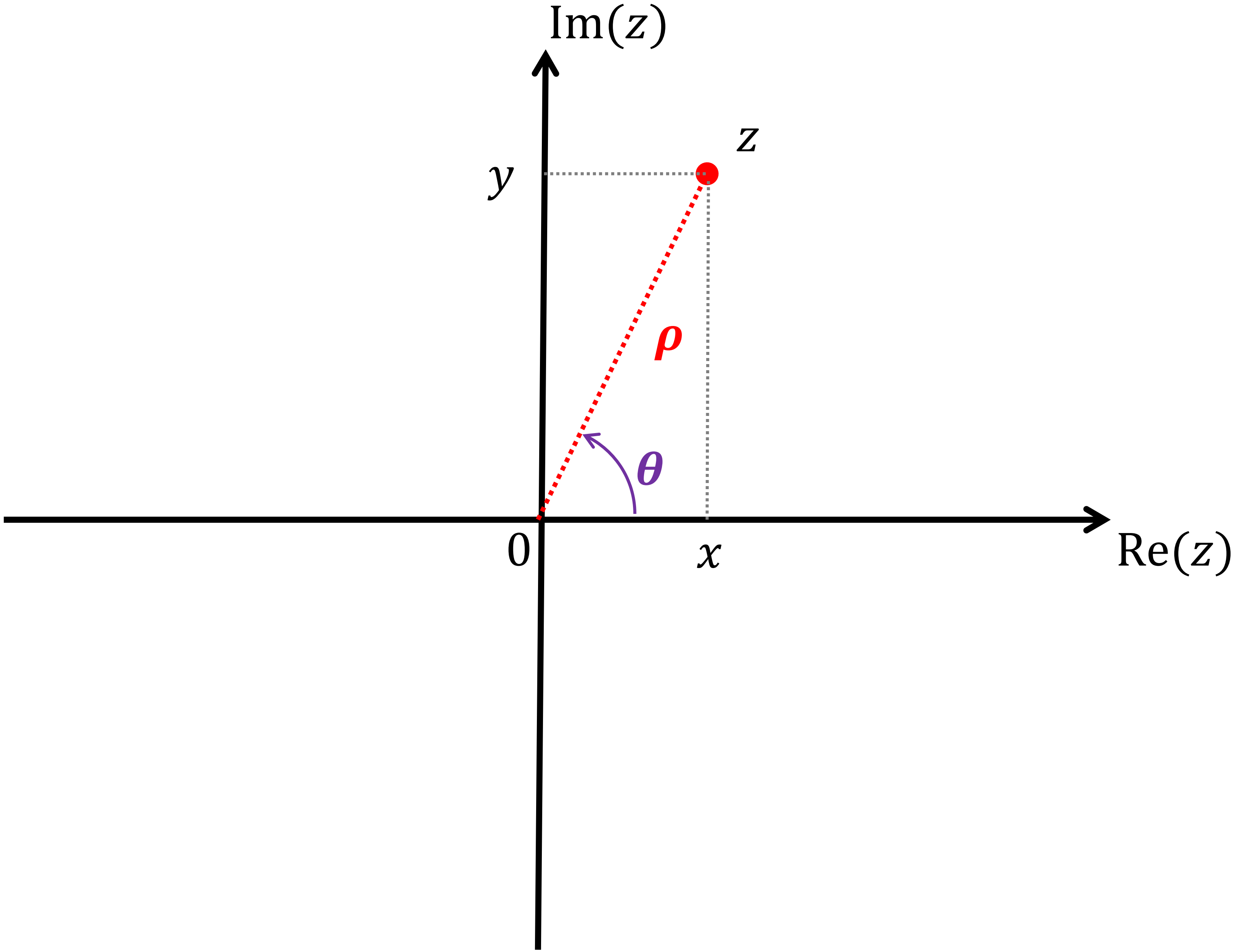
$$z = \rho e^{j\theta} \Rightarrow z^* = \rho e^{-j\theta}$$

- $\mathrm{Re}(z^*) = x = \rho \cdot \cos(\theta) \quad ; \quad \mathrm{Im}(z^*) = -y = -\rho \cdot \sin(\theta)$

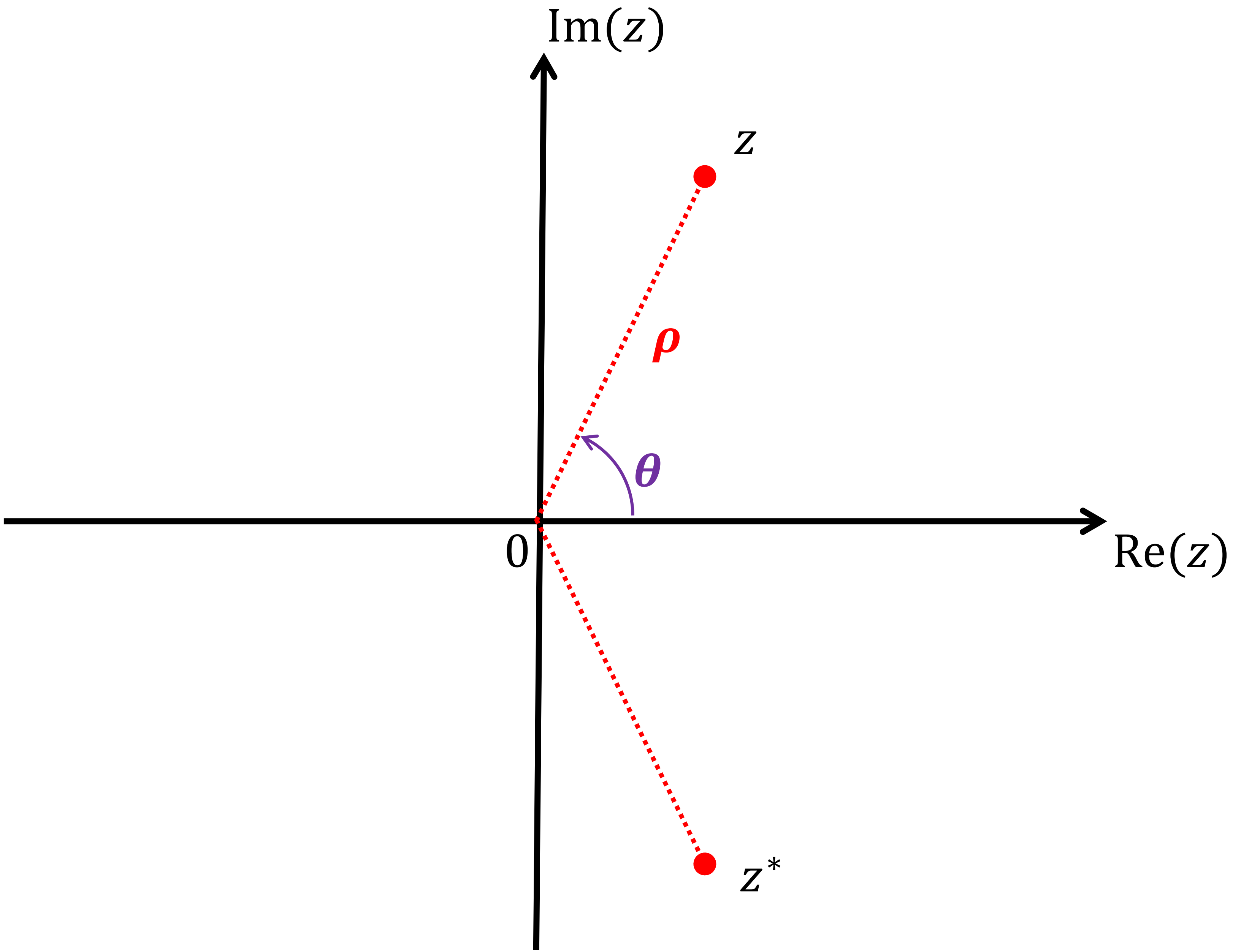
  $|z^*| = \rho = \sqrt{x^2 + y^2} \quad ; \quad \arg(z^*) = -\theta$

- $|z|^2 = z \cdot z^*$

Plan complexe

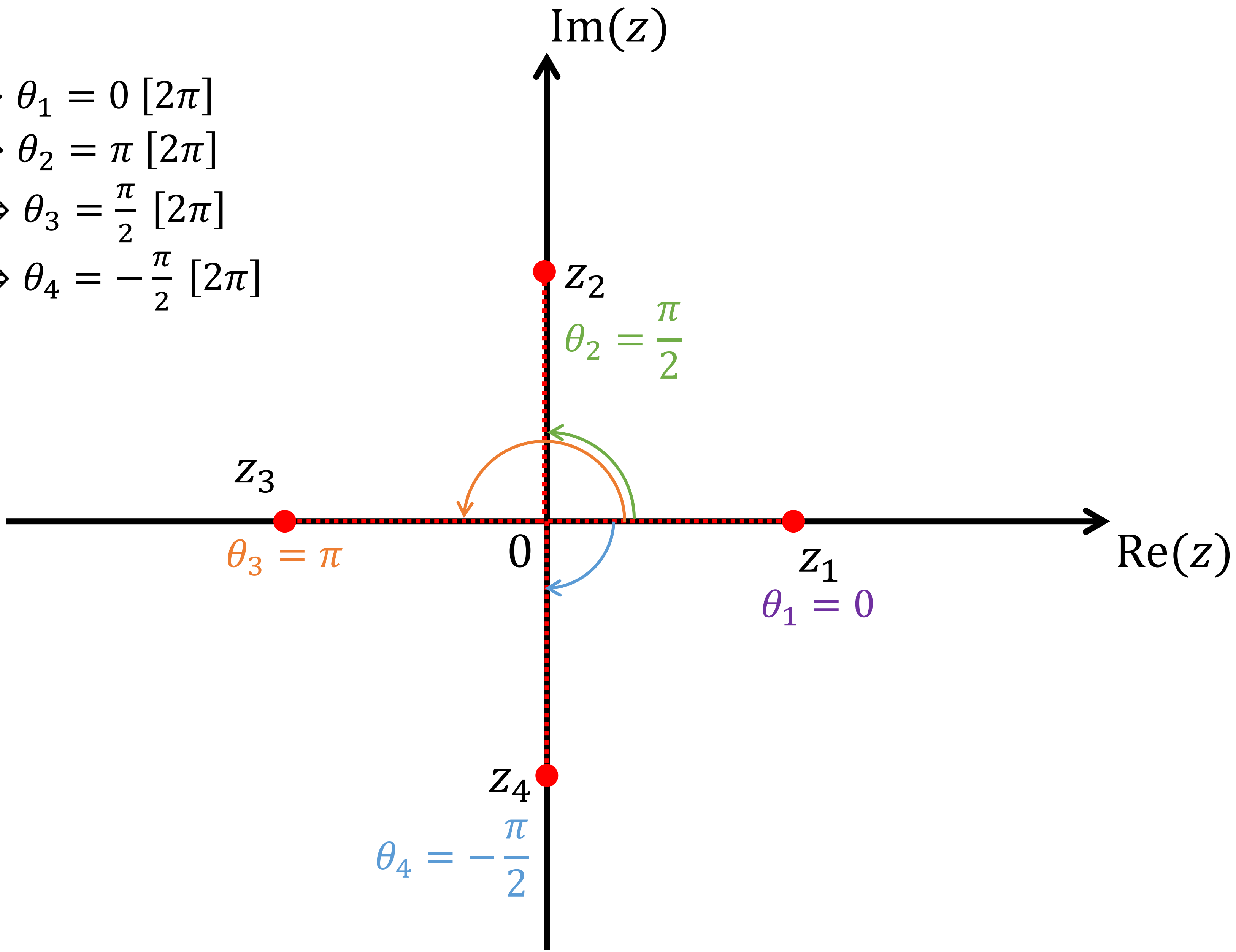


Plan complexe



Cas particuliers

- $z_1 \in \mathbb{R}_+^* \Rightarrow \theta_1 = 0\ [2\pi]$
- $z_2 \in \mathbb{R}_-^* \Rightarrow \theta_2 = \pi\ [2\pi]$
- $z_3 \in j\mathbb{R}_+^* \Rightarrow \theta_3 = \frac{\pi}{2}\ [2\pi]$
- $z_4 \in j\mathbb{R}_-^* \Rightarrow \theta_4 = -\frac{\pi}{2}\ [2\pi]$



Formules

- $\mathrm{Re}(z_1 + z_2) = \mathrm{Re}(z_1) + \mathrm{Re}(z_2)$
- $\mathrm{Im}(z_1 + z_2) = \mathrm{Im}(z_1) + \mathrm{Im}(z_2)$
- $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
- $(z_1 \cdot z_2)^* = z_1^* \cdot z_2^*$
- $\arg(z_1 \cdot z_2) = \arg(z_1) + \arg(z_2)$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
- $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$
- $|z^n| = |z|^n$
- $\arg(z^n) = n \cdot \arg(z)$

Cas particuliers

$z = x + j \cdot y = \rho \cdot e^{j\theta}$

- ❑ $x \in \mathbb{R}_+^* \Rightarrow \theta = \arctan\left(\frac{y}{x}\right)$
- ❑ $x \in \mathbb{R}_-^*, y \in \mathbb{R}_+^* \Rightarrow \theta = \arctan\left(\frac{y}{x}\right) + \pi$
- ❑ $x \in \mathbb{R}_-^*, y \in \mathbb{R}_-^* \Rightarrow \theta = \arctan\left(\frac{y}{x}\right) - \pi$