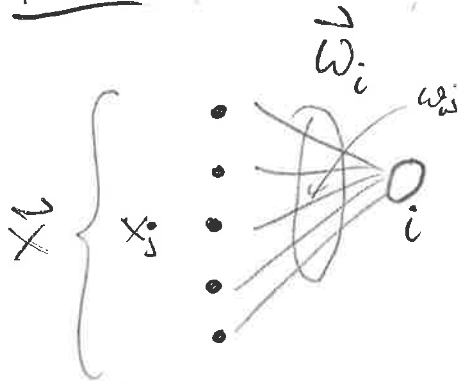
PCA



linear neuron

$$y_i = \sum_k w_{ik} x_k = \vec{w}^T \vec{x} \qquad (1)$$

simple Hebb rule: ν_i^{post}, ν_j^{pre}

$$\Delta w_{ij} = a_2^{corr} \cdot y_i \cdot x_j \qquad (2)$$

insert (1)

$$= a_2^{corr} \cdot \sum_k w_{ik} x_k x_j$$

we present patterns randomly, each for a time Δt

expected change:

$$\underset{\uparrow \text{ all patterns}}{E[\Delta w_{ij}]} = a_2^{corr} \cdot \sum_k w_{ik} \underbrace{E[x_k x_j]} \qquad (3)$$

$$= a_2^{corr} \sum_k w_{ik} C_{kj} \qquad (4)$$

correlation matrix C with element $C_{jk} = \frac{1}{P} \sum_{\mu=1}^{P} x_j^\mu x_k^\mu = C_{kj}$

$$E[\Delta w_{ij}] = \gamma \cdot \Delta t \; w_{ik} C_{jk}$$

$$\frac{d}{dt} w_{ij} = \lim_{\Delta t \to 0} E\left[\frac{\Delta w_{ij}}{\Delta t}\right] = \gamma \sum_k C_{jk} \cdot w_{ik}$$

in the limit, all patterns are presented before a significant change of $\vec{w}_i$. Hence "self-averaging"

$$\boxed{\frac{d}{dt} \vec{w} = \gamma \cdot C \cdot \vec{w}} \qquad (5)$$

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Lemma: $\vec{\omega}$ aligns with eigenvector $\vec{e}_1$ of C.

$\lambda_1 \geq \lambda_2 \geq \dots 0$

Proof: express $\vec{\omega}$ in coordinate system of eigenvectors

$$\vec{\omega}(t) = \sum_n a_n(t) \cdot \vec{e}_n \qquad \text{with } \lambda_n \vec{e}_n = C \cdot \vec{e}_n$$

with (5):

$$\frac{d\vec{\omega}}{dt} = \sum_n \frac{d}{dt} a_n(t) \cdot \vec{e}_n = \gamma \cdot \lambda_n \cdot a_n(t) \cdot \vec{e}_n$$

$$\Rightarrow \boxed{a_n(t) = e^{\gamma \cdot \lambda_n \cdot t} \cdot a_n(0)}$$

therefore

$$\vec{\omega}(t) = e^{\gamma \lambda_1 \cdot t} \Big[a_1(0) \vec{e}_1 + a_2(0) \cdot e^{\gamma(\lambda_2 - \lambda_1) \cdot t} \vec{e}_2 + \dots + \underbrace{a_n(0) \cdot e^{\gamma(\lambda_n - \lambda_1) \cdot t}}_{\text{decays!}} \vec{e}_n$$

$$= e^{\gamma \lambda_1 \cdot t} a_1(0) \cdot \vec{e}_1 \left[1 + \text{small correction} \right]$$

$$\Rightarrow \boxed{\vec{\omega}(t) \text{ becomes parallel to } \vec{e}_1}$$

But: $\|\vec{\omega}(t)\|$ "explodes"