Concurrent Computing
Mock Midterm Exam

November 25th, 2024

Time: 1h45

- This midterm is "closed book": no notes, electronics, or cheat sheets allowed.
- When solving a problem, do not assume any known result from the lectures, unless we explicitly state that you might use some known result.
- Keep in mind that only one operation on one shared object (e.g., a read or a write of a register) can be executed by a process in a single step. To avoid confusion (and common mistakes) write only a single atomic step in each line of an algorithm.
- Remember to write which variables represent shared objects (e.g., registers).
- Unless otherwise stated, we assume atomic multi-valued MRMW shared registers.
- Unless otherwise stated, we ask for *wait-free* algorithms.
- Unless otherwise stated, we assume a system of n asynchronous processes which might crash.
- For every algorithm you write, provide a short explanation of why the algorithm is correct.

- Make sure that your name and SCIPER number appear on **every** loose sheet of paper you hand in.
- You are **only** allowed to use additional pages handed to you (available upon request).

Good luck!

Problem	Max Points	Score
1	2	
2	2	
3	3	
4	3	
5	4	
Total	14	

Problem 1 (2 points)

Tasks.

1. Write a wait-free algorithm that implements a safe MRSW binary register using (any number of) SRSW safe binary registers.
2. Write a wait-free algorithm that implements a regular MRSW binary register using (any number of) safe MRSW binary registers.

Problem 2 (2 points)

A *snapshot* object maintains an array of registers R of size n, has operations $scan()$ and $update_i()$ and the following sequential specification:

```
1 upon update_i(v) do
2 |  R_i ←v
3 upon scan do
4 |  return R
```

Figure 1: Sequential specification of the snapshot object.

The following algorithm (incorrectly) implements an atomic *snapshot* object using an array of shared registers R:

```
1  upon update_i(v) do
2  |  ts ← ts + 1
3  |  R_i ← (v, ts, scan())
4  upon scan do
5  |  t_1 ← collect(), t_2 ← t_1
6  |  while true do
7  |  |  t_3 ← collect()
8  |  |  if t_3 = t_2 then return ⟨ t_3[1].val, ..., t_3[N].val ⟩
9  |  |
10 |  |  for k ← 1 to N do
11 |  |  |  if t_3[k].ts ≥ t_1[k].ts + 1 then return t_3[k].snapshot
12 |  |  |
13 |  |  t_2 ← t_3
14 upon collect do
15 |  for j ← 1 to N do
16 |  |  x_j ← R_j;
17 |  return x
```

Figure 2: Incorrect implementation of the snapshot object.

Task. Give an execution of the algorithm which violates atomicity of the *snapshot* object.

Problem 3 (3 points)

Consider the linearizable and wait-free log object. The log object supports two operations: append and getLog. The sequential specification of the log object is shown below:

```
1  Given:
2  Sequential linked list L that is initially empty.
3
4  procedure append(obj)
5      L.append(obj)
6
7  procedure getLog()
8      result[] ← ⊥
9      k ← length(L)
10     i ← 1
11     while i ≤ k do
12     |   result[i] ← element(L, i) // the element(L, i) function call returns the i-th element of list L
13     |   i ← i + 1
14     return result
```

Figure 3: Sequential specification of the log object.

Furthermore, consider a linearizable and wait-free fetch-and-increment object where its sequential specification is shown below:

```
1  Given:
2  Register R that is initially 0.
3
4  procedure fetchAndIncrement()
5      old ← R
6      R ← old + 1
7      return old
```

Figure 4: Sequential specification of the fetch-and-increment object.

Is it possible to implement the linearizable and wait-free log object by using any number of read-write registers and fetch-and-increment objects? Explain your answer.

Problem 4 (3 points)

An atomic shared counter maintains an integer x, initially 0, and has two operations `inc()` and `read()`. The sequential specification is as follows:

```
1  x integer, initially 0
2  upon read(x) do
3  |  return x
4  upon inc(x) do
5  |  x ← x + 1
```

Consider the following, *incorrect*, implementation of an obstruction-free consensus object from shared counters:

```
   uses: C_0, C_1 – atomic shared counters initialized to 0
1  upon propose(v) do
2  |  while true do
3  |  |  (x_0, x_1) ← readCounters()
4  |  |  if x_0 > x_1 then
5  |  |  |  v ← 0
6  |  |  else if x_1 > x_0 then
7  |  |  |  v ← 1
8  |  |  if |x_0 − x_1| ≥ 1 then
9  |  |  |  return v
10 |  |  C_v.inc()
11 upon readCounters() do
12 |  while true do
13 |  |  x_0 ← C_0.read()
14 |  |  x_1 ← C_1.read()
15 |  |  x'_0 ← C_0.read()
16 |  |  if x_0 = x'_0 then
17 |  |  |  return (x_0, x_1)
```

Give an execution of the above algorithm that shows that the algorithm is not a correct implementation of an obstruction-free consensus object, i.e. an execution in which some property (obstruction-freedom, validity, or agreement) of obstruction-free consensus is violated.

Problem 5 (4 points)

An atomic 0**-set-once** object is a shared object that has three states $\perp$, 0, and 1. $\perp$ is the initial state. It provides only one operation *set(v)* where $v \in \{0, 1\}$, such that:

- If the object is in state $\perp$, then *set(v)* changes the state of the object to v and returns v.
- If the object is in state s where $s \in \{0, 1\}$, then *set(v)* changes the state of the object to $s \wedge \neg v$ and returns the new state of the object (i.e., $s \wedge \neg v$).

Tasks.

1. Explain what it means for a shared object to have infinite consensus number.
2. Write an algorithm that solves consensus among two processes using any number of 0**-set-once** objects and registers.
3. Prove that the atomic 0**-set-once** object has infinite consensus number.