CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 6 Solution   Date: 24/10/2023

Topics: Boolean operators, recursive paradigm (cf. slide set 8)

Problem 1

Given the Boolean function $F = \bar{a}\bar{c}d + \bar{a}cd + a\bar{b}\bar{c} + abc + ac$

(a) Check if F is negative or positive unate in the variables a, b, c and d.
Ans: First, compute the cofactors:

$$
\begin{aligned}
F_a &= 0 + 0 + \bar{b}\bar{c} + bc + c = \bar{b} + c \\
F_{\bar{a}} &= \bar{c}d + cd + 0 + 0 + 0 = d \\
F_b &= \bar{a}\bar{c}d + \bar{a}cd + 0 + ac + ac = \bar{a}d + ac \\
F_{\bar{b}} &= \bar{a}\bar{c}d + \bar{a}cd + a\bar{c} + 0 + ac = a + d \\
F_c &= 0 + \bar{a}d + 0 + ab + a = a + d \\
F_{\bar{c}} &= \bar{a}d + 0 + a\bar{b} + 0 + 0 = \bar{a}d + a\bar{b} \\
F_d &= \bar{a}\bar{c} + \bar{a}c + a\bar{b}\bar{c} + abc + ac = \bar{a} + a\bar{b}\bar{c} + c \\
F_{\bar{d}} &= 0 + 0 + a\bar{b}\bar{c} + abc + ac = a\bar{b}\bar{c} + ac
\end{aligned}
$$

- When $b = 1, c = 0, d = 1$, $F_a = 0 < F_{\bar{a}} = 1$; when $b = 0, d = 0$, $F_a = 1 > F_{\bar{a}} = 0$. Thus, F is *binate* in a.

- To have $F_{\bar{b}} = 0$, we need $a = 0, d = 0$, which implies $F_b = 0$. However, we can have $F_b = 0 < F_{\bar{b}} = 1$ when, e.g., $a = 1, c = 1$. Thus, $F_b \leq F_{\bar{b}}$ under all assignments, so F is *negative unate* in b.
- To have $F_c = 0$, we need $a = 0, d = 0$, which implies $F_{\bar{c}} = 0$. However, we can have $F_{\bar{c}} = 0 < F_c = 1$ when, e.g., $a = 1, b = 1$. Thus, $F_{\bar{c}} \leq F_c$ under all assignments, so F is *positive unate* in c.
- Observe that all minterms of $F_{\bar{d}}$ are contained by F_d, but $\bar{a}$ in F_d is not contained by $F_{\bar{d}}$. Thus, $F_{\bar{d}} \leq F_d$ and F is *positive unate* in d.

(b) Is F negative or positive unate?
Ans: F is binate because it is binate in at least one variable.

Problem 2

Given the Boolean function $G = \bar{a}\bar{b} + \bar{a}bc + \bar{a}b\bar{c}\bar{d} + ab\bar{c}d + a\bar{b} + abc$, compute:
Cofactors of G with respect to a:

$$G_a = 0 + 0 + 0 + b\bar{c}d + \bar{b} + bc = \bar{b} + c + d = \overline{(b\bar{c}\bar{d})}$$
$$G_{\bar{a}} = \bar{b} + bc + b\bar{c}\bar{d} + 0 + 0 + 0 = \bar{b} + c + \bar{d} = \overline{(b\bar{c}d)}$$

(a) The Boolean difference $\partial G/\partial a$.
Ans: $\partial G/\partial a = G_a \oplus G_{\bar{a}} = b\bar{c}$.

(b) The smoothing $S_a(G)$.
Ans: $S_a(G) = G_a + G_{\bar{a}} = 1$.

(c) The consensus $C_a(G)$.
Ans: $C_a(G) = G_a \cdot G_{\bar{a}} = \bar{b} + c$.

Problem 3

Given the Boolean function $H = \bar{a}d + ac + a\bar{b}\bar{c}$, use the positional cube notation and recursive paradigm to show if the following cubes are contained in H:

$$H = \begin{pmatrix} 10 & 11 & 11 & 01 \\ 01 & 11 & 01 & 11 \\ 01 & 10 & 10 & 11 \end{pmatrix} = \begin{pmatrix} \bar{a}d \\ ac \\ a\bar{b}\bar{c} \end{pmatrix}$$

- cd

$$cd = \begin{pmatrix} 11 & 11 & 01 & 01 \end{pmatrix}$$

$$\Rightarrow H_{cd} = \begin{pmatrix} 10 & 11 & 11 & 11 \\ 01 & 11 & 11 & 11 \\ 01 & 10 & 00 & 11 \end{pmatrix} = \begin{pmatrix} \bar{a} \\ a \\ \emptyset \end{pmatrix} = \top$$

Ans: Cube cd is contained in F.

- ad

$$ad = \begin{pmatrix} 01 & 11 & 11 & 01 \end{pmatrix}$$

$$\Rightarrow H_{ad} = \begin{pmatrix} 00 & 11 & 11 & 01 \\ 11 & 11 & 01 & 11 \\ 11 & 10 & 10 & 11 \end{pmatrix} = \begin{pmatrix} \emptyset \\ c \\ \bar{b}\bar{c} \end{pmatrix} \neq \top$$

Ans: Cube ad is not contained in F.