



Exercise I, Sublinear Algorithms for Big Data Analysis 2024-2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students, and solve as many problems as you can. Problems marked (*) are more difficult, but also more rewarding. These problems have been taken from various sources on the Internet, too numerous to cite individually.

- 1 In class we saw a constant factor approximate randomized counting algorithm with space complexity $O(\log \log n)$, where n is the maximum value of the counter. Prove that any *deterministic* algorithm that provides a factor 2 approximation to the count must use $\Omega(\log n)$ space.
- 2 Let X_n denote the random variable maintained by Morris' algorithm after n events have happened. Prove that $\mathbf{E}[2^{2X_n}] = O(n^2)$.