First Name: **Signature:**

Last Name:

EPFL Student Number: ____________________

EPFL - Geometric Computing Laboratory

Exam 2024 (C)

CS-341 Computer Graphics

Procedures:

Leave your **student ID card** on the table during the examination for verification. Write your **first and last name** as well as your **EPFL student number** on the first page and then write your initials on all subsequent pages. Use only black or blue ink pens and write legibly. Exam questions done in pencil or other colored pens will **NOT** be graded. Write your solutions on these sheets. Check if the exam is complete by verifying that no page is missing.

Regulations:

You are allowed to use one A4 sheet with hand-written notes on both sides for the exam. Course notes, textbooks, other books/printed materials, calculators are **NOT** permitted during the examination. The use of electronic devices such as cell phones, laptops, etc. is strictly **PROHIBITED** during the examination.

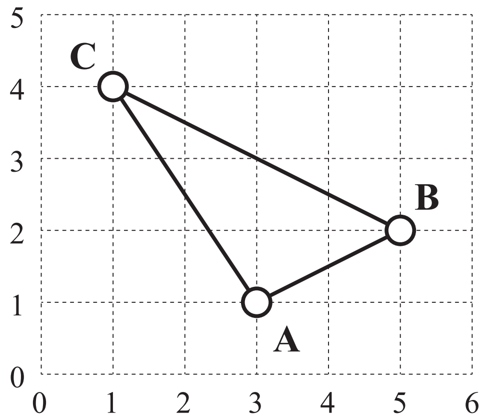
Students found cheating or violating any of the above regulations will fail the class.

Exercise	*Max. points*	*Earned points*
1. Raytracing	11	
2. Rasterization	4	
3. Transformations and Projections	9	
4. Textures and Shadows	16	
5. GPU Pipeline	11	
6. Procedural Models	16	
7. Affine Combinations	6	
8. Bezier Curves	10	
Total		

Initials:

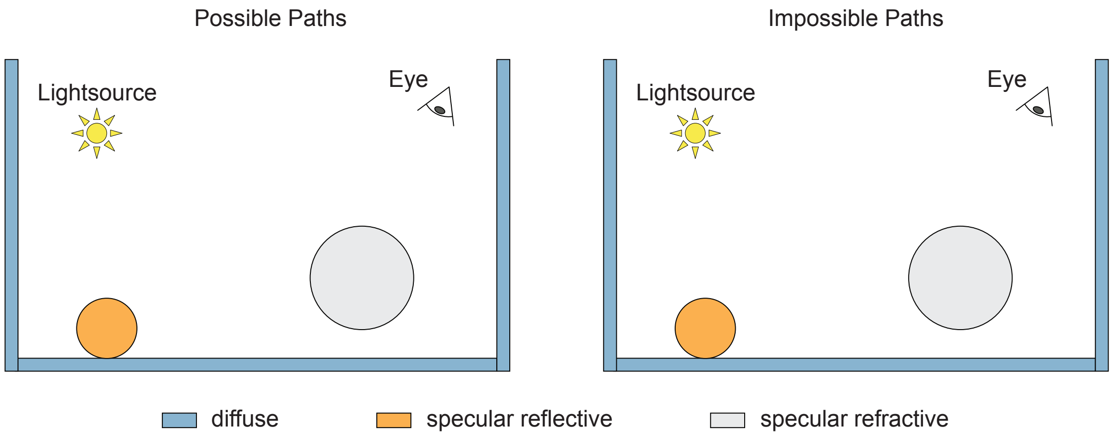
1 Raytracing

- **(5 points)** In the figure below, a 2D triangle is defined with three vertices in the plane, $\mathbf{A} = (3,1)$, $\mathbf{B} = (5,2)$, and $\mathbf{C} = (1,4)$. A point in the plane can be represented with barycentric coordinates as $\mathbf{P} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$, where $\alpha + \beta + \gamma = 1$. Consider a ray starting at the origin $\mathbf{O} = (0,0)$ with direction vector $\mathbf{d} = (1,1)$. Find the barycentric coordinates with respect to triangle **ABC** of the point where the ray intersects with the triangle for the first time.



Initials:

- **(6 points)** In the scene sketched below, the light source is a point light, and we use the standard pinhole camera model. The blue walls are purely diffuse, and the spheres are purely specular (reflective and refractive, respectively). Assume a path of length k consists of k line segments. For example, a path going from the light to a sphere to the eye is of length 2. For the basic recursive raytracing algorithm discussed in class, sketch two paths per image with different material interactions of length at least 3 that the algorithm can generate (left image) and cannot generate (right image). Briefly explain why a path is possible or not possible.



Initials:

2 Rasterization

- **(2 points)** Complete the pseudo code to implement the z-buffer algorithm. Assume the color of the polygon P is constant and equal to c_P.

```
// zbuffer[x,y]: stores the depth value at position x, y.
// framebuffer[.,.]: stores the color value at position x, y.

for (each polygon P):
    for (each pixel (x,y,z) in P):
```

- **(2 points)** Barycentric coordinates can be used for simple 2D rasterization of a triangle. Complete the following pseudo code for rasterizing one triangle.

```
// You can use
// set_pixel(x,y) to fill a pixel
// (a,b,c)=bary_coords(x,y) to compute barycentric coordinates wih respect to the triangle

for (all x):
  for (all y):
```

Initials:

3 Transformations and Projections

- **(4 points)** You are given a triangle in 2D defined by the vertices $(0,0)$, $(1,0)$, and $(0,1)$. Find an affine transformation that transforms this triangle into an equilateral triangle of side length 1. Write down the 3×3 matrix of your transformation in homogenous coordinates.

- **(1 point)** What kind of projection do you use to generate the shadow map for a *point* light source?

Initials:

- **(2 points)** Consider the 2D affine transformation in homogenous coordinates:

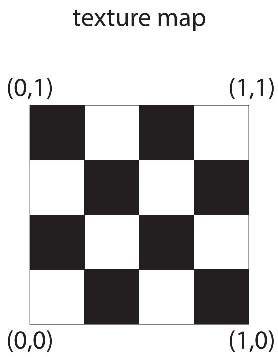
$$M = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

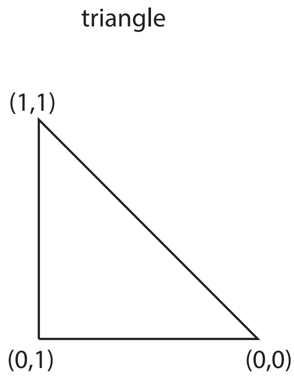
  If a triangle is transformed with this matrix, how will its area change?

- **(2 points)** Given two vectors **p** and **q**. Show that their angle is preserved under rotation.

4 Textures and Shadows

- **(4 points)** Given a texture map with a checkerboard pattern as illustrated below. We are rendering the triangle shown on the right using this texture with nearest neighbor texture sampling and no anti-aliasing. The corresponding texture coordinates are indicated at the vertices. Assume we are rendering the triangle under some arbitrary projective transformation. Let *k* be the percentage of rendered pixels of the triangle that are black. What range can *k* assume? Explain your answer.



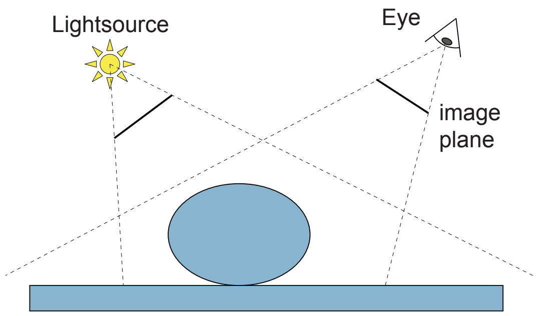


Initials:

- Which of the following statements is true? Please check the corresponding box on the right. (correct answer = 1 point, no answer = 0 points, wrong answer = -0.5 points)

Statement	**True**	**False**
Mipmapping selects the suitable image resolution based on the average z value of a triangle.		
There can be no mapping from the half-sphere to the plane that preserves length everywhere.		
There can be no mapping from the half-sphere to the plane that preserves area everywhere.		
Bilinear texture filtering eliminates aliasing.		
Alpha mapping selects which pixels to render in the fragment shader.		

- Consider the scene sketched below with two opaque objects. Assume we have computed a shadow map S from the point light source using the indicated view angle. Now we multiply every value in the shadow map by a factor of 2.



Which of the following statements is true? Please check the corresponding box on the right. (correct answer = 1 point, no answer = 0 points, wrong answer = -0.5 points)

Statement	**True**	**False**
Some points that should not be in shadow are now in shadow.		
Some points that should be in shadow are no longer in shadow.		
All points are still correctly in shadow or not, but the shadows are less dark.		

Initials:

- **(4 points)** Sketch a 2D scene with a point light source and some shadows, for which the above procedure of multiplying the value of the shadow map by a factor of 2 would not change the result for any viewpoint. Briefly explain why your scene has the required behavior.

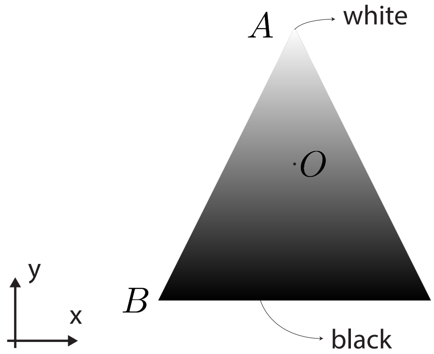
Initials:

5 GPU Pipeline

- **(5 points)** Complete the definition of the pipeline that renders the triangle from the figure. For each component, indicate the corresponding code snippet in the table.

```
const pipeline = ...
const vert_src = ...
const frag_src = ...
pipeline()
```

Component	Code
pipeline	
vert_src	
frag_src	



Code 1

```
attribute vec2 V;
varying vec3 color;

void main() {
  color = vec3(V.y, V.y, V.y);
  gl_Position = vec4(V.xy, 0.0, 1.0);
}
```

Code 2

```
attribute vec3 V;
varying vec3 color;

void main() {
  color = vec3(V.x, V.y, V.z);
  gl_Position = vec4(V, 1.0);
}
```

Code 3

```
attribute vec3 V;
varying vec3 color;

void main() {
  color = vec3(V.z, V.z, V.z);
  gl_Position = vec4(V, 1.0);
}
```

Code 4

```
precision highp float;

varying vec3 color;
uniform vec3 c_scale;

void main() {
  gl_FragColor = vec4(c_scale * color, 1.0);
}
```

Code 5

```
regl({
  attributes: {
    V: [
      [-1.0, -1.0],
      [ 0.0,  1.0],
      [ 1.0, -1.0],
    ],
  },
  elements: [[0, 1, 2]],
  uniforms: {
    c_scale: [1.0, 1.0, 1.0],
  },
  vert: vert_src,
  frag: frag_src,
})
```

Code 6

```
regl({
  attributes: {
    V: [
      [-1.0, -1.0, 0.0],
      [ 0.0,  1.0, 1.0],
      [ 1.0, -1.0, 0.0],
      [ 0.0,  0.0, 0.5],
    ],
  },
  elements: [
    [0, 1, 3],
    [0, 2, 3],
    [1, 2, 3],
  ],
  uniforms: {
    c_scale: [1.0, 1.0, 1.0],
  },
  vert: vert_src,
  frag: frag_src,
})
```

Initials:

- **(1 point)** What does the fourth component of the `gl_Position` vector represent?

- **(1 point)** What does the fourth component of the `gl_FragColor` vector represent?

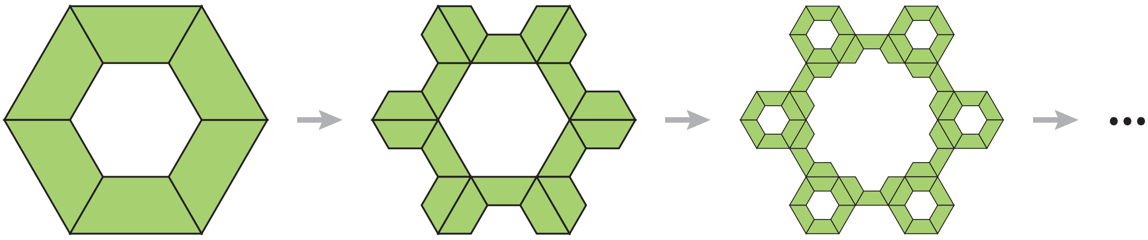
- **(4 points)** Imagine you want to change the value of the `c_scale` uniform variable to obtain a desired color at vertices A, B, C, and at the origin O. For each of the coloring options in the table below, write YES if they can be achieved by modifying `c_scale` alone, NO otherwise. For those that are feasible, indicate the corresponding value of `c_scale`. The first row refers to the figure above and it is filled in as an example.

Output	**Feasible**	`c_scale`
B black	YES	[1, 1, 1]
A red		
A black		
B white		
C blue		
O green		

Initials:

6 Procedural Models

- **Fractals**: Assume we have a fractal in the plane constructed with the procedure sketched below.

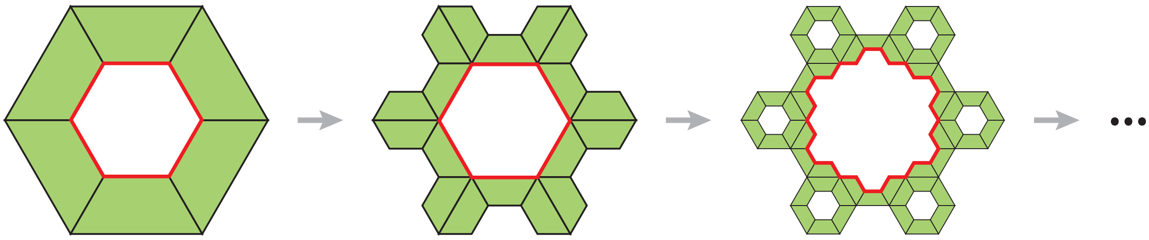


a) **(3 points)** What is the Haussdorff dimension of this fractal?

b) **(2 points)** Give formula for the area of the above fractal as a function of recursion level k, where $k = 0$ corresponds to the leftmost image above. We assume that the total area of the green polygons on the left is 1. What does this expression converge to as $k \to \infty$?

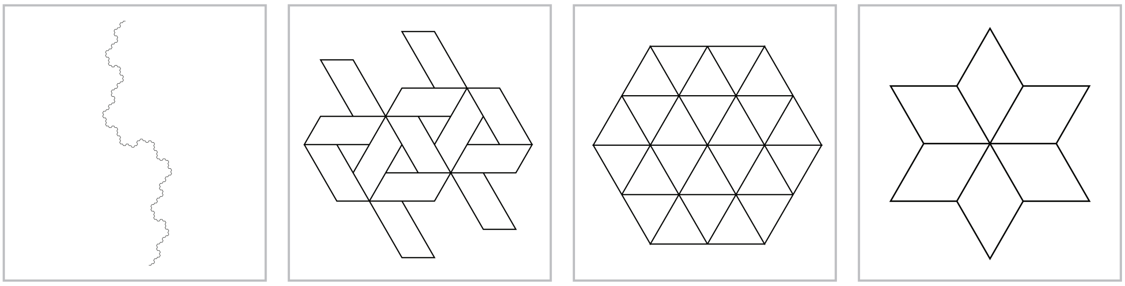
Initials:

c) **(3 points)** Give the formula for the length of the interior polygon marked in red below as a function of recursion level k, where $k = 0$ corresponds to the leftmost image. We assume that the total length of the red curve on the left is 6. What does this expression converge to as $k \to \infty$?



Initials:

- **L-Systems (8 points)**



A B C D

The figure above shows the turtle graphic output (scaled to fit into the frame) at recursion level 5 of four different L-Systems that all start with the same axiom F using a 60 degree turning angle for the $+$ and $-$ symbols. In the table below, assign the correct letter (A, B, C, D) to the rule defined in the left column.

Rule	**Figure Label**
$F \rightarrow FF+F+$	
$F \rightarrow F+F-F$	
$F \rightarrow F+F+$	
$F \rightarrow F+F+F+$	

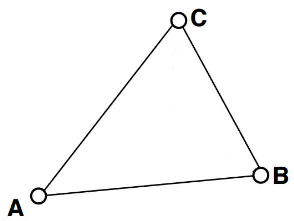
Initials:

7 Affine Combinations

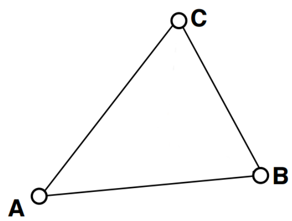
Let **A**, **B**, **C** be points in 3D.

- **(3 points)** Is the expression $\cos^2(t)\mathbf{A} + \sin^2(t)\mathbf{B}$ an affine combination for all $t \in [0,1]$? If yes, which curve does it define? If no, why not?

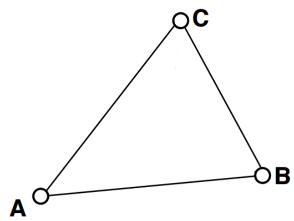
- **(3 points)** We have a 2D triangle with vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$. A point in the plane can be represented with barycentric coordinates as $\mathbf{P} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$. In each figure below, sketch the region of the plane that corresponds to the constraints listed below the triangle.



$\beta = 0$



$\gamma < 0$



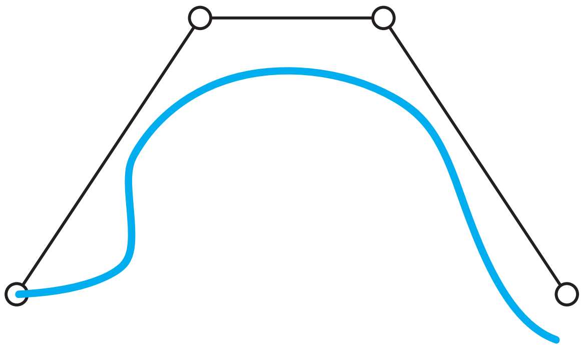
$\alpha > 0 \cap \gamma > 0$

Initials:

8 Bezier Curves

- **(3 points)**

  The figure below shows in black a control polygon for a Bezier curve. The drawn blue curve violates a number of properties that Bezier curves have in relation to their control polygon. List three of these properties that are violated for the curve shown in the figure.



1.

2.

3.

- **(2 points)** Prove the partition of unity property for Bernstein polynomials of degree two.

Initials:

- **(5 points)** Assume we are given a Bezier curve in 2D as $\mathbf{c}(t) = \sum_{i=0}^{2} \mathbf{b}_i B_i^2(t)$ with three control points $\mathbf{b}_0 = (0,0)$, $\mathbf{b}_1 = (1,y)$, $\mathbf{b}_2 = (3,0)$. Find the unknown y-coordinate of $\mathbf{b}_1$ such that the curve passes through the point $(1,1)$ for some parameter value $t \in [0,1]$.

  Hint: The quadratic equation of the form $ax^2 + bx + c = 0$ has the solutions: $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Initials:

Initials: