Exercises for week 11
Spike-time dependent plasticity

There are two versions of the same exercise. You are free to choose the formalism that you prefer to solve the exercise.

The goal of this exercise is to show that it is possible to account for the asymmetry in the STDP window using a simple microscopic model of synaptic plasticity.

Exercise 1: Spike-time dependent plasticity by local variables: "Physics version"

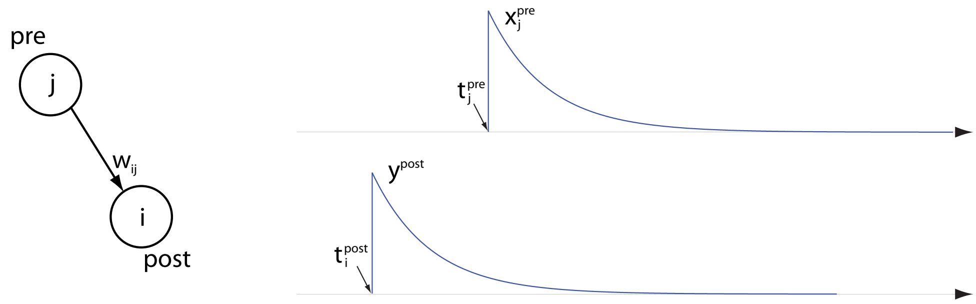

Figure 1: Memory traces of pre- and post-synaptic spike trains.

Suppose that the change in synaptic weight is controlled by the local concentration of two molecules x^{pre} and y^{post}. The substance x^{pre} acts as a memory trace of presynaptic spikes in the sense that each presynaptic spike triggers an increase in the concentration of x^{pre}:

$$\tau_+ \frac{d}{dt} x_j^{\text{pre}} = -x_j^{\text{pre}} + \delta(t - t_j^{\text{pre}})\,. \tag{1}$$

Similarly, y^{post} is the trace left by the postsynaptic spike train,

$$\tau_- \frac{d}{dt} y_i^{\text{post}} = -y_i^{\text{post}} + \delta(t - t_i^{\text{post}})\,. \tag{2}$$

Here t_j^{pre} and t_i^{post} represent the time of the spike of the pre- and post-synaptic neurons respectively.

Calculate the form of the learning window $\Delta w = f(\Delta t)$ – where $\Delta t = t_j^{\text{pre}} - t_i^{\text{post}}$ assuming that the synaptic weights are updated according to the rule

$$\frac{d}{dt} w_{ij} = a_+ x_j^{\text{pre}} \delta(t - t_i^{\text{post}}) - a_- y_i^{\text{post}} \delta(t - t_j^{\text{pre}})\,. \tag{3}$$

The constants a_+ and a_- are both positive.

Hint: Calculate the weight change for an isolated pair of pre/post spikes. Consider the two cases $\Delta t > 0$ and $\Delta t < 0$.

Exercise 2: Spike-time dependent plasticity by local variables: "Computer science version"

Suppose that the change in synaptic weight is controlled by the local concentration of two molecules x^{pre} and y^{post}. The substance x^{pre} acts as a memory trace of presynaptic spikes in the sense that each presynaptic spike triggers an increase in the concentration of x^{pre}:

$$x_j(t+1) = \begin{cases} \beta_x x_j(t) + 1 & \text{if spike } (t = t_j^{\text{pre}}) \\ \beta_x x_j(t) & \text{if no spike} \end{cases} \tag{4}$$

Similarly, y^{post} is the trace left by the postsynaptic spike train,

$$y_i(t+1) = \begin{cases} \beta_y y_i(t) + 1 & \text{if spike } (t = t_i^{\text{post}}) \\ \beta_y y_i(t) & \text{if no spike} \end{cases} \tag{5}$$

Here t_j^{pre} and t_i^{post} represent the time of the spike of the pre- and post-synaptic neurons respectively.

Calculate the form of the learning window $\Delta w = f(\Delta t)$ – where $\Delta t = t_j^{\text{pre}} - t_i^{\text{post}}$ assuming that the synaptic weights are updated according to the rule

$$w_{ij}(t+1) = \begin{cases} w_{ij}(t) + a_+ x_j^{\text{pre}} & \text{if postsynaptic spike } (t = t_i^{\text{post}}) \\ w_{ij}(t) - a_- y_i^{\text{post}} & \text{if presynaptic spike } (t = t_j^{\text{pre}}) \\ w_{ij}(t) & \text{if no spike} \end{cases} \tag{6}$$

The constants a_+ and a_- are both positive.

Hint: Calculate the weight change for an isolated pair of pre/post spikes. Consider the two cases $\Delta t > 0$ and $\Delta t < 0$.