CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 9 Solution — Date: 21/11/2024

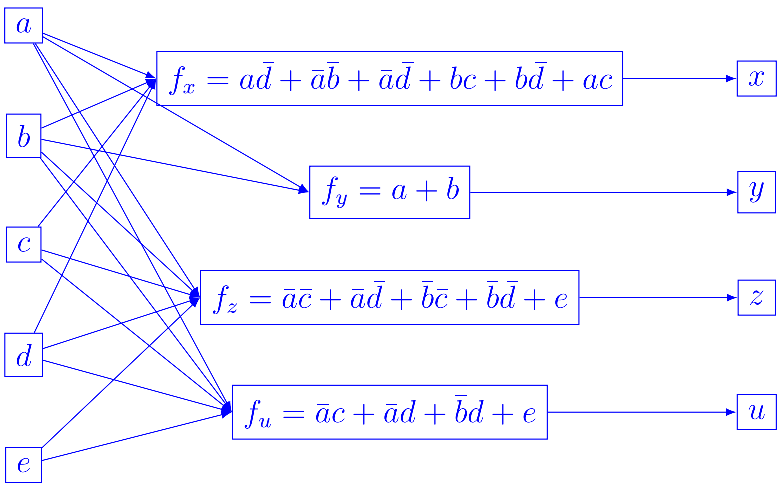
Topic: Algebraic methods (cf. slide set 11)

Problem 1

Consider the following relations among Boolean variables $a, b, c, d, e, x, y, z, u$:

$$
\begin{aligned}
x &= a\bar{d} + \bar{a}\bar{b} + \bar{a}\bar{d} + bc + b\bar{d} + ac \\
y &= a + b \\
z &= \bar{a}\bar{c} + \bar{a}\bar{d} + \bar{b}\bar{c} + \bar{b}\bar{d} + e \\
u &= \bar{a}c + \bar{a}d + \bar{b}d + e
\end{aligned}
$$

(a) Draw the logic network, where inputs are $\{a, b, c, d, e\}$, outputs are $\{x, y, z, u\}$, and each node of the network can compute arbitrary function.

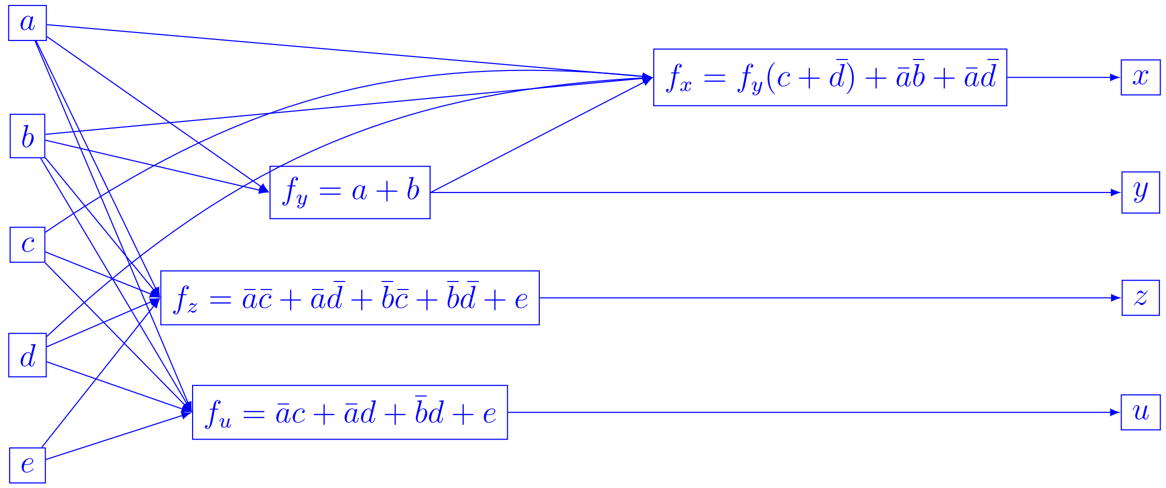


(b) Perform the algebraic division of f_x/f_y (f_x denotes the Boolean function of x in terms of a, b, c, d and e).
cf: Algorithm in textbook pp. 362.
Ans:

$$
\begin{aligned}
&A = \{a\bar{d}, \bar{a}\bar{b}, \bar{a}\bar{d}, bc, b\bar{d}, ac\} \\
&B = \{a, b\} \\
&\quad i = 1 : C_1^B = a. \;\; D = \{a\bar{d}, ac\}, D_1 = \{\bar{d}, c\} = Q. \\
&\quad i = 2 : C_2^B = b. \;\; D = \{bc, b\bar{d}\}, D_2 = \{c, \bar{d}\}, Q = \{c, \bar{d}\} \\
&R = \{\bar{a}\bar{b}, \bar{a}\bar{d}\} \\
&\therefore f_x = f_y(c + \bar{d}) + \bar{a}\bar{b} + \bar{a}\bar{d}
\end{aligned}
$$

Note that in algebraic division, a and $\bar{a}$, etc., are treated as unrelated symbols.

(c) Substitute y in f_x and redraw the network.



Problem 2

For the following functions:

$$F = abc + abd + c\bar{d}$$
$$G = \bar{b}d + a\bar{b} + \bar{b}c$$

(a) Compute all kernels and co-kernels of F and G.
cf: Algorithm in textbook pp. 369; slide 11 pp. 47.

sup(F)	a	b	c	d	$\bar{d}$
$j =$	1	2	3	4	5

KERNALS(F, 1):
$i = 1$: $x_i = a$, $C = \{a, b\}$, $F/(ab) = c + d$, KERNALS($c + d$, 2)= $\{c + d\}$
$i = 2$: $x_i = b$, $C = \{a, b\}$, C has $x_k = a$ where $k < i$, skip
$i = 3$: $x_i = c$, $C = \{c\}$, $F/c = ab + \bar{d}$, KERNALS($ab + \bar{d}$, 4)= $\{ab + \bar{d}\}$
$i = 4$: $x_i = d$, CUBES(F, d)= $\{abd\}$, skip
$i = 5$: $x_i = \bar{d}$, CUBES($F, \bar{d}$)= $\{c\bar{d}\}$, skip
F is cube-free, so F itself is also a kernel with co-kernel being 1.
Ans:

$$K(F) = \{ \quad c + d, \quad ab + \bar{d}, \quad abc + abd + c\bar{d}\}$$
$$CoK(F) = \{ \quad ab, \quad c, \quad 1\}$$

sup(G)	$\bar{b}$	d	a	c
$j =$	1	2	3	4

KERNALS(G, 1):
$i = 1$: $x_i = \bar{b}$, $C = \{\bar{b}\}$, $G/\bar{b} = d + a + c$, KERNALS($a + c + d$, 2)= $\{a + c + d\}$
$i = 2$: $x_i = d$, CUBES(G, d)= $\{\bar{b}d\}$, skip
$i = 3$: $x_i = a$, CUBES(G, a)= $\{a\bar{b}\}$, skip
$i = 4$: $x_i = c$, CUBES(G, c)= $\{\bar{b}c\}$, skip
G is not cube-free ($\bar{b}$), so G itself is not a kernel.
Ans:

$$K(G) = \{a + c + d\}$$
$$CoK(G) = \{\bar{b}\}$$

(b) Extract a multiple-cube sub-expression common to F and G.
cf: slide 11 pp. 53.
Let $x_c = c, x_d = d, x_{ab} = ab, x_{\bar{d}} = \bar{d}, x_{abc} = abc, x_{abd} = abd, x_{c\bar{d}} = c\bar{d}, x_a = a$.
$f_{\text{aux}} = f x_c x_d + f x_{ab} x_{\bar{d}} + f x_{abc} x_{abd} x_{c\bar{d}} + g x_a x_c x_d$
$CoK(f_{\text{aux}}) = x_c x_d$
Ans: Extract $c + d$ from F and G. Now $F = (c + d)ab + c\bar{d}, G = (c + d)\bar{b} + a\bar{b}$.