Introduction Computer Graphics

Triangle Meshes

Mark Pauly

Geometric Computing Laboratory

Triangles

Barycentric Coordinates

- *Affine combination* of two points

$$\mathbf{x} = \alpha\mathbf{A} + \beta\mathbf{B}$$

with $\alpha + \beta = 1$.

- *Convex combination* of two points

$$\mathbf{x} = \alpha\mathbf{A} + \beta\mathbf{B}$$

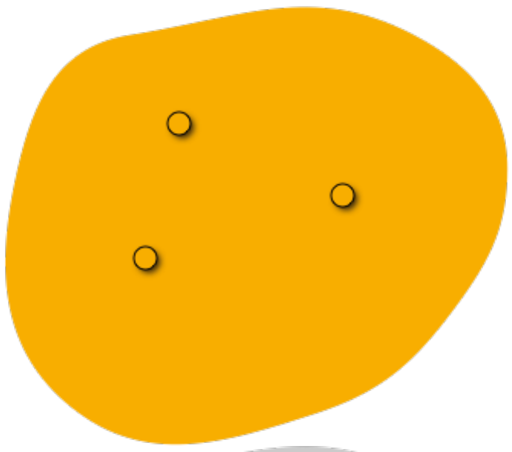
with $\alpha + \beta = 1$ **and** $\alpha, \beta \geq 0$.

Barycentric Coordinates

- *Affine combination* of three points

$$\mathbf{x} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$$

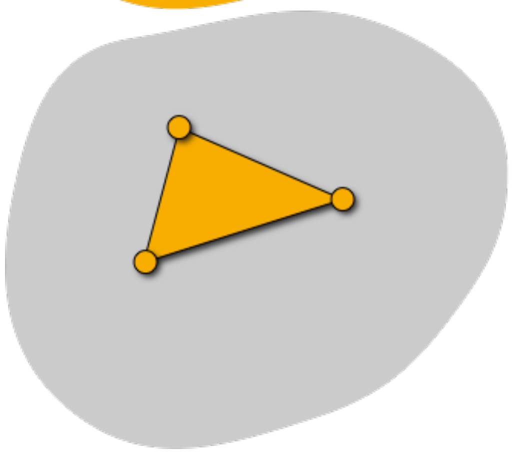
with $\alpha + \beta + \gamma = 1$.



- *Convex combination* of three points

$$\mathbf{x} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$$

with $\alpha + \beta + \gamma = 1$ **and** $\alpha, \beta, \gamma \geq 0$.

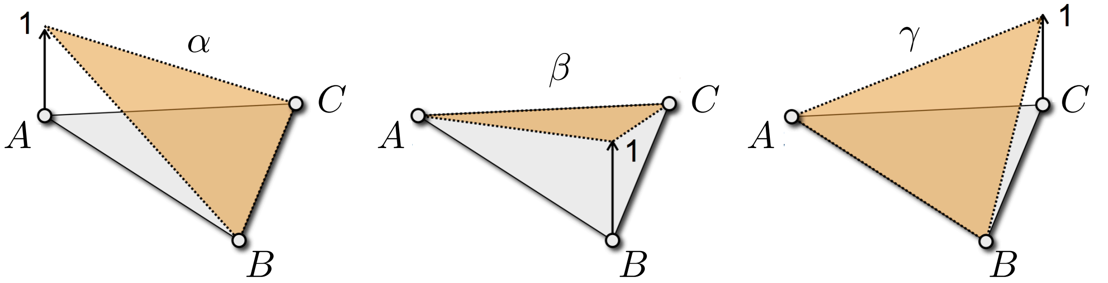


Barycentric Coordinates

- Barycentric representation: $\mathbf{x} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C} \quad$ with $\alpha + \beta + \gamma = 1$

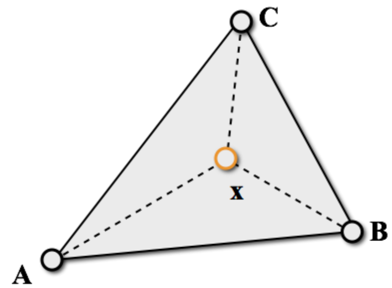
Barycentric Coordinates

- Barycentric representation: $\mathbf{x} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$ with $\alpha + \beta + \gamma = 1$
- For any point inside the triangle $0 \leq \alpha, \beta, \gamma \leq 1$.
  - For example, points with barycentric coordinates $(\alpha, \beta, 0)$ and $\alpha, \beta \geq 0$ lie on edge AB.



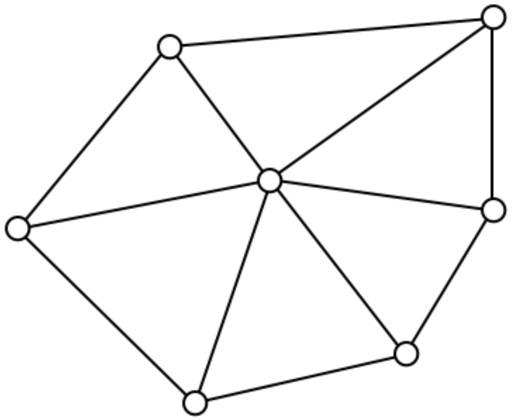
- Geometric interpretation: ratio of areas

$$\alpha(\mathbf{x}) = \frac{\text{area}(\mathbf{x}, \mathbf{B}, \mathbf{C})}{\text{area}(\mathbf{A}, \mathbf{B}, \mathbf{C})}, \quad \beta(\mathbf{x}) = \frac{\text{area}(\mathbf{A}, \mathbf{x}, \mathbf{C})}{\text{area}(\mathbf{A}, \mathbf{B}, \mathbf{C})}, \quad \gamma(\mathbf{x}) = \frac{\text{area}(\mathbf{A}, \mathbf{B}, \mathbf{x})}{\text{area}(\mathbf{A}, \mathbf{B}, \mathbf{C})}$$

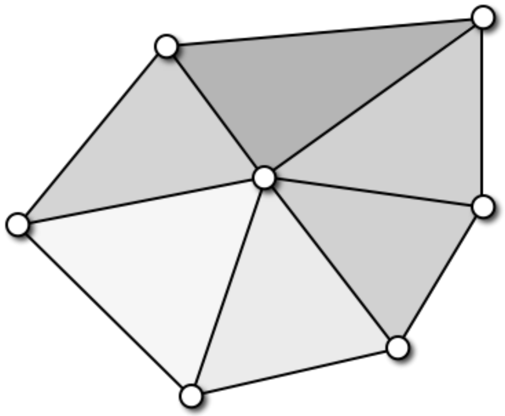


Triangle Meshes

What is a triangle mesh?



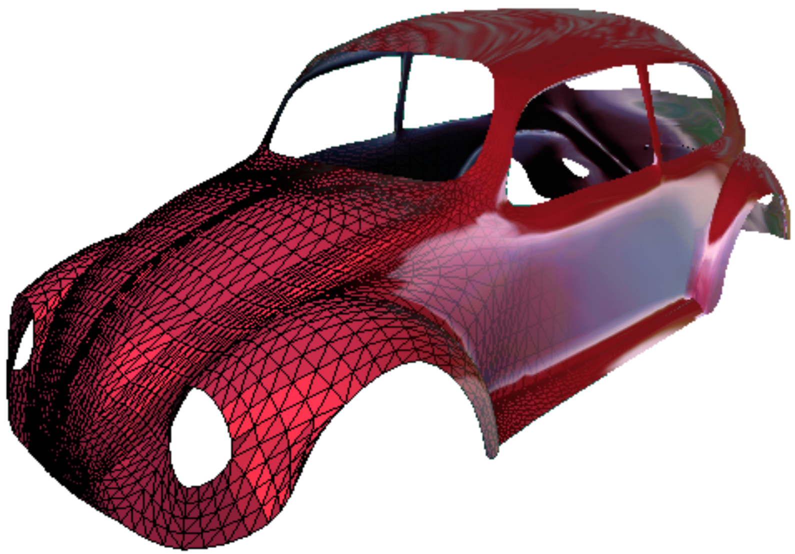
- **Connectivity / Topology**
  - Vertices $\mathcal{V} = \{v_1, \ldots, v_n\}$
  - Edges $\mathcal{E} = \{e_1, \ldots, e_k\}$, $e_i \in \mathcal{V} \times \mathcal{V}$
  - Faces $\mathcal{F} = \{f_1, \ldots, f_m\}$, $f_i \in \mathcal{V} \times \mathcal{V} \times \mathcal{V}$



- **Geometry**
  - Vertex positions $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, $\mathbf{x}_i \in \mathbb{R}^3$

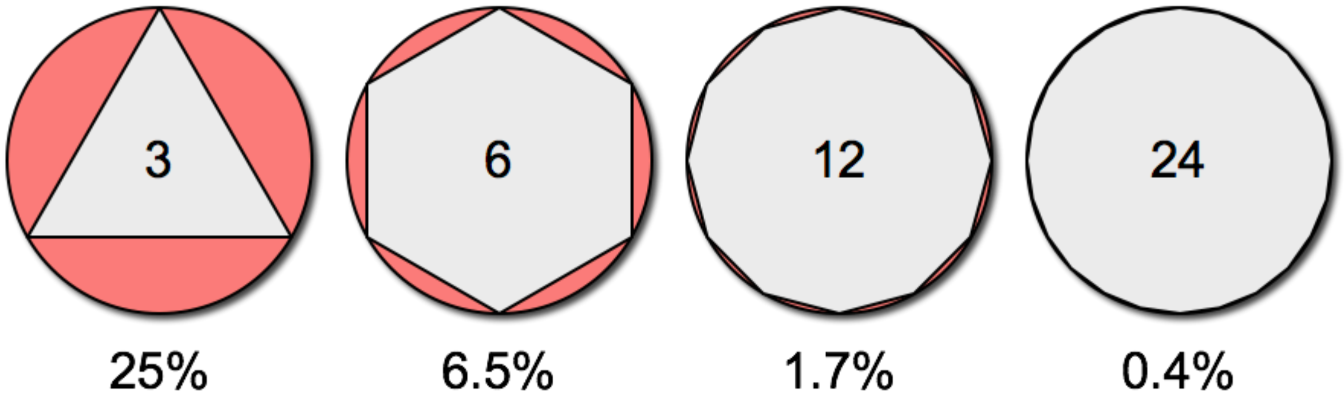
Triangle Meshes

- Triangle meshes can represent arbitrary surfaces



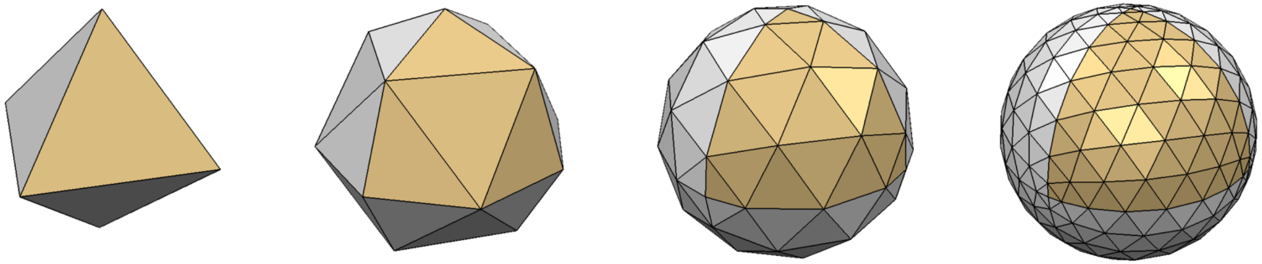
Triangle Meshes

- Triangle meshes can represent arbitrary surfaces
- Piecewise linear approximation → error is $\mathcal{O}(h^2)$



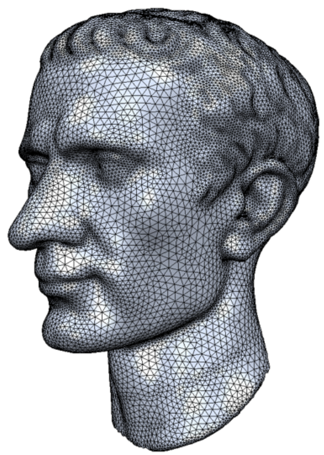
Triangle Meshes

- Triangle meshes can represent arbitrary surfaces
- Piecewise linear approximation → error is $\mathcal{O}(h^2)$
- Approximation error inversely proportional to #triangles

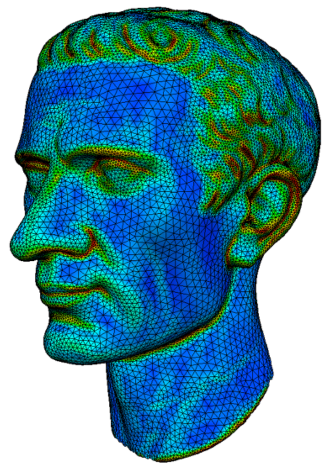


Triangle Meshes

- Triangle meshes can represent arbitrary surfaces
- Piecewise linear approximation → error is $\mathcal{O}(h^2)$
- Approximation error inversely proportional to #triangles
- Adaptive tessellation can adapt to surface curvature



adaptive meshing

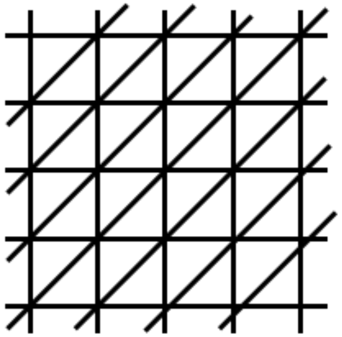


curvature visualization

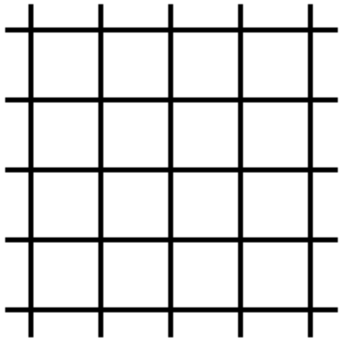
Triangle Meshes

- Triangle meshes can represent arbitrary surfaces
- Piecewise linear approximation → error is $\mathcal{O}(h^2)$
- Approximation error inversely proportional to #triangles
- Adaptive tessellation can adapt to surface curvature
- Simple primitives can be processed efficiently by CPU/GPU

Mesh Statistics



- **Triangle Meshes**
  - $F \approx 2V$
  - $E \approx 3V$
  - Average valence 6



- **Quad Meshes**
  - $F \approx V$
  - $E \approx 2V$
  - Average valence 4

Example: Triangle Mesh

5676 vertices, 17022 edges, 11348 faces

Example: Quad Mesh

2012 vertices, 3978 edges, 1968 faces

Storing Meshes

- Face Set
  - standard file format for triangle meshes (e.g. STL format)
  - Memory consumption: 36 B/f = 72 B/v

Triangles		
x_{11} y_{11} z_{11}	x_{12} y_{12} z_{12}	x_{13} y_{13} z_{13}
x_{21} y_{21} z_{21}	x_{22} y_{22} z_{22}	x_{23} y_{23} z_{23}
...	...	...
...	...	...
...	...	...
x_{F1} y_{F1} z_{F1}	x_{F2} y_{F2} z_{F2}	x_{F3} y_{F3} z_{F3}

Storing Meshes

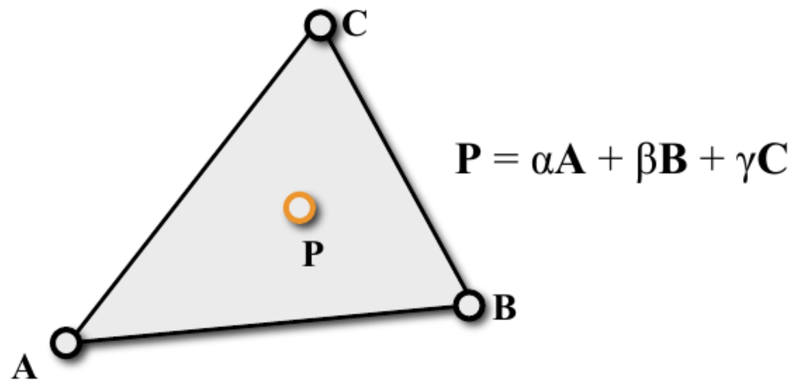
- Indexed Face Set
  - used for many file formats (e.g. OFF, OBJ, VRML)
  - Memory consumption: 12 B/v + 12 B/f = 36 B/v

Vertices
x_1 y_1 z_1
...
x_V y_V z_V

Triangles
i_{11} i_{12} i_{13}
...
...
...
...
i_{F1} i_{F2} i_{F3}

Quiz: Barycentric Coordinates

- How can you test that $\mathbf{P} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$, which lies in the plane of the triangle $(\mathbf{A}, \mathbf{B}, \mathbf{C})$, lies **within** this triangle?
- Which condition is violated if the point is outside the triangle?



A: $\alpha + \beta + \gamma = 1$

B: $\alpha < \beta < \gamma$

C: $\alpha, \beta, \gamma \leq 1$

D: $\alpha, \beta, \gamma \geq 0$