ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Exam II, Theory of Computation 2018-2019

- Books, notes, communication, calculators, cell phones, computers, etc... are not allowed.
- Your explanations and proofs should be clear enough and in sufficient detail so that they are easy to understand and have no ambiguities.
- You are allowed to refer to material covered in the class including theorems without reproving them.
- **Do not touch until the start of the exam.**

Good luck!

Name: ______________________ **N° Sciper:** ______________

Problem 1 / 7 points	Problem 2 / 6 points	Problem 3 / 6 points	Problem 4 / 6 points

Total / 25

1 *(consisting of subproblems **a-b**, 7 pts)* **Basic questions.**

1a *(4 pts)* Consider two languages A and B. Which of the following statements are true?

1. A is regular $\implies$ A is decidable $\implies$ A is recognizable.
2. A and B are decidable $\implies$ $A \cup B$ is decidable.
3. A and B are unrecognizable $\implies$ $A \cup B$ is unrecognizable.
4. A is undecidable and recognizable $\implies$ $\overline{A}$ is unrecognizable.
5. $\overline{A}$ is decidable $\implies$ A is decidable.
6. $\{0^n 1^n : n \geq 0\}$ is decidable.
7. A is decidable and $A \leq_m B$ $\implies$ B decidable.
8. $\{\langle M, w \rangle \mid$ M is a TM and M doesn't halt on input w$\}$ is recognizable.

*(A complete solution identifies **all** true statements. A fully correct solution is worth* 4 *points. A solution with one mistake is worth* 3 *points. A solution with two mistakes is worth* 1 *point. Solutions with more mistakes are worth* 0 *points. A mistake is to either indicate falsely that a false statement is true or to not indicate that a true statement is true.)*

1b *(3 pts)* Let B be an undecidable language. Show that $B \leq_m \overline{B}$ implies that B and $\overline{B}$ are unrecognizable.

(In this problem you are asked to provide a formal proof of the statement. Recall that you are allowed to refer to material covered in the class including theorems without reproving them.)

2 *(6 pts)* A DFA $D = (Q, \Sigma, \delta, q, F)$ is defined to be minimal if it has the minimum number of states among all DFAs that recognize the same language as D. More formally, D is minimal if

$$|Q| = \min_{\substack{D'=(Q',\Sigma,\delta',q',F'):\\ L(D)=L(D')}} |Q'|.$$

Show that the following language is decidable.

$$L_2 = \{\langle D \rangle \mid D \text{ is a minimal DFA}\}.$$

(In this problem you are asked to show that the language L_2 is decidable. Recall that you are allowed to refer to material covered in the class including theorems without reproving them.)

3 *(6 pts)* Consider the following language

$$L_3 = \{\langle M \rangle \mid \ M \text{ is a TM that halts on every input}\}\,.$$

Show that L_3 is unrecognizable by giving a mapping reduction from the unrecognizable language NR defined as follows:

$$NR = \{\langle M, w \rangle \mid \ M \text{ is a TM and M does not reject input } w\}\,.$$

In other words, show that $NR \leq_m L_3$.

*(In this problem you are asked to give a mapping reduction from the language NR to the language L_3 and to provide a formal proof of its correctness. A mapping reduction from another unrecognizable language, such as $\overline{A_{TM}}$, only gives partial points. You are **not** required to prove that L_3 and NR are unrecognizable. Recall that you are allowed to refer to material covered in the class including theorems without reproving them.)*

4 *(6 pts)* Classify the following language into one of: decidable, undecidable but recognizable, unrecognizable.

$$L_4 = \{\langle M, D \rangle \mid \ M \text{ is a TM, } D \text{ is a DFA and } L(D) \cap L(M) \neq \phi\}\,.$$

Justify your answer with a formal proof.

(In this problem, you are asked to identify whether L_4 is (decidable and recognizable), (undecidable and recognizable), or (unrecognizable) and provide a formal correctness proof. Recall that you are allowed to refer to material covered in the class including theorems without reproving them.)