Exercise V, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked * are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

1 Prove that the problem of checking whether a given DFA accepts a finite language is decidable.

2 Prove that the language

$$L = \big\{\langle D\rangle : D \text{ is a DFA over alphabet } \{0,1\} \text{ that only rejects a single string}\big\}$$

is decidable.

3 Prove that the following questions about pairs of Turing machines (M, N) are undecidable.

3a Is $L(M) \cap L(N)$ empty?

3b Is $L(M) \cap L(N)$ finite?

4* We say that a Turing machine has property $\mathcal{U}$ if for all $n \in \mathbb{N}$, at most one string of length n is accepted by M. Assuming that $|\Sigma| > 1$, prove that the language

$$L = \big\{\langle M\rangle : M \text{ has property } \mathcal{U}\big\}$$

is undecidable. What happens if $|\Sigma| = 1$?

5* Is the following language recognizable?

$$L = \big\{\langle M\rangle : M \text{ is a Turing machine that accepts at most 2025 strings}\big\}$$