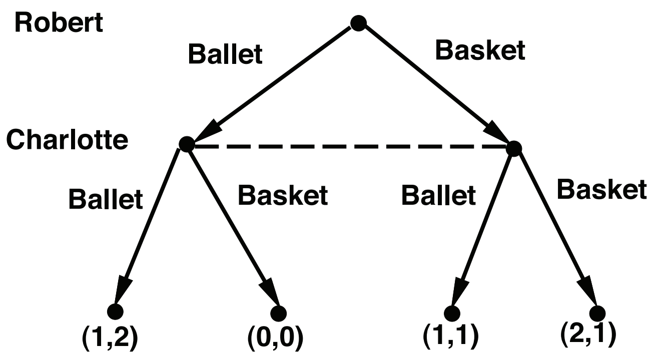
Intelligent Agents

Paper Exercise: Introduction to Game Theory

Solutions

Question 1: Robert and Charlotte like each other and are thinking of what to do on Saturday evening. Robert would like to attend a Basketball game, while Charlotte would like to attend a Ballet performance. But most of all, they would like to do something together. Suppose that each gets a utility of 1 for attending his/her most preferred activity, and another utility of 1 for being at the same place as the other person. Model this situation as a game, both in extensive and normal form

Answer:



Charlotte

Robert	Basket	Ballet
Basket	2,1	1,1
Ballet	0,0	1,2

Question 2: Consider the game in Figure 1. Does this game have a dominant strategy equilibrium? What is it? Explain your answer.

Player B

Player A	B1	B2
A1	-1, 1	0, 4
A2	2, 2	3, 3
A3	0, 1	2, 2

Figure 1.

Answer: yes, A plays A2 and B plays B2.

Question 3: Can a game have multiple dominant equilibria? Motivate.

Answer: Yes, but only if there are multiple weakly dominant strategies. In this case, any combination of them is an equilibrium, and they all have the same payoff.

Question 4: Consider the game in Figure 2. Does it have a dominant strategy equilibrium? Do the players have pure minimax strategies? What are these strategies? Motivate your answer.

Player A	Player B: B1	Player B: B2
A1	-1, 1	0, 0
A2	3, -3	2, -2
A3	4, -4	-1, 1

Figure 2.

Answer: No. A3 is best against B1 but A2 is best against B2. For B, B1 is best against A1 but B2 is best against A2 and A3. The minimax strategy for A is A2 (worst case gain of 2), for B it is B2 (worst case loss of 2).

Question 5: Consider the game in Figure 3. What are the minimax strategies (pure or mixed) of the two players? Motivate your answer.

Playe	Player B: Head	Player B: Tail
Head	1, -1	-1, 1
Tail	-1, 1	1, -1

Figure 3. *The Matching Pennies Game*. Each of two players chooses either Head or Tail. If the choices differ, player A pays 1 Franc to player B. If they are the same, player B pays 1 Franc to player A.

Answer: there are only mixed minimax strategies, they are for both players to play Head and Tail with equal probability.

Question 6: We would like to characterize an agent's preferences among the following 4 events by a utility function that assigns a numerical utility to each of them, where the utility of the least preferred event should be equal to 1:

1. it obtains a low quality image of the Cervin.
2. it obtains a low quality image of the Mont Blanc.
3. it obtains a high quality image of the Cervin.
4. It obtains a high quality image of the Mont Blanc.

Given that we know that the following are equally good to the agent:

a) a lottery that gives it 2 or 4 with 50% probability each vs. outcome 1 with certainty.
b) a lottery that gives it 1 or 3 with 50% probability each vs. outcome 4 with 60% and 2 with 40%.
c) 3 vs. 4 with 80% probability.

Answer: Let u1,u2,u3,u4 be the utilities of the 4 events. The statements translate to the following equations:

a) $0.5\ u2 + 0.5\ u4 = u1$

b) $0.5\ u1 + 0.5\ u3 = 0.6\ u4 + 0.4\ u2$

c) $u3 = 0.8\ u4$

using a) to replace u1 in b) gives:

$0.25u2 + 0.25u4 + 0.5u3 = 0.6u4 + 0.4u2 \Leftrightarrow 0.5u3 = 0.35u4 + 0.15u2$

Further using c) to replace u3 results in: $0.05u4 = 0.15u2$
$u4 = 3u2 = 3u2$
$u3 = 0.8\ u4 = 2.4u2$
$u1 = 0.5\ u2 + 0.5\ u4 = 2u2$
So we see that u2 is the lowest value =1, and we have:
U1=2, u2=1, u3=2.4 and u4=2.

Question 7: Do the games in Figures 1, 2 and 3 have a Nash Equilibrium? What is it? Motivate. Is it true that any dominant equilibrium is also a Nash equilibrium?

Answer:
Figure 1: yes, the dominant strategy equilibrium is also a Nash equilibrium, and this holds in general.
Figure 2: (A2,B2) is a Nash equilibrium.
Figure 3: the minimax strategies ([0.5,0.5],[0.5,0.5]) form a Nash equilibrium.

Question 8: Find all Nash equilibria of the game in Figure 5 using the Algorithm given in class.

		Player B			
Player A		B0	B1	B2	B3
	A0	1, 2	1,2	0,3	1,0
	A1	2,1	0,0	2,1	4,2
	A2	1,1	1,2	3,0	1,1
	A3	2,1	2,4	2,1	2,2

Figure 5.

Answer: First delecte dominated strategies:

- Delete A0 since it is dominated by A3.
- Delete B2 since it is dominated by B3.
- Delete A2 since it is dominated by A3.
- Delete B0 since it is dominated by B3.

Nash equilibria of the remaining game:
Pure: (A1,B3) (A3,B1)
Mixed: could be ([0.5,0.5],[0.5,0.5]) with revenue (2,2)
This is not (trembling-hand) perfect since agents would switch to the pure equilibria.

Question 9: We have seen that finding Nash equilibria in zero-sum games is significantly easier than in general games. Now consider the problem of finding Nash equilibria in a zero-sum game with 3 (not 2) players. Show how to reduce the problem of finding Nash equilibria in general 2 player games to Nash equilibria of 3 player zero sum games, and thus prove the hardness of this problem.

Answer: We just add a dummy player in the general game. Then, we can show that NEs in (n-1)-players general games are the same as in n-players zero-sum games. NEs of 2-players zero-sum game can be computed in polynomial time. Moreover, finding NEs of n-players zero-sum game and (n-1)-players general game is "polynomial parity argument, directed version" (4.2.1, Shoham & Leyton-Brown).