Lecture 9

Expander codes

We start studying error-correcting codes with the goal of eventually designing local Hamiltonians that have the NLTS property described in the previous lecture.

We first review classical codes. In class, we reviewed elementary definitions around linear binary error-correcting codes and then saw a general class of codes based on bipartite expander graphs. These codes are well described in Section 1 of these notes, so we will not copy the definitions here. The key lemma we used is lemma 3 in the notes. We used this lemma to argue that expander codes have the following "clustering of approximate codewords" property, which we will revisit later in the context of quantum codes.

Lemma 9.1. *Let G be an $(n, m, D, \gamma, (1-\varepsilon))$ bipartite expander, and let H be the parity check matrix associated with the corresponding expander code (i.e. H has a row for each right vertex of G, which is the indicator of its left neighbors). Then for every δ and every $y \in \{0,1\}^n$ such that $|Hy| \leq \delta m$, it holds that either $|y| \geq \gamma n$ or $|y| \leq \frac{\delta}{D(1-2\varepsilon)}\frac{m}{n}n$.*

We refer to this property as "clustering" because it implies that for any y, y' such that both $|Hy|, |Hy'| \leq \frac{1}{2}\delta m$, we have that either y, y' are quite close, $|y - y'| \leq O(1)\delta n$, or y, y' are far, $|y - y'| \geq \gamma n$.