Learning Agents

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Machine learning for agents

Often, effects of actions are not known a priori:

- self-driving car: how steering/acceleration/braking affects trajectory.
- financial markets: market reaction.
- recommendation: does the user like this item?
- ad placement: select one of possible ads to display.

$\Rightarrow$ agent has to learn them from observation.

Learning to act

Different from learning from data:

- gathering data requires *exploration* which may be costly or even dangerous (e.g. medical trials).
- actions may have to be part of a sequence of steps to be beneficial.
- world is often not static and model has to adapt.
- other agents may change their behavior and invalidate the model.

Different form of learning: *reinforcement learning*.

Motivating Application - Medical Trial

- Treat patients effectively for a new strain of a disease.
- Actions: $drug_1, drug_2, \ldots, drug_n$
- Reward:
  - 1 / 0 for success or failure,
  - Patient's condition improvement (%)
  - 1 / time to heal, etc.

- Goal:
  - Identify the efficiency of each treatment (exploration).
  - Give each patient the best possible treatment (exploitation)
- Trials:
  - expensive (find qualified patients, cost of drugs, etc.)
  - dangerous (patients may die or be ill)

$\Rightarrow$ minimize needs for trials.

Model for learning agents

- Agents maintain a *Q-table* that estimates the expected cumulative future reward of each action:

$$Q(s,a) \simeq r(s,a) + \gamma \sum_{s'} Pr(s',s,a) \max_a Q(s',a)$$

- Q-table is learned from the agent observing effects of its actions.
- optimal action $a^* = argmax_a Q(s,a)$.
- Simpler model than MDP: state transitions do not need to be explicitly modelled.

First consider model without state: learn $Q(a) = r(a)$.

Updating the Q-table

- Bayesian update: given observation $r(a_t)$ (e.g. 1 / time to heal):

$$Q_{t+1}(a) = \begin{cases} \alpha r(a) + (1-\alpha) Q_t(a) & \text{for } a = a_t \\ Q_t(a) & \text{otherwise} \end{cases}$$

- Moving average: $\alpha = 1/n(a,t)$ where $n(a,t)$ = number of times a was taken up to time t.
- Larger α gives more importance to recent observations.
- Every action gets played a large number of times
  $\Rightarrow$ estimates become precise
  $\Rightarrow$ Q-table converges to correct values.

Exploration-Exploitation Tradeoff

- We don't want to do random moves infinitely often!
- Need a strategy so that we can balance the need for *exploration* with the desire for *exploitation* what was already learned!
- Goal is to minimize overall *regret*: difference between the payoff with the optimal policy vs. payoff with the current policy.
  - exploration: improve the model to reduce future regret.
  - exploitation: use the model to reduce current regret.

Objective: minimize regret

- Agent can only do as good as the environment lets it.
- Regret l = difference in reward between
  - action a^* taken by the best static policy.
  - action taken by the agent's policy π.
- Goal is to minimize *cumulative regret* L_T:

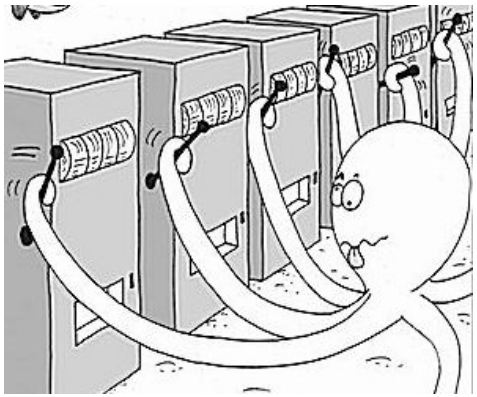
$$L_T = \sum_{t=1}^{T} l_t = \underbrace{max_a \sum_{t=1}^{T} r_t(a)}_{=T \cdot r_t(a^*)} - \sum_{t=1}^{T} r_t(\pi)$$

- Learning without state
  $\Rightarrow$ optimal action does not change over time
  $\Rightarrow$ compare with best single action a^* taken at every step.

The Multi-Armed Bandit (MAB) Problem

A slot machine (bandit) with multiple arms:

- Each arm (move) generates rewards with an unknown probability distribution.
- Rewards are only observed when playing the i^{th} arm.
- A player uses a policy to choose the next arm based on previous plays.

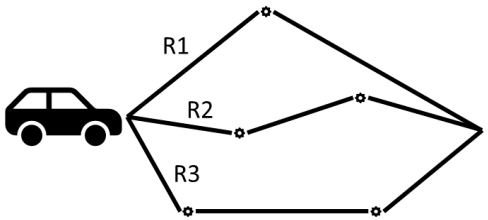


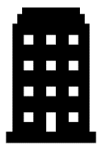
Objective

Maximize the sum of the rewards over many plays.

Example

- Reward for each action is independent and identically distributed.
- E.g. route selection:
  - $$Q_{t+1}(a) = \begin{cases} \alpha r(a) + (1-\alpha) Q_t(a) & \text{for } a = a_t \\ Q_t(a) & \text{otherwise} \end{cases}$$
  - Assume max travel time 20 min $\Rightarrow r(a) = 1 - \frac{time}{20}, \alpha = 0.1$





	Time (min)			
R1	-	-	12	14
R2	18	-	-	-
R3	-	9	-	-

➡

Q table				
1	1	0.94	0.88	
0.91	0.91	0.91	0.91	...
1	0.96	0.96	0.96	

Example (2)

- E.g. slot machines:
  - $$Q_{t+1}(a) = \begin{cases} \alpha r(a) + (1-\alpha) Q_t(a) & \text{for } a = a_t \\ Q_t(a) & \text{otherwise} \end{cases}$$
  - Assume max prize 20CHF $\Rightarrow r(a) = \frac{prize}{20}, \alpha = 0.1$

Prize (CHF)					Q table				
-	10	-	11	15	1	0.95	0.95	0.91	0.894
9	-	20	-	-	0.945	0.945	0.951	0.951	0.951

Epsilon-greedy Strategy

- Choose best action according to current Q-table with probability $1 - \epsilon$, choose a random action otherwise.
- As $t \to \infty$, clearly every action is tried an infinite number of times.

$\Rightarrow$ Q-learning will converge to the true table.

$\Rightarrow$ eventually, the average regret when selecting the optimal action will approach 0.

Motivating Application - Medical Trial (2)

- ϵ-greedy exploration:
  - High probability: prescribe the most successful drug.
  - Periodically prescribe random ones.

Performance of ϵ-greedy

After convergence:

- With probability $1 - \epsilon$, agent takes optimal action.
- However, takes a suboptimal action with probability ϵ.

$\Rightarrow$ cumulative regret increases linearly with time ($O(c\epsilon t)$).

- Note: $c = max_{a,a'} r(a) - r(a')$: bigger differences between actions cause bigger regret.

Average regret for an action only decreases as ϵ.

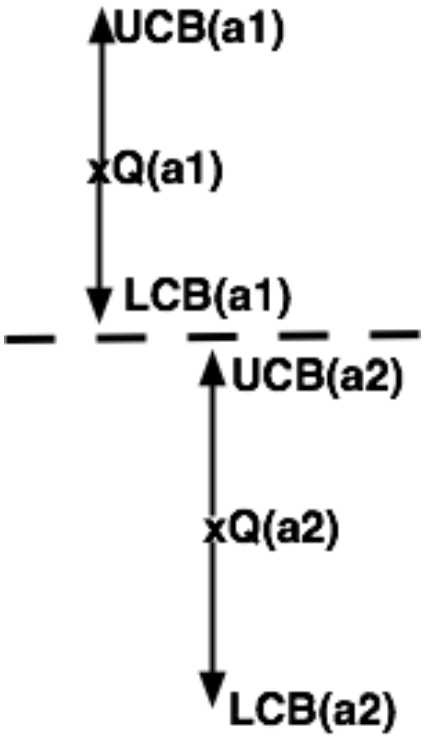
Decreasing ϵ

- Growth of regret depends on ϵ: decreasing ϵ with time can reduce regret growth!
- Example: $\epsilon \sim 1/t$: cumulative regret $L_t = \int_t c/t = O(\log t)$.
- However, need to also ensure that Q-learning receives sufficient samples to converge.

$\Rightarrow$ rate should depend on an estimate of convergence.

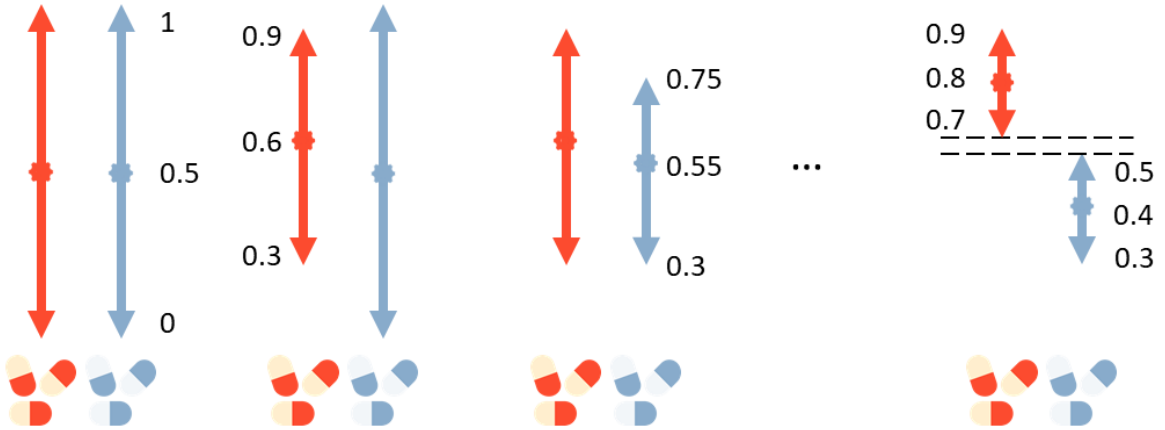
Confidence Bounds

- Maintain *confidence bounds* on the rewards $r(a)$ of each action a based on observations:
  $Pr[LCB(a) < r(a) < UCB(a)] \geq 1 - \delta$
- Successive elimination: alternate a_1, a_2 until $UCB_T(a_2) < LCB_T(a_1) \Rightarrow$ eliminate a_2 since optimal only with small probability $< 2\delta$.
- Optimism under uncertainty: pick the action with the best upper bound.

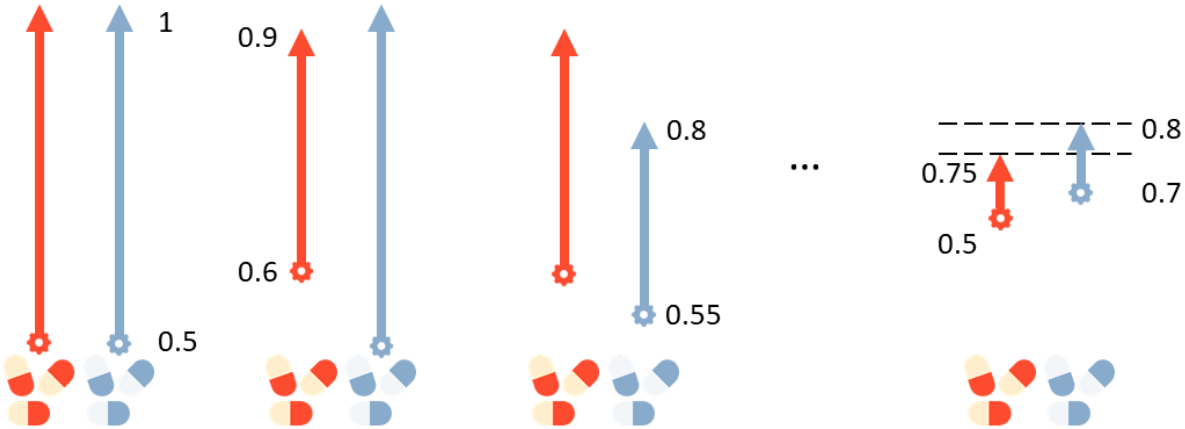


Motivating Application - Medical Trial (3)

Successive Elimination:



Upper confidence bound (optimistic):



Upper confidence bounds

- Hoeffding bound (for random X scaled to [0..1], t samples):

$$Pr[X_{t+1} > \bar{X}_t + u] \leq e^{-2tu^2}$$

- Suppose we want $Pr[Q^* > Q_t + u] \leq \delta$, N_t samples:

$$u(a) = \sqrt{\frac{-\log \delta}{2N_t(a)}} = \sqrt{\frac{2 \log t}{N_t(a)}}$$

where second equality holds when choosing $\delta = t^{-4}$:

- (optimistic) UCB1 algorithm: assume actual reward close to upper bound; select best action according to $Q_t(a) + u(a)$.

$\Rightarrow$ actions with small N_t are played more often.

Bounding expected regret

- Theorem: UCB1 algorithm achieves expected cumulative regret:

$$lim_{t\to\infty} L_t \leq \frac{8 \log t}{\sum_a \Delta_a}$$

  where *gap* Δ_a is the difference $max_{a'} r(a') - r(a)$.
- Logarithmic in t
- Very similar rewards $\Rightarrow$ learning takes longer to distinguish.
- No matter what the (fixed) distribution of rewards.
- Agent only needs to observe reward for the action it has actually taken.

Average regret of each action tends to zero: *no-regret* learning.

Complex rewards

Rewards can have complex structure, for example recommending an event:

- Choices $= \{\underbrace{F(ootball), B(asketball)}_{type=S(ports)}, \underbrace{O(pera), C(oncert)}_{type=C(ulture)}\}$
- Actions of same type have similar rewards.
- Can be modelled statistically with covariances...
- ...but UCB model difficult to extend.

Thompson sampling

- Principle: play each action with the probability that it is optimal.
- Implementation:
  1. sample rewards for each action according to distribution of observed rewards.
  2. then choose action that gives the highest reward.
- Extends to cases where rewards are a function of an underlying model: model can be fit to observations and sampled.
- Regret bound is $O(\sqrt{t \log t})$, worse than UCB.
- However, performance in practice observed to be much better!

Example (Thompson sampling)

- Observe rewards for each user and recommendation and fit statistical model (of any complexity).
- Sample one specific set of rewards from this model.
- Choose action with maximum reward according to this sample.
- Probabilistic choice means that each action has some probability of being chosen.
- Actions are reinforced in the statistical model when they succeed.

Adversarial bandits

- What if rewards can change over time:
  - randomly?
  - set by an adversary to fool the learning algorithm?
    e.g. rewards: (0, 1), (1, 0), (0, 1), ...
- Focus on adversarial setting as worst case.
- Insight: play needs to be random, otherwise adversary can always make the chosen action have the worst payoff.

Motivating Examples

- Medical trial:
  - Same treatment to everyone $\Rightarrow$ antibiotic resistant bacteria $\Rightarrow$ low reward.
  - $\Rightarrow$ maintain a mixture of antibiotics.
- Financial investments:
  - Want to minimize worst-case loss
  - $\Rightarrow$ diversify into different investments.

Adversarial rewards

- Adversary knows policy $\pi(a) = Pr(a)$ and maximizes regret.
- Baseline policy: agent always plays the same action a^*
- Regret of policy $\pi(a)$:

$$L_T = max_{a^* \in A} \sum_{t=1}^{T} \left[r_t(a^*) - \sum_{a \in A} \pi(a) r_t(a) \right]$$

- Deterministic policy $(\pi(a_t) = 1)$:
  adversary sets $r_t(a_t) = 0, r_t(a^*) = 1$ for some $a^* \neq a_t$.
- if $\exists a^*$ with $\pi(a^*) = 0$, sets $r(a^*) = 1, r(a \neq a^*) = 0$.

Randomized strategy

- Uniform policy ($\pi(a_t) = 1/k$): expected reward $1/k$ no matter what the reward function, adversary has no influence.
- To maximize regret, adversary picks an a^* and sets $r(a^*) = 1$.
- All choices of a^* give the same regret $(k-1)/k$: no good strategy.
- However, regret is large: can we find better randomized strategies?

Adversarial strategies

- Counterfactual regret of action a when agent played π_t:

$$l(a)_t = r(a)_t - r(\pi_t)_t$$

  (= regret for not having taken action π_t instead of a)
- Cumulative regret of action a:

$$L_T(a) = \sum_{t=1}^{T} l(a)_t$$

- Make all actions have the same cumulative regret $\Rightarrow$ adversary cannot pick a good a^*.
- Insight: whenever agent plays a, cumulative regret of a is unchanged, and decreases relative to the rest.

$\Rightarrow$ Regret matching: play a with the largest positive cumulative regret to reduce its relative cumulative regret.

Regret matching

- Assumption: can observe rewards that would be obtained for all actions (even if the action was not taken).
- Regret matching: play action a with probability proportional to its cumulative regret.
- Adversary can choose rewards to hurt policy π (before learning); policy converges to the best response.
- But actions are deterministic and predictable: adversary could exploit the exploration.

Multiplicative weight update

- Policy = probability distribution P(a): action a is taken with probability P.

$$P(a) = \frac{w(a)}{\sum_a w(a)}$$

- Initialize weights as $w(a) = 1$.
- Update weight of each action according to its reward $r_t(a)$:

$$w(a)_{t+1} = w(a)_t(1 + \epsilon r_t(a)/B)$$

then adjust probabilities through normalization (B = upper bound on possible rewards).

- Bound on cumulative regret (assuming B=1):

$$L_T = \sum_{t=1}^{T} E_{P(t,a)} r_t(a) - E_{P^*(t,a)} r_t(a) \leq \frac{\log |A|}{\epsilon} + \epsilon T$$

Regret bounds

- Choose $\epsilon = 1/\sqrt{T} \Rightarrow \mathit{regret} = O(\log |A| \sqrt{T})$
- Weaker bound than for UCB ($O(\log T)$), but can handle adversarial rewards.
- Note: regret is against a single best policy played throughout all T steps (not varied by adversary).

Exponential weight update (exp-3)

- Learning from only the rewards of the action actually taken $\Rightarrow$ distribution of samples is unbalanced.
- Instead of multiplicative update, multiply weight $w(a)$ by an exponential:

$$w(a)_{t+1} = w(a)_t e^{\epsilon r_t(a)/p_t(a)}$$

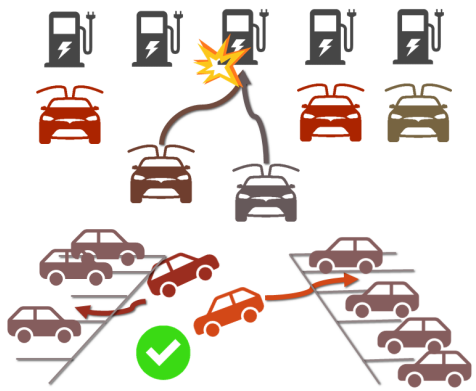
$\Rightarrow$ changes are stronger when action is not very likely, to even out convergence.

- best cumulative regret bound is $O(\sqrt{T|A|\log|A|})$ (when setting $\epsilon \propto 1/\sqrt{T}$).

Example

- Autonomous vehicles trying to acquire a charging station, or parking spot (two available resources: s_1, s_2).
- Repeated interactions until successful access.
- Binary reward 1 / 0 (success / failure).

- UCB1 *fails*:
  - Initially $UCB(s_1) = UCB(s_2)$.
  - Assume both initially select s_1 (collision) $\Rightarrow UCB(s_1) < UCB(s_2)$ for both agents.
  - Since $UCB(s_1) < UCB(s_2)$, both select s_2 (collision).
  - Next, both select s_1 (collision).
  - ...
- EXP3 *randomizes*!

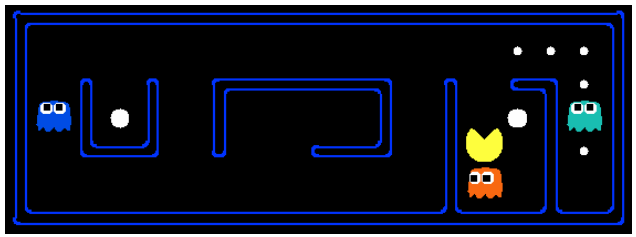


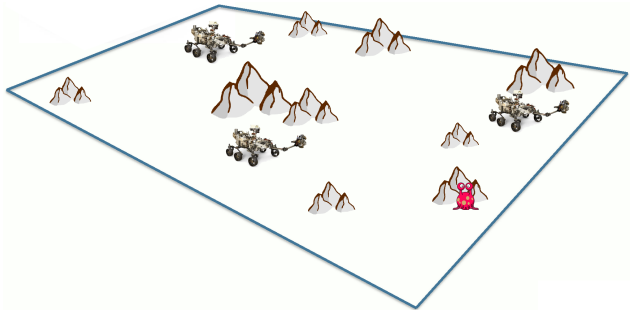
Learning with state

- Models so far learn to optimize instantaneous rewards.
- What if:
  - rewards also depend on state, and
  - actions also cause unknown state transitions?

Examples

- Video Games
  - Position of pac-man, ghosts, dots
  - Active power pellet?
  - Time, #lives
- Robotics
  - Position
  - Goal
  - Teammates
  - Obstacles





Q-learning

- Learn table $Q(s, a) =$ sum of instantenous reward plus discounted value of successor state.
- start with arbitrary initial values.
- Observations:
  (s,a,r,s')

  action a in state s gives reward r and leads to state s'.
- $\Rightarrow$ improve current value of $Q(s, a)$ using *Q-learning rule*:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + \alpha(r + \gamma max_{a'} Q(s', a'))$$

  $\alpha =$ learning factor $\in [0..1)$, must decrease over time
- Q-learning is guaranteed to converge to the optimum, as long as sample is representative.

Q ⇒ optimal policy

Optimal policy: take action that maximizes Q:

$$\pi(s) = \overset{argmax}{a} \; Q(s, a)$$

However, as before there is a tradeoff between *exploration* and *reward*.

Convergence of Q-learning

Let

- $Q^*(s,a)$ be the true maximum reward reachable from state s by taking action a.
- $Q_t(s,a)$ the estimate at time t.
- $\Delta_t = max_{s,a}|Q_t(s,a) - Q^*(s,a)|$.
- $s' =$ state following $s \Rightarrow \Delta_{t+1} = (1-\alpha)\Delta_t + \alpha\delta_t$, where:

$$\begin{aligned}
\delta_t &= |\{R + \gamma max_{a'} Q_t(s',a')\} - \{R + \gamma max_{a''} Q^*(s',a'')\}| \\
&= \gamma|max_{a'} Q_t(s',a') - max_{a''} Q^*(s',a'')| \\
&\leq \gamma max_{a'''}|Q_t(s',a''') - Q^*(s',a''')| \\
&\leq \gamma max_{s'',a'''}|Q_t(s'',a''') - Q^*(s'',a''')| \\
&= \gamma\Delta_t
\end{aligned}$$

so that:

$$\Delta_{t+1} \leq (1 - \alpha + \alpha\gamma)\Delta_t$$

Convergence of Q-learning (2)

- $\alpha, \gamma < 1 \Rightarrow (1 - \alpha + \alpha\gamma) < 1 \Rightarrow \Delta_{t+1} < \Delta_t$
- Thus, in the limit, the difference between $Q_t(s, a)$ and $Q^*(s, a)$ goes to zero!
- However, requires that all states and actions are visited sufficiently often.

Exploration-Exploitation tradeoff in Q-learning

- Convergence of Q-learning requires that all states and actions are visited a sufficient number of times.

$\Rightarrow$ same issue as in bandit problems.

- However, complicated by state transitions: not all states are easily reachable.
- Transitions cause dependencies between successive states that contradict the independence assumptions of bandit algorithms.
- Extreme case: transition graph is not connected $\Rightarrow$ complete exploration impossible.

Initializing the Q-table

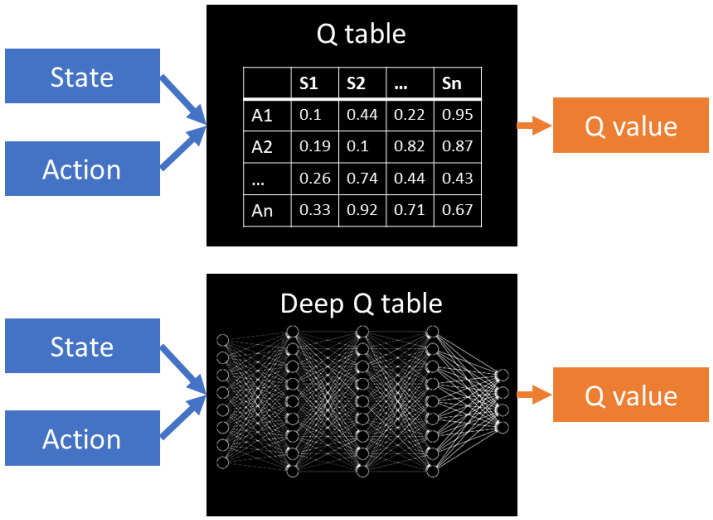
- Initialize all rewards to very high values $\Rightarrow$ action selection will pick actions that have not been tried a lot.
- However, space is very large $\Rightarrow$ will take a long time to converge.
- ϵ-greedy may be a good alternative.
- hard to prove optimality or even convergence: depends on state transitions.

Deep q-learning

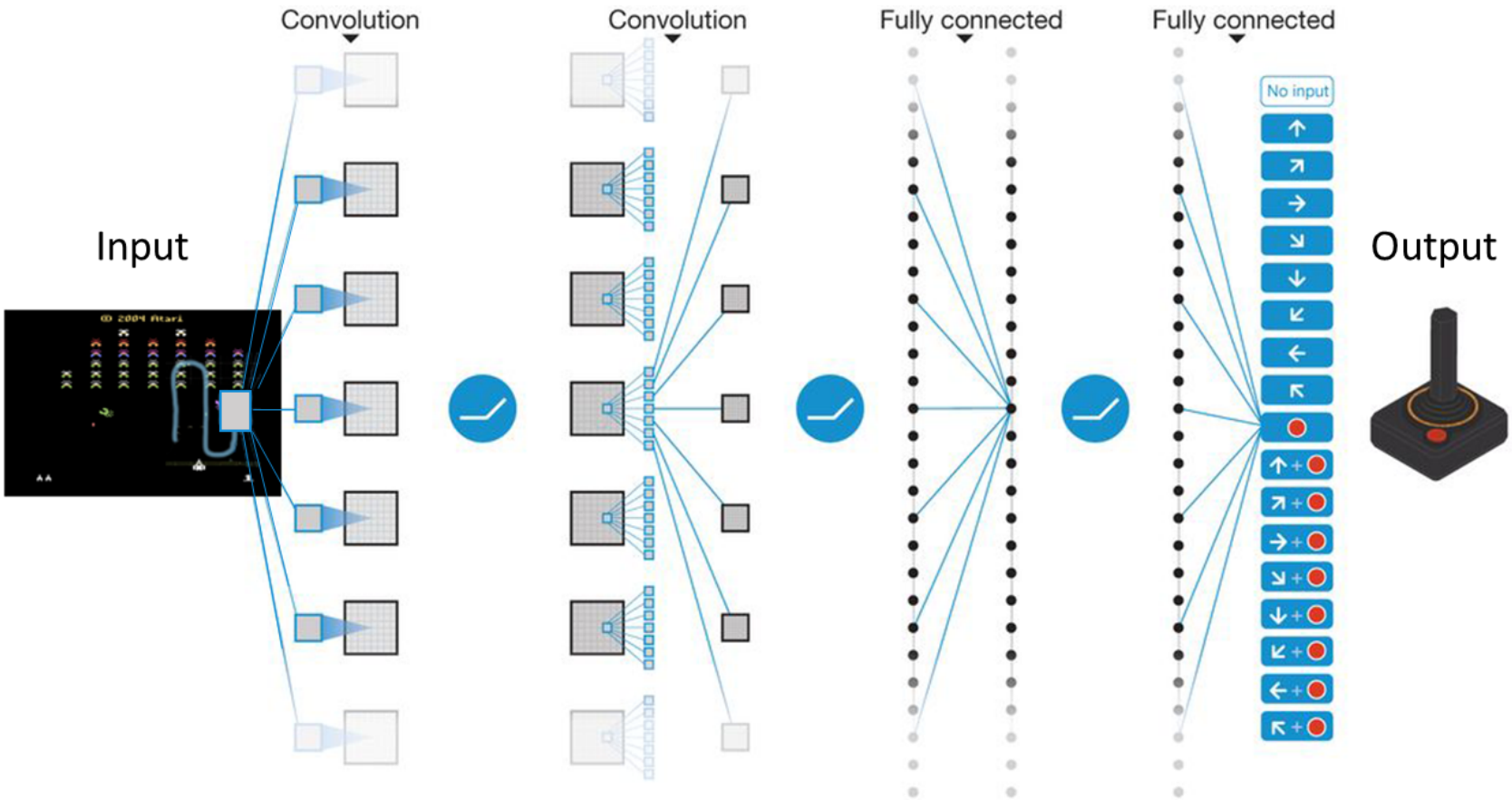
- Q-table can be very large, and take a lot of data to fill in.
- Assumption: entries in the Q-table have a regular distribution.

$\Rightarrow$ represent by a deep neural net.

- Advantage: generalization reduces need to try many similar actions.



Example


Convolution
Convolution
Fully connected
Fully connected
Input
Output
No input

Experience replay

- Sequences of observed states follow state transitions.

$\Rightarrow$ creates dependencies that destroy convergence of stochastic gradient descent.

- Avoid by rearranging observation tuples in random order.
- Developed by Google Deep Mind to play Atari videogames.

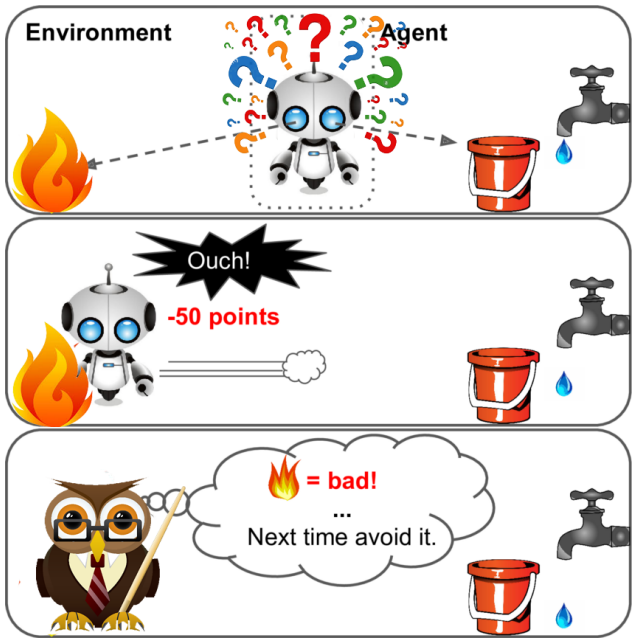
Most difficult example

Learning from a Teacher

- Key to efficient Q-learning: avoid trying bad actions.
- Observing a teacher shows only "good" transitions.

$\Rightarrow$ learning is much more efficient.

- However, agent will not become better than the teacher!



Exploration/exploitation with state

- Straightforward solution: one bandit per state.
- However, requires collecting data for each state: convergence time multiplied by the number of states.
- Not clear how to ensure that all states are reached.
- Usually not feasible.

Contextual Bandits

- Define *contexts* = groups of states with similar features. (e.g. medical trials: patient history, recommendation: user profile, etc.)
- Instead of one bandit per state, have one bandit per context.

$\Rightarrow$ Learning requires less data, but quality depends on how well contexts fit the problem.

- Example: EXP4 algorithm.
- However, no consideration of state transitions between states in same context!

Summary

- Bandit problems: learn effect (reward) of each action.
- Exploration-Exploitation tradeoff.
- Fixed stochastic reward structure: greedy/UCB algorithms, cumulative regret $= O(\log T)$.
- Adversarial rewards: learn randomized strategy.
  - regret matching/multiplicative weight update
  - exponential weight update

  cumulative regret $= O(\sqrt{T})$.
  No-regret learning: cumulative regret grows less than $O(T)$: average regret goes to zero.
- Learning with state
  - Q-learning
  - contextual bandits